Introduction to Political Economy 14.770
Problem Set 2 Solutions

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Question 1:

A society is a two party democracy with population normalized to 1, with political parties R and D competing to maximize their vote share. Parties compete by proposing a tax rate $\tau \in [0,1]$ with proceeds distributed lump-sum to each member of society. Taxing income introduces distortions, so the tax revenue is $(\tau - v(\tau))\bar{y}$, where $\bar{y}$ is average income in society and $v(0) = v'(0) = 0$ and $v'(1) = \infty$, and the government budget constraint is

$$T \leq (\tau - v(\tau))\bar{y}$$

The society is stratified into n groups. The size of each group varies, but members of the same group have the same income, denoted by y_j, with differing political ideologies. Let the political leaning towards party R of individual i in group j be σ_j^i and the size of group j be α_j, with $\sum_{j=1}^n \alpha_j = 1$ and naturally $\sum_{j=1}^n \alpha_j y_j = \bar{y}$. Assume that σ_j^i is drawn from a distribution $F_j(x)$ symmetric around 0.

Assume that individuals of the society all share a common utility function

$$U_j^i(c_i, \sigma_j^i) = c_i + [\sigma_j^i + \delta]I_R$$

where I_R is an indicator for party R coming to power, and δ is a random popularity measure for party R, drawn from distribution $G(\cdot)$.

1. *First, ignore the ideological leanings of each group and the relative popularity measure (i.e., $\sigma_j^i = \delta = 0$). Find the equilibrium in the party competition game and the tax rate announced by the two parties. Does a pure strategy equilibrium always exist?*

   This is the standard Downsian convergence case (analyzed in Lectures 1 and 2), and should be relatively easy to solve (given your experience from Problem Set 1).

Since $\sigma_j^i = \delta = 0$, we have:

$$U_j^i\left(c_i\right) = c^j$$

where $c^j = y_j\left(1-\tau\right) + \left(\tau - v\left(\tau\right)\right)\overline{y}$. Consequently,the indirect utility function for each individual i in group j is:

$$W_j^i(\tau) = y_j\left(1-\tau\right) + \left(\tau - v\left(\tau\right)\right)\overline{y}$$

Assume that $v(.)$ is strictly increasing, strictly convex, $v'\left(0\right) = v\left(0\right) = 0,\ v'\left(1\right) = \infty$ – standard assumptions by now. By an argument similar to the one we made in Question 4 of Problem Set 1, then, each individual i in group j has single-peaked preferences over the tax rate with a bliss point τ_j^*. Moreover,

$$\tau_j^* = \begin{cases} 0, & \text{if } y_j \geq \overline{y} \\ v'^{-1}\left(1 - \frac{y_j}{\overline{y}}\right), & \text{if } y_j < \overline{y}. \end{cases}$$

That is, the groups with above-average incomes prefer zero taxes, and the groups with below-average incomes prefer some taxes, with bliss points decreasing in income.[1] Let $j = m$ be the group with the median income. Assume that if parties announce the same platform they win the election with equal probabilities. In this case, since the preferences are single-peaked, Downsian Policy Convergence Theorem (page 30 of Lecture 1 and 2 Notes) applies and the equilibrium is:

$$\tau^R = \tau^D = \tau_m^*$$

2. *Now characterize the equilibrium with the ideological leanings (still with $\delta = 0$). Does a pure strategy equilibrium always exist?*

   In this case, the indirect utility function for individual i in group j is:

$$W_j^i\left(\tau;\sigma_j^i\right) = y_j\left(1-\tau\right) + \left(\tau - v\left(\tau\right)\right)\overline{y} + \sigma_j^i I_R$$

   Let τ^R, τ^D be the announced tax rates by the two parties. Individual i in group j will vote for party R if and only if

$$\sigma_j^i \geq \left(\tau^R - \tau^D\right)\left(y_j - \overline{y}\right) + \left(v\left(\tau^R\right) - v\left(\tau^D\right)\right)\overline{y}$$

[1] This follows from strict convexity of $v(.)$ – check Problem Set 1 Solutions if you need a refresher.

