# 14.662 (Spring 2015): Problem Set 2 

David Autor and Peter Hull

March 9, 2015

## 1. Inefficient Learning-by-Doing

As some of you noted in class, talent discovery is not the only setting where a worker's inability to accept a negative wage may lead to inefficiency. This question asks you to consider the market for careers where profession-specific learning-by-doing is crucial to productivity (e.g. lawyers, financiers....or academic economists!) and certain long-term contracts (that is, indentured servitude) are not enforceable.

Suppose workers in a given profession live for two periods and are employed via a spot labor market. The profession is competitive, so neither firms nor workers may earn rents over their outside option and both take market wages as given. Industry wages are given by $w_{t}=p y_{t}-\phi$, where $y$ is the output of the worker in period $t, p$ is the market price of output, and $\phi$ is a fixed per-worker cost. Suppose worker output is exogenously higher in the second period: $y_{2}>y_{1}>0$ and their lifetime utility is given by $u\left(c_{1}\right)+u\left(c_{2}\right)$ where $c_{t}$ denotes consumption in period $t$ and $u^{\prime \prime}(c)<0<u^{\prime}(c)$. Young workers may borrow (with zero interest) an amount $b<L$ against their future wages, where $L$ reflects possible liquidity constraints.
(a) (5 points) Young workers are indifferent between entering the profession and going outside the industry, where they would earn a constant wage $w_{0}$. Write an equilibrium condition reflecting this, and use it to derive an expression for prices and wages when $L=\infty$ (i.e. workers are unconstrained). Under what conditions must a worker pay to enter the profession (that is, $w_{1}^{*}<$ 0 )?
A worker earning no lifetime rents has

$$
2 u\left(w_{0}\right)=u\left(c_{1}\right)+u\left(c_{2}\right)=u\left(p y_{1}-\phi+b\right)+u\left(p y_{2}-\phi-b\right)
$$

When the worker is unconstrained, concave utility implies perfect consumption smoothing, so that

$$
\begin{aligned}
p y_{1}-\phi+b & =p y_{2}-\phi-b \\
b & =p\left(y_{2}-y_{1}\right) / 2
\end{aligned}
$$

and

$$
\begin{aligned}
p^{*} y_{1}-\phi+p^{*}\left(y_{2}-y_{1}\right) / 2 & =w_{0} \\
\Longrightarrow p^{*} & =\frac{w_{0}+\phi}{\left(y_{1}+y_{2}\right) / 2}
\end{aligned}
$$

which is the average cost of production. Therefore

$$
w_{t}^{*}=\frac{w_{0}+\phi}{\left(y_{1}+y_{2}\right) / 2} y_{t}-\phi
$$

A novice pays to work when

$$
\begin{aligned}
\frac{w_{0}+\phi}{\left(y_{1}+y_{2}\right) / 2} y_{1}-\phi & <0 \\
\frac{y_{1}}{\left(y_{1}+y_{2}\right) / 2} & <\frac{\phi}{\phi+w_{0}}
\end{aligned}
$$

This is the case when either the effect of experience on output is large (that is, the LHS is small) or when the non-labor costs of production are relatively high (that is, the RHS is large).
(b) (6 points) Derive and interpret an expression for optimal borrowing $b^{*}$. Suppose $L=b^{*}$, so that workers are just able to borrow their desired amount. Derive an expression for how prices and wages respond to a marginal decrease in L. Discuss.
We have from above that

$$
b^{*}=\left(w_{0}+\phi\right) \frac{y_{2}-y_{1}}{y_{1}+y_{2}}
$$

The worker's required borrowing is increasing in $y_{2}-y_{1}$, or the deficiency of productivity that an inexperienced worker has compared to an experienced worker. It is also increasing in $w_{0}$, the worker's outside option, and $\phi$, the workers fixed cost to the firm, and decreasing in the total productivity of the worker. A marginal decrease in $L$ decreases equilibrium borrowing one-for-one, and we can see from differentiating the equilibrium condition that