The swing voter in group j is therefore the voter with:

$$\sigma_j^i = \left(\tau^R - \tau^D\right)(y_j - \overline{y}) + \left(v\left(\tau^R\right) - v\left(\tau^D\right)\right)\overline{y}$$

The expected vote share of party R in group j is

$$\pi_j^R = 1 - F_j\left(\left(\tau^R - \tau^D\right)(y_j - \overline{y}) + \left(v\left(\tau^R\right) - v\left(\tau^D\right)\right)\overline{y}\right)$$

and the total vote share of party R is

$$\pi^R(\tau^R, \tau^D) := \sum_j \alpha_j \pi_j^R = 1 - \sum_j \alpha_j F_j\left(\left(\tau^R - \tau^D\right)(y_j - \overline{y}) + \left(v\left(\tau^R\right) - v\left(\tau^D\right)\right)\overline{y}\right)$$

Now, in a pure strategy equilibrium (τ^R, τ^D):

- Party R takes τ^D as given and solves the following problem:

$$\max_{\tilde{\tau}} \pi^R(\tilde{\tau}, \tau^D)$$

  Assuming differentiability of $F_j(.)$'s, the first order condition for optimality is:

$$\sum_j \alpha_j f_j\left(\left(\tau^R - \tau^D\right)(y_j - \overline{y}) + \left(v\left(\tau^R\right) - v\left(\tau^D\right)\right)\overline{y}\right)\left[(y_j - \overline{y}) + v'\left(\tau^R\right)\overline{y}\right] = 0$$

- Party D takes τ^R as given and solves:

$$\max_{\tilde{\tau}} 1 - \pi^R(\tau^R, \tilde{\tau})$$

  Once again assuming differentiability of $F_j(.)$'s, the first order condition is:

$$\sum_j \alpha_j f_j\left(\left(\tau^R - \tau^D\right)(y_j - \overline{y}) + \left(v\left(\tau^R\right) - v\left(\tau^D\right)\right)\overline{y}\right)\left[(y_j - \overline{y}) + v'\left(\tau^D\right)\overline{y}\right] = 0$$

Due to the symmetry of these two first order conditions, when an equilibrium exists it is given by

$$\tau^R = \tau^L = v'^{-1}\left(\frac{\sum_j \alpha_j f_j(0)\left(1 - \frac{y_j}{\overline{y}}\right)}{\sum_j \alpha_j f_j(0)}\right)$$

The equilibrium has the look of a "weighted average" of bliss points, where weights depend on the group size (α_j) and the responsiveness to policy changes around equilibrium ($f_j(0)$). That is, unlike part 1, parties now care about how responsive the groups are and they cater towards groups with more "swing voters".

Note, however, that the conditions for existence of a pure strategy equilibrium is quite stringent, and the existence of a pure strategy equilibrium is in general not guaranteed. A sufficient condition for existence is given by Equation (9) in page 57 of Lecture 1 and 2 Notes. As you recall from class, this condition is roughly equivalent to $F_j(.)$'s being approximately uniform distribution.

3. *Now assume the parties can offer both a lump-sum redistribution T and a group-specific redistibution to all members of each group, denoted by $\omega_j \geq 0$, so the government budget constraint becomes*

$$\sum_{j=1}^{n} \alpha_j \omega_j + T \leq (\tau - v(\tau))\, \bar{y}$$

Show that there is no pure strategy equilibrium for the game when δ is known in advance to be 0. Determine conditions for an equilibrium to exist when δ is random with distribution G, and characterize such an equilibrium. Will the two parties necessarily offer the same policy platform?

A party's policy now is given by the $n+2$ tuple: $(\tau, \{\omega_j\}_{j=1}^{n}, T)$. The indirect utility function is:

$$W_i^j(\tau, \{\omega_j\}_{j=1}^{n}, T; \sigma_j^i) = y_j (1 - \tau) + \omega_j + T + \sigma_j^i I_R$$

where

$$\sum_{j=1}^{n} \alpha_j \omega_j + T \leq (\tau - v(\tau))\, \bar{y}$$

Begin by realizing that any policy $(\tau, \{\omega_j\}_{j=1}^{n}, T)$ with $T > 0$ can be implemented with an alternative policy $(\tau, \{\omega_j'\}_{j=1}^{n}, T')$ with $T' = 0$, where

$$\omega_j' = \omega_j + T$$

That is, the lump-sum transfer T can be embedded in the group-specific transfers. Therefore, there is no loss in generality in assuming $T = 0$ and a policy is given by the $n+1$-tuple

$$\gamma := (\tau, \{\omega_j\}_{j=1}^{n})$$

with the feasibility condition

$$\sum_{j=1}^{n} \alpha_j \omega_j \leq (\tau - v(\tau))\, \bar{y}$$

Assume $\delta = 0$**.** I'll argue the non-existence of a pure strategy equilibrium from first principles: it is really similar to the argument made in page 49 of Lecture 1 and 2 notes. Suppose, to get a contradiction, that there exists a pure strategy equilibrium (γ^R, γ^D). The total vote share of party R is

$$\pi^R(\gamma^R, \gamma^D) := 1 - \sum_j \alpha_j F_j\left(\left(\tau^R - \tau^D\right) y_j + \omega_j^D - \omega_j^R\right)$$

Clearly, in this equilibrium both the parties must be getting half the vote share, i.e. $\pi^R(\gamma^R, \gamma^D) = \frac{1}{2}$ – otherwise the losing party can just offer the winning party's policy and get half of the vote share. Now, order the groups by the sensitivity of their votes to the transfers. Then one party can transfer ε extra to the groups most sensitive to policy and take out ε from the contribution to the least sensitive groups, which is a strictly profitable deviation.