$$
\begin{aligned}
0 & =u^{\prime}\left(p y_{1}-\phi+b\right)\left(\frac{\partial p}{\partial b} y_{1}+1\right)+u\left(p y_{2}-\phi-b\right)\left(\frac{\partial p}{\partial b} y_{2}-1\right) \\
\Longrightarrow \frac{\partial p}{\partial b}=\frac{\partial p}{\partial L} & =-\frac{u^{\prime}\left(c_{1}\right)-u^{\prime}\left(c_{2}\right)}{u^{\prime}\left(c_{1}\right) y_{1}+u^{\prime}\left(c_{2}\right) y_{2}}<0
\end{aligned}
$$

Thus prices, and therefore wages, increase when liquidity constraints bind. Intuitively, a binding liquidity constraint makes the marginal utility of consumption for inexperienced workers higher than that of experienced workers; in order to attract novices to the profession, wages must increase via a higher output price. Note that this is an inefficient way to hire inexperienced workers, since most of the increase in prices is captured by experienced workers, who already have a lower marginal utility of consumption.
(c) (8 points) Let us now endogenize worker productivity. Suppose $y_{t}=\theta_{t} e_{t}$, where $e_{t}$ denotes a worker's effort in period $t$ and $\theta_{2}>\theta_{1}>0$. To simplify the analysis assume worker utility is given by

$$
V\left(c_{1}, c_{2}, e_{1}, e_{2}\right)=\sum_{t=1}^{2} \alpha \ln \left(c_{t}\right)+(1-\alpha) \ln \left(1-e_{t}\right)
$$

for $\alpha \in(0,1)$. Again suppose workers are unconstrained in their borrowing. Write an expression for how workers choose effort in the two periods and solve for effort and consumption in terms of output prices. How are prices determined?
The unconstrained worker again perfectly smooths consumption, so that

$$
\begin{aligned}
c_{t}=c & =p\left(y_{1}+y_{2}\right) / 2-\phi \\
& =p\left(\theta_{1} e_{1}+\theta_{2} e_{2}\right) / 2-\phi
\end{aligned}
$$

Her effort choice thus solves

$$
\max _{e_{1}, e_{2}} 2 \alpha \ln \left(p\left(\theta_{1} e_{1}+\theta_{2} e_{2}\right) / 2-\phi\right)+(1-\alpha) \ln \left(\left(1-e_{1}\right)\left(1-e_{2}\right)\right)
$$

The first order condition is, for each $t \in\{1,2\}$

$$
\begin{aligned}
2 \alpha \frac{p \theta_{t} / 2}{p\left(\theta_{1} e_{1}^{*}+\theta_{2} e_{2}^{*}\right) / 2-\phi} & =\frac{(1-\alpha)}{\left(1-e_{t}^{*}\right)} \\
\frac{\alpha p}{c^{*}} \theta_{t} & =\frac{(1-\alpha)}{\left(1-e_{t}^{*}\right)}
\end{aligned}
$$

Thus

$$
\begin{aligned}
e_{t}^{*} & =1-\frac{(1-\alpha)}{\alpha p \theta_{t}} c^{*} \\
c^{*} & =p\left(\theta_{1}\left(1-\frac{(1-\alpha)}{\alpha p \theta_{1}} c^{*}\right)+\theta_{2}\left(1-\frac{(1-\alpha)}{\alpha p \theta_{2}} c^{*}\right)\right) / 2-\phi \\
c^{*}\left(1+\frac{1-\alpha}{\alpha}\right) & =p\left(\theta_{1}+\theta_{2}\right) / 2-\phi \\
c^{*} & =\alpha\left(p\left(\theta_{1}+\theta_{2}\right) / 2-\phi\right) \\
e_{t}^{*} & =1-\frac{1-\alpha}{p \theta_{t}}\left(p\left(\theta_{1}+\theta_{2}\right) / 2-\phi\right)
\end{aligned}
$$

Note that worker effort in each period $t$ is increasing in $\theta_{t}$; that is, they are working harder when they are most productive, as would be expected from dynamic labor supply with perfect credit markets (recall the taxi drivers and stadium vendors from 14.661!). Equilibrium output prices are pinned down by the worker's outside option; their indirect utility from prices $p$ must equal $2 w_{0}$, as before.
(d) (7 points) Again suppose $L=b^{*}$ so that workers are just able to make their borrowing requirements. How do prices respond to a marginal decrease in L? How does effort respond? Discuss.
Workers now solve

$$
\max _{b, e_{1}, e_{2}} \alpha \ln \left(p \theta_{1} e_{1}-\phi+b\right)+\alpha \ln \left(p \theta_{2} e_{2}-\phi+b\right)+(1-\alpha) \ln \left(\left(1-e_{1}\right)\left(1-e_{2}\right)\right)
$$

A liquidity-constrained worker faces lower indirect utility for a given output price; as before prices must therefore increase when workers become constrained such that they are indifferent to their outside option, and $\partial p / \partial L<0$. We still have optimal effort solving

$$
e_{t}=1-\frac{(1-\alpha)}{\alpha p \theta_{t}} c_{t}
$$

Since $\partial p / \partial L<0$ and

$$
e_{2}=1-\frac{(1-\alpha)}{\alpha}+\frac{(1-\alpha)}{\alpha p \theta_{2}}(\phi+b)
$$

we can immediately see that $\partial e_{2} / \partial L=\partial e_{2} / \partial b>0$; a lower ability to borrow leads to less effort from experienced workers. Intuitively, the increased output price caused by liquidity constraints increases the value of the endowment of effective labor for experienced workers, while the constraints themselves force experienced workers to carry less debt. This combined wealth effect in period-2 leads to experienced workers (think tenured professors) consuming more leisure in the form of reduced effort. On the other hand for inexperienced workers we have

$$
\begin{aligned}
\frac{\partial e_{1}}{\partial b} & =-\frac{(1-\alpha)}{\alpha \theta_{1}} \frac{\partial}{\partial b}\left(\frac{c_{1}}{p}\right) \\
& =-\frac{(1-\alpha)}{\alpha \theta_{1} p}\left(\frac{\partial c_{1}}{\partial L}-\frac{c_{1}}{p} \frac{\partial p}{\partial L}\right)
\end{aligned}
$$

Since $\partial p / \partial L<0$ and $\partial c / \partial L>0$ (a relaxing of credit constraints increases first-period consumption), we have that $\partial e_{1} / \partial b<0$. Intuitively, the more constrained novices are (think assistant professors), the less able they are to accept a lower wage to compete for the future rents to experience - instead, they simply work harder. This is, of course, inefficient.

## 2. A Brief History of Gravity

Trade economists were quite aware of the empirical deficiencies of Heckscher-Ohlin-style models well
 trade between Canadian provinces to be more than 22 times higher than trade between Canada and the U.S., despite a high degree of integration between these two economies. This is strong motivation for the kinds of "iceberg" costs that gravity models take seriously.

McCallum's estimating equation is of the form

$$
\ln X_{i j}=\pi+\alpha \ln G D P_{i}+\beta \ln G D P_{j}+\gamma \ln d i s t_{i j}+\delta D_{i j}+\epsilon_{i j}
$$

where $X_{i j}$ denotes the value of exports from region $i$ (either a U.S. state or a Canadian province) to region $j$, dist $i_{i j}$ is the distance between region $i$ and $j$, and $D_{i j}$ equals 1 if both $i$ and $j$ are Canadian provinces, zero otherwise (the coefficient $\delta$ measures the importance of the U.S.-Canada border to trade that McCallum reports). Our goal is to motivate this sort of regression by a simple gravity model.