In particular, given $\gamma^D = (\tau^D, \{\omega_j^D\}_{j=1}^n)$, take two groups k, l with $\alpha_k f_k(0) > \alpha_l f_l(0)$ and $\omega_l^D > 0$.[2] Party R can offer $(\gamma^R)' = (\tau^D, \{(\omega_j^R)'\}_{j=1}^n))$ with

$$(\omega_k^R)' = \omega_k^D + \varepsilon$$

and

$$(\omega_l^R)' = \omega_l^D - \varepsilon$$

whereas

$$(\omega_j^R)' = \omega_j^D \quad \text{for } j \neq k, l$$

For ε sufficiently small, $\pi((\gamma^R)', \gamma^D) > \frac{1}{2}$, i.e. it is a strictly profitable deviation – a contradiction. We conclude that there can't be a pure strategy equilibrium in this case.

The argument above concludes that we need δ to be random in order to have a pure strategy equilibrium. The conditions on $G(.)$ to guarantee the existence of a pure strategy equilibrium is once more complicated –

[2]There's the "missing case" of what happens if $\omega_j^D = 0$ for each group except the most sensitive one. To ensure that there is a deviation for R in that case, we just need to assume that the group sizes are not too far away from each other.

in many practical applications, we'll assume that it is uniform around 0.

Note that, for a general (realized) value of δ, the vote share of Party R is:

$$\pi^R(\gamma^R,\gamma^D) := 1 - \sum_j \alpha_j F_j\left(\left(\tau^R - \tau^D\right) y_j + \omega_j^D - \omega_j^R - \delta\right)$$

For a general distribution $G(.)$, the probability of Party R winning is:

$$\begin{aligned} Pr\{\pi^R(\gamma^R,\gamma^D) > \frac{1}{2}\} &= Pr_{\delta\sim G}\{1 - \sum_j \alpha_j F_j\left(\left(\tau^R - \tau^D\right) y_j + \omega_j^D - \omega_j^R - \delta\right) > \frac{1}{2}\} \\ & Pr_{\delta\sim G}\{\sum_j \alpha_j F_j\left(\left(\tau^R - \tau^D\right) y_j + \omega_j^D - \omega_j^R - \delta\right) < \frac{1}{2}\} \end{aligned}$$

Once again, a pure strategy equilibrium is[3] (γ^R,γ^D) such that:

- $\gamma^R \in \arg\max_{\tilde{\gamma}} Pr\{\pi^R(\tilde{\gamma},\gamma^D) > \frac{1}{2}\}$
- $\gamma^D \in \arg\min_{\tilde{\gamma}} Pr\{\pi^R(\gamma^R,\tilde{\gamma}) > \frac{1}{2}\}$

It is difficult to move on from here and get a clean analytical result, unless one is willing to assume particular distributions. This is what we do next.

4. *Now fully characterize the equilibrium in this probabilistic voting model assuming that σ_j^i is uniform over $\left[-\phi_j^{-1},\phi_j^{-1}\right]$ for all j and δ is uniform over $\left[-\psi^{-1},\psi^{-1}\right]$.*

   Now, we have $F_j(x) = \frac{1}{2} + \frac{\phi_j}{2}x$ for each j, $G(x) = \frac{1}{2} + \frac{\psi}{2}x$.

   We can continue by plugging these in. For a given $\delta \in \left[-\frac{1}{\psi},\frac{1}{\psi}\right]$, vote share for party R under policies (γ^R,γ^D) is:

$$\pi^R(\gamma^R,\gamma^D) = \frac{1}{2} + \sum_j \frac{\alpha_j\phi_j}{2}\left(\left(\tau^D - \tau^R\right) y_j + \omega_j^R - \omega_j^D + \delta\right)$$

[3] I've made the transition from "parties maximizing vote share" to "parties maximizing the probability of winning" here, because it is more or less customary to assume that the parties maximize probability of winning in probabilistic voting models – so I wanted to be consistent. Needless to say, qualitative insights would not change.

The probability of party R winning is therefore

$$\begin{aligned}
\Pr\{\pi^R(\gamma^R,\gamma^D) > 1/2\} &= \Pr\{\sum_j \frac{\alpha_j\phi_j}{2}\left(\left(\tau^R-\tau^D\right)y_j+\omega_j^D-\omega_j^R\right) < \delta\sum_j\frac{\alpha_j\phi_j}{2}\} \\
&= \frac{1}{2}+\frac{\psi}{2}\left[\frac{\sum_j \alpha_j\phi_j\left(\left(\tau^D-\tau^R\right)y_j+\omega_j^R-\omega_j^D\right)}{\sum_j\alpha_j\phi_j}\right]
\end{aligned}$$

In equilibrium, Party R takes $(\tau^D, \{\omega_j^D\}_{j=1}^n)$ as given and solves the following maximization problem:

$$\max_{\tau,\{\omega_j\}_{j=1}^n} \frac{1}{2}+\frac{\psi}{2}\left[\frac{\sum_j \alpha_j\phi_j\left(\left(\tau^D-\tau\right)y_j+\omega_j-\omega_j^D\right)}{\sum_j\alpha_j\phi_j}\right]$$

subject to

$$\begin{aligned}
\sum_{j=1}^n \alpha_j\omega_j &\le \left(\tau-v\left(\tau\right)\right)\bar{y} && && (\lambda) \\
0 &\le \omega_j && \forall j && (\mu_j)
\end{aligned}$$

The first order conditions are

$$\begin{aligned}
-\frac{\psi}{2}\sum_j\frac{\alpha_j\phi_j y_j}{\sum_j\alpha_j\phi_j}+\lambda\left(1-v'\left(\tau\right)\right)\bar{y}=0 \\
\frac{\psi}{2}\frac{\alpha_j\phi_j}{\sum_j\alpha_j\phi_j}-\lambda\alpha_j+\mu_j=0 && \forall j
\end{aligned}$$

By the complementary slackness condition $\omega_j > 0 \Rightarrow \mu_j = 0$. This implies that groups that receive transfer must have

$$\frac{\psi}{2}\frac{\phi_j}{\sum_j\alpha_j\phi_j}=\lambda$$

Yet, clearly, this equation cannot hold for more than one j as long as ϕ_j's are different. Moreover, since $\mu_j \ge 0$, this condition must hold for the group with highest ϕ_j only. We conclude that, in equilibrium, only the group with the highest vote responsiveness to policy will get a transfer.

Question 2:

Consider the following environment: There are two states of nature, $\theta \in \{0,1\}$. The ex-ante probability of state 0 is $\alpha := Pr\{\theta = 0\}$, with $\alpha < \frac{1}{2}$.

There are $N+1$ voters, where N is even. The voters vote over two policies, $x \in \{0,1\}$. The implemented policy is chosen via simple majority rule: the alternative that receives $\frac{N}{2}+1$ votes wins.

A voter may have one of the three types, $t \in \{0,1,i\}$. A voter with type $t=0$ always votes for $x=0$, and similarly, a voter with type $t=1$ always votes for $x=1$. A voter with type i (an independent voter) has preferences given by:

$$U_i(x,\theta) = -\mathbf{1}(x \neq \theta)$$

where x is the chosen policy and θ is the state of the world.

Each voter's type is drawn randomly and independently, according to the following distribution: each voter has probability $\frac{\gamma}{2}$ of being $t=0$, probability $\frac{\gamma}{2}$ of being $t=1$, and probability $1-\gamma$ of being $t=i$. Here, $\gamma \in [0,1]$ parametrizes the expected share of partisans in the population. Conditional on being $t=i$, a voter is informed (i.e. learns the true value of θ) with probability $q \in [0,1]$ and uninformed with the complementary probability. We will consider the Bayesian Nash Equilibrium of the game induced by this setup.

Warm-Up. *Consider the case where $N=0$. Observe that in the unique Bayesian Nash Equilibrium, an uninformed independent voter votes for $x=1$ with probability one. To do this, you first need to observe*

$$Pr\{\theta = 0 | t = i\} = \alpha < \frac{1}{2}$$

The crucial point in this exercise is recognizing that the event of being uninformed and and independent is independent of the state. Therefore, conditional on being uninformed and independent is not informative at all, so the prior about the state equals the posterior.

We will now argue that this behavior by uninformed independent voters does not always arise when N is larger, i.e. the voter no longer votes in isolation.