For simplicity, suppose each region $i$ specializes in a single good, the total supply of which is fixed. Preferences for goods are homothetic and identical across regions, given by

$$
U_{j}=\left(\sum_{i} q_{i j}^{\rho}\right)^{1 / \rho}
$$

where $q_{i j}$ denotes the quantity of imports to country $j$ from country $i$. The price of region $i$ 's good to region $j$ is given by $p_{i j}=\tau_{i j} p_{i}$ where $\tau_{i j}$ is an exogenous transport cost. The value of exports from $i$ to $j$ can thus be written $X_{i j}=p_{i j} q_{i j}$. Assume all countries are small, and so take prices as given.
(a) (5 points) Write the constrained problem country $j$ solves in deciding how much to import from each country.
Imports solve

$$
\begin{aligned}
& \max _{q_{i j}}\left(\sum_{i} q_{i j}^{\rho}\right)^{1 / \rho} \\
& \text { s.t. } \sum_{i} p_{i j} q_{i j}=Y_{j}
\end{aligned}
$$

where $Y_{i}=\sum_{j} X_{i j}$ is the nominal income of region $i$.
(b) (6 points) Derive the nominal demand for region i's goods by region $j$, and use this to write an expression for $X_{i j}$.
Solving the utility-maximization problem gives us, for each $i, i^{\prime}$

$$
\begin{aligned}
\left(\frac{q_{i j}^{*}}{q_{i^{\prime} j}^{*}}\right)^{\rho-1} & =\frac{p_{i j}}{p_{i^{\prime} j}} \\
q_{i j}^{*} & =\left(\frac{p_{i j}}{p_{i^{\prime} j}}\right)^{1 /(\rho-1)} q_{i^{\prime} j}^{*}
\end{aligned}
$$

Multiplying both sides by $p_{i j}$ and summing over $i$ gives

$$
\begin{aligned}
\sum_{i} p_{i j} q_{i j}^{*} & =\sum_{i} p_{i j}^{\rho /(\rho-1)} p_{i^{\prime} j}^{-1 /(\rho-1)} q_{i^{\prime} j}^{*} \\
\frac{p_{i^{\prime} j}^{1 /(\rho-1)}}{\sum_{i} p_{i j}^{\rho /(\rho-1)}} Y_{j} & =q_{i^{\prime} j}^{*}
\end{aligned}
$$

[^0]where the second line substitutes in the income constraint. Plugging this back into the original FOC gives
$$
q_{i j}^{*}=\frac{\left(\tau_{i j} p_{i}\right)^{1 /(\rho-1)}}{\sum_{i}\left(\tau_{i j} p_{i}\right)^{\rho /(\rho-1)}} Y_{j}
$$

Finally, we can write exports as

$$
\begin{aligned}
X_{i j} & =p_{i j} q_{i j}^{*} \\
& =\left(\frac{\tau_{i j} p_{i}}{P_{j}}\right)^{\rho /(\rho-1)} Y_{j} \\
\text { where } P_{j} & \equiv\left(\sum_{i}\left(\tau_{i j} p_{i}\right)^{\rho /(\rho-1)}\right)^{(\rho-1) / \rho}
\end{aligned}
$$

(c) (10 points) Use market clearing to show there exists an equilibrium with symmetric trading costs $\left(\tau_{i j}=\tau_{j i} \forall i, j\right)$ where

$$
P_{j}=\left(\sum_{i}\left(\frac{\tau_{i j}}{P_{i}}\right)^{\rho /(\rho-1)} \frac{Y_{i}}{Y_{W}}\right)^{(\rho-1) / \rho}
$$

where $Y_{i}$ is the nominal income of region $i, P_{i}$ is the price index of region $i$, and $Y_{W}=\sum_{i} Y_{i}$. Use this to derive a simple gravity equation for bilateral trade flows $X_{i j}$ in terms of $Y_{i}, Y_{j}, P_{i}, P_{j}$, $Y_{W}$, and $\tau_{i j}$.
Market clearing and the above expression implies

$$
\begin{aligned}
Y_{i}=\sum_{j} X_{i j} & =\sum_{j}\left(\frac{\tau_{i j} p_{i}}{P_{j}}\right)^{\rho /(\rho-1)} Y_{j} \\
& =p_{i}^{\rho /(\rho-1)} \sum_{j}\left(\frac{\tau_{i j}}{P_{j}}\right)^{\rho /(\rho-1)} Y_{j} \\
p_{i} & =\left(\frac{Y_{i}}{\sum_{j}\left(\frac{\tau_{i j}}{P_{j}}\right)^{\rho /(\rho-1)} Y_{j}}\right)^{(\rho-1) / \rho}
\end{aligned}
$$

Substituting this into the equation for $P_{j}$ found in (b) gives

$$
\begin{aligned}
P_{j} & =\left(\sum_{i} \tau_{i j}^{\rho /(\rho-1)}\left(\frac{Y_{i}}{\sum_{j}\left(\frac{\tau_{i j}}{P_{j}}\right)^{\rho /(\rho-1)} Y_{j}}\right)\right)^{(\rho-1) / \rho} \\
& =\left(\sum_{i}\left(\frac{\tau_{i j}}{\Pi_{i}}\right)^{\rho /(\rho-1)} \frac{Y_{i}}{Y_{W}}\right)^{(\rho-1) / \rho} \\
\text { where } \Pi_{i} & \equiv\left(\sum_{j}\left(\frac{\tau_{i j}}{P_{j}}\right)^{\rho /(\rho-1)} \frac{Y_{j}}{Y_{W}}\right)^{(\rho-1) / \rho}
\end{aligned}
$$

Under symmetric transaction costs there exists a solution to this system of equations with $P_{i}=\Pi_{i}$ for all $i$. In this case the equation for $p_{i}$ becomes

$$
p_{i}=\left(\frac{Y_{i} / Y_{W}}{P_{i}}\right)^{(\rho-1) / \rho}
$$

Plugging this solution back into the demand equation gives

$$
X_{i j}=\frac{Y_{i} Y_{j}}{Y_{W}}\left(\frac{\tau_{i j}}{P_{i} P_{j}}\right)^{\rho /(\rho-1)}
$$

This is a "gravity" equation; exports are increasing in the relative size of both trading partners.
(d) (6 points) Suppose $\rho \in(0,1)$. What is the effect of increased trading costs on trade flows between country $i$ and $j$ ? What is the effect of increased $P_{i}$ or $P_{j}$ holding $\tau_{i j}$ constant? Explain.
The larger the bilateral trade barrier $\tau_{i j}$ between any two countries, the less they will trade with one another. Increases in $P_{i}$ and $P_{j}$, which we can think of as "multilateral resistance" variables as they depend on all bilateral trade resistances $\left\{t_{i j}\right\}$, will increase trade flows holding $\tau_{i j}$ constant. Intuitively, for a given transaction barrier between countries $i$ and $j$, higher barriers between the importer $j$ and its other trading partners will reduce the relative price of goods from $i$ and thus raise imports from $i$. At the same time, higher barriers between the exporter $i$ and its other trading partners lowers the demand for $i$ 's goods and therefore its supply price $p_{i}$, which also increases trade.
(e) (5 points) Suppose we model trading costs as

$$
\tau_{i j}=\left(1+b_{i j}\right) d i s t_{i j}^{c}
$$

where $b_{i j}$ equals the tariff equivalent of the border between country $i$ and $j$ and $c$ is a scalar. Use your gravity equation to write down an estimating equation similar to McCallum's. What issues with McCallum's empirical strategy does this exercise raise?
Taking logs, we have

$$
\ln \left(\frac{X_{i j}}{Y_{i} Y_{j}}\right)=-\ln Y_{W}+\frac{\rho}{\rho-1} c \ln d i s t_{i j}+\frac{\rho}{\rho-1} \ln \left(1+b_{i j}\right)-\ln P_{i}^{\rho /(\rho-1)}-\ln P_{j}^{\rho /(\rho-1)}+\epsilon_{i j}
$$

where we've tacked on an error term $\epsilon_{i j}$ (which we could choose to interpret as classical measurement error). This is quite similar to McCallum's equation, except that he relegates the $\ln \left(P_{i} P_{j}\right)^{\rho /(\rho-1)}$ term to the residual. This is highly problematic, as the $P_{j}$ terms form a nonlinear system of $\left\{\tau_{i j}\right\}$, causing McCallum's residual to be correlated with his regressors. One way to overcome this, if we take our model seriously, would be to jointly estimate each $P_{i}$ and $P_{j}$ along with the above regression by nonlinear least squares. This is what Anderson and van Wincoop (2003) do.

## 3. The Market for Worker Harassment

A labor market institution that we did not discuss in class but which is widespread throughout the developing and developed world is mandatory labor "standards" that place constraints on the type of contracts that can be struck (for example, indentured servitude). While injunctions on debt bondage or, say, child labor are relatively non-controversial, there are many other labor standards for which the economic or moral case may be less unambiguous; for example: maximum hours limitations, minimum safety requirements, or rules against certain types of treatment by employers (e.g. sexual harassment). The neoclassical economist in you naturally chafes at these regulations. If a worker is willing to accept somewhat less safe working conditions in exchange for a higher rate of pay, why shouldn't she be allowed to? A simple laissez faire argument says that market transactions among consenting adults that don't produce negative externalities on others should be permissible.

In this question you'll explore one aspect of this discussion. Consider a market for sexual harassment. Firms produce output using labor n, where $n$ is the number of workers. Production occurs according to $Y=f(n)$, where $f(n)$ is strictly increasing and strictly concave. Employers get perverse gratification $\theta>0$ (measured in units of output) from each worker they are allowed to harass. Consider two cases. In the first, sexual harassment is illegal. The wage of each worker is given by $w_{N}^{B}$ and profits are

$$
\pi\left(n_{N}\right)=f\left(n_{N}\right)-n_{N} w_{N}^{B}
$$

In the second, sexually harassment is permitted, and the wages of harassed and non-harassed workers are $w_{H}^{A}$ and $w_{N}^{A}$, respectively. The firm's payoff is then

$$
\pi\left(n_{N}, n_{H}\right)=f\left(n_{N}+n_{H}\right)+n_{H} \theta-n_{N} w_{N}^{A}-n_{H} w_{H}^{A}
$$

Assume a unit mass of workers each with labor supply function $s(w)$, where $s^{\prime}(w)>0$. Write the monetized cost of harassment to worker $i$ as $c(i)$; workers are indexed such that the individual with the highest disutility of harassment is identified by $i=0$ and the worker with the lowest disutility is $i=1$. Assume $c(\cdot)$ is strictly increasing and continuously differentiable, and let $\phi(k)$ be its inverse. Assume $c(0)>\theta>c(1)$.
(a) (5 points) Derive an expression for equilibrium labor supply in the regimes where harassment is illegal $(B)$ and where it is legal $(A)$. Draw a graph depicting the labor market equilibrium in the $A$ regime.
When harassment is illegal, profit maximization implies $w_{N}^{B}=f^{\prime}\left(n_{N}^{B}\right)$ and labor supply dictates $s\left(w_{N}^{B}\right)=n_{N}^{B}$, so that $n_{N}^{B}$ solves for the intersection of supply and demand:

$$
s\left(f^{\prime}\left(n_{N}^{B}\right)\right)=n_{n}^{B}
$$

When harassment is legal, profit maximization implies

$$
w_{N}^{A}=f^{\prime}\left(n_{N}^{A}+n_{H}^{A}\right)=w_{H}^{A}-\theta
$$

Note that in equilibrium every worker $i \in[0, \phi(\theta))$ would strictly prefer a non-harassment contract, every worker $i \in(\phi(\theta), 1]$ would strictly prefer a harassment contract, and the worker with $c(\theta)=$ $w_{H}^{A}-w_{N}^{A}$ is exactly indifferent. Total labor supply is thus given by

$$
n_{N}^{A}+n_{H}^{A}=\phi(\theta) s\left(w_{N}^{A}\right)+\int_{\phi(\theta)}^{1} s\left(w_{N}^{A}+\theta-c(i)\right) d i
$$

and $n_{N}^{A}+n_{H}^{A}$ again solves for the intersection of supply and demand:

$$
s\left(f^{\prime}\left(n_{N}^{A}+n_{H}^{A}\right)\right)=n_{N}^{A}+n_{H}^{A}
$$

We can see these two equilibria in Figure 1. Note that since $\theta-c(i)>0$ for all $i \in(\phi(\theta), 1]$ the harassment-allowed supply schedule lies to the right of the banned schedule for all $w_{N}^{A}$.


Figure 1: Labor market equilibria with and without harassment


Figure 2: A Pareto-improving "workplace sex worker" scheme
(b) (6 points) Can you sign $w_{N}^{\prime} \equiv w_{N}^{B}-w_{N}^{A}$ or is its sign indeterminate?

We can sign $w_{N}^{\prime}>0$ by contradiction. Assume $w_{N}^{A} \geq w_{N}^{B}$. Then from above,

$$
\begin{aligned}
f^{\prime}\left(n_{N}^{A}+n_{H}^{A}\right) & \geq f^{\prime}\left(n_{N}^{B}\right) \\
n_{N}^{A}+n_{H}^{A} & \leq n_{N}^{B} \\
\phi(\theta) s\left(w_{N}^{A}\right)+\int_{\phi(\theta)}^{1} s\left(w_{N}^{A}+\theta-c(i)\right) d i & \leq s\left(w_{N}^{B}\right)
\end{aligned}
$$

since $f(w)$ is concave. Furthermore since $s(w)$ is increasing, $w_{N}^{A} \geq w_{N}^{B}$ implies

$$
\int_{\phi(\theta)}^{1} s\left(w_{N}^{A}+\theta-c(i)\right) d i \leq(1-\phi(\theta)) s\left(w_{N}^{B}\right)
$$

But note that for harassed workers we have $\theta>c(i)$, and as $s(w)$ is increasing

$$
\begin{aligned}
\int_{\phi(\theta)}^{1} s\left(w_{N}^{A}+\theta-c(i)\right) d i & >\int_{\phi(\theta)}^{1} s\left(w_{N}^{A}+\theta\right) d i \\
& =(1-\phi(\theta)) s\left(w_{N}^{A}+\theta\right)
\end{aligned}
$$

leading to a contradiction. Thus we must have $w_{N}^{\prime}>0$.
(c) (6 points) Are any workers better off in the A regime than they would be in the $B$ regime? Any worker $i$ for which $w_{H}^{A}-w_{N}^{B}>c(i)$ would be better off if harassment were allowed. These workers would earn $w_{N}^{B}$ in the $B$ regime, but as they have

$$
\begin{aligned}
w_{H}^{A}-c(i) & >w_{N}^{B} \\
& >w_{N}^{A} \\
\Longrightarrow \theta & >c(i)
\end{aligned}
$$

they would choose to be harassed in the $A$ regime, earning $w_{N}^{A}+\theta-c(i)$. This is strictly better than $w_{N}^{B}$ as

$$
\begin{aligned}
w_{H}^{A}-c(i) & >w_{N}^{B} \\
\Longrightarrow w_{N}^{A}+\theta-c(i) & >w_{N}^{B}
\end{aligned}
$$

(d) (5 points) Are there any non-harassed workers in the $A$ regime who are worse off than they would be in the $B$ regime?
Yes. As we've shown, $w_{N}^{A}<w_{N}^{B}$. Thus all workers with $i \in[0, \phi(\theta))$ in the $A$ regime stand to earn a higher wage in a regime where harassment were banned.
(e) (5 points) Are there any harassed workers in the A regime who are worse off than they would be in the $B$ regime?
Yes. Any worker with $c(i) \in\left(\phi(\theta), w_{H}^{A}-w_{N}^{B}\right)$ will be harassed and strictly worse off in the $A$ regime, as is easily shown by reversing the proof in (c).
(f) (7 points) We noted above that an individual worker's agreeing to tolerate sexual harassment in exchange for payment does not generate negative externalities. Interpret your answers to (d) and (e) in light of this observation.

Although an individual worker's agreement to accept harassment in exchange for a higher wage (represented by an atomless change in our continuum of agents) must constitute a Pareto improvement for the price-taking agent, any non-zero measure of agent that sign up for harassment will change equilibrium wages and thus generate a negative pecuniary externality for other agents. In particular, such a shift corresponds to a lower equilibrium wage for non-harassed workers, so that the multiplicity of agents agreeing to accept harassment no longer constitutes a Pareto improvement.
(g) (8 points) Congress wants to write a law that legalizes sexual harassment while guaranteeing that the law is Pareto improving. Assuming all of the parameters of the model above are known, can this law be written, and if so how? Illustrate diagrammatically.
One straightforward way to legalize firms hiring workers to sexually harass while guaranteeing a Pareto improvement relative to the harassment-banned regime is to break the link between contracts for employment and contracts for harassment and to, with Coasian logic, define and enforce harassment "property rights." In effect the negative externality here arises from a missing market - the fact that workers with low $c(i)$ cannot separately contract for providing labor and sexual services, so that workers more averse to the latter must accept a lower equilibrium wage and be made worse off. If sexual harassment were perfectly observable and Congress were able to separately enforce (1) labor contracts that do not have harassment provisions and (2) contracts over, effectively, prostitution for firms (that is independent of promises for productive labor, but may be freely used by employees), these low- $c(i)$ workers could bargain to make themselves better off by providing harassment services to firms without affecting the equilibrium wage, constituting a Pareto improvement. Specifically, firms would demand contracts to maximize

$$
\pi=f\left(n_{N}\right)-n_{N} w_{N}+\theta n_{H}-n_{H} w_{H}
$$

which additively separates the profit maximization problem, while workers would supply contracts separately according to $s_{N}\left(w_{N}\right)=s\left(w_{N}\right)$ and $s_{H}\left(w_{H}\right)=\left\{s^{p}\left(w_{H}-c(i)\right)\right\}^{+}$for some increasing supply of harassment schedule $s^{p}(\cdot)$. Perfect competition guarantees $w_{H}=\theta$ in equilibrium, so that only agents with $c(i) \geq \theta$ will supply harassment (while possibly also supplying non-harassed work contracts) and those with $c(i)<\theta$ will not be harassed. Importantly, the decision to be or not be harassed for these agents is independent of the equilibrium wage paid for non-harassed work (which is $f^{\prime}\left(n_{N}\right)$ ), so that the same number of non-harassed workers will be employed at the same wage as in the harassment-banned regime and those workers will all be as well off. Moreover, any provider of a harassment contract will be by construction better off as they receive a wage over their disutility of the service, constituting a Pareto improvement relative to the harassmentbanned regime. Figure 2 illustrates this "workplace sex worker" scheme.

MIT OpenCourseWare
http://ocw.mit.edu

### 14.662 Labor Economics II

Spring 2015

For information about citing these materials or our Terms of Use, visit: http://ocw.mit.edu/terms.


[^0]:    ${ }^{1}$ McCallum, John, (1995) "National Borders Matter: Canada-U.S. Regional Trade Patterns," American Economic Review 85(3): 615-623.