1. *Now, consider the case $N=2$. Derive a condition to ensure that there can not be a Bayesian Nash Equilibrium (BNE) where all uninformed independent voters vote for $x=1$ with probability one.*

Assume, to get a contradiction, that there is a BNE in which all uninformed independent voters vote for $x = 1$ with probability one.

- Letting $\sigma_{x,\theta}$ denote the probability that a voter votes for $x \in \{0, 1\}$ in state $\theta \in \{0, 1\}$, one can easily derive:

$$\begin{aligned}
\sigma_{0,0} &= \frac{\gamma}{2} + (1-\gamma)q \\
\sigma_{0,1} &= \frac{\gamma}{2} \\
\sigma_{1,0} &= \frac{\gamma}{2} + (1-\gamma)(1-q) \\
\sigma_{1,1} &= \frac{\gamma}{2} + (1-\gamma)
\end{aligned}$$

  One thing to note is that, by construction, $\sigma_{0,0} + \sigma_{1,0} = 1$ and $\sigma_{1,0} + \sigma_{1,0} = 1$.

- Note that an an uninformed independent voter is pivotal only when one of the other agents votes for $x = 0$ and one votes for $x = 1$, i.e. when there is a tie. In state $\theta = 0$ (w.p. α), the probability of this happening is $2\sigma_{0,0}\sigma_{1,0}$. In state $\theta = 1$ (w.p. $1 - \alpha$), the probability of this happening is $2\sigma_{0,1}\sigma_{1,1}$. Bayesian updating (and the fact that being uninformed and independent is not informative about the state) then gives the posterior probability conditional on a tie and being uninformed and independent:

$$\begin{aligned}
Pr\{\theta = 0|t = i, t \text{ pivotal}\} &= \frac{\alpha 2\sigma_{0,0}\sigma_{1,0}}{\alpha 2\sigma_{0,0}\sigma_{1,0} + (1-\alpha)2\sigma_{0,1}\sigma_{1,1}} \\
&= \frac{\alpha\sigma_{0,0}\sigma_{1,0}}{\alpha\sigma_{0,0}\sigma_{1,0} + (1-\alpha)\sigma_{0,1}\sigma_{1,1}}
\end{aligned}$$

- Realize that when $Pr\{\theta = 0|t = i, t \text{ pivotal}\} \geq \frac{1}{2}$, an uninformed independent voter strictly prefers to vote for $x = 0$ instead. Given the equation above, this is the case if and only if

$$\alpha\sigma_{0,0}\sigma_{1,0} > (1-\alpha)\sigma_{0,1}\sigma_{1,1}$$

  Or, more succinctly:

$$\frac{\sigma_{0,0}\sigma_{1,0}}{\sigma_{0,1}\sigma_{1,1}} > \frac{1-\alpha}{\alpha} \Leftrightarrow \frac{(\frac{\gamma}{2} + (1-\gamma)q)(\frac{\gamma}{2} + (1-\gamma)(1-q))}{\frac{\gamma}{2}(\frac{\gamma}{2} + (1-\gamma))} > \frac{1-\alpha}{\alpha}$$

  There are a couple of things to be noted about this inequality. Note that, since $\alpha < \frac{1}{2}$, $\frac{1-\alpha}{\alpha} > 1$, so the right hand-side is always greater than one. Moreover, recognizing that $\sigma_{1,0} = 1 - \sigma_{0,0}$ and $\sigma_{1,1} = 1 - \sigma_{0,1}$, the left hand-side is

$$\frac{\sigma_{0,0}(1 - \sigma_{0,0})}{\sigma_{0,1}(1 - \sigma_{0,1})}$$

Note that $\sigma_{0,0} \in [\sigma_{0,1}, 1 - \sigma_{0,1}]$. Since $f(x) = x(1-x)$ is a strictly decreasing function around $\frac{1}{2}$ for any $x \in [0,1]$, we know that for any $y \in [x, 1-x]$, $f(y) \geq f(x)$. This implies that $\sigma_{0,0}(1-\sigma_{0,0}) \geq \sigma_{0,1}(1 - \sigma_{0,1})$, i.e. the left hand-side is also greater than one.

It doesn't necessarily mean that the left hand-side is larger than the right hand-side, though! Whether the exact inequality holds or not depends on the parameters. In particular,

- An increase in α makes it easier to satisfy this condition. Consequently, for sufficiently high values of α there is no BNE in which all uninformed independent voters vote for $x = 1$ with probability one. (Indeed, when $\alpha = \frac{1}{2}$ we cannot have this equilibrium for any γ and any $q \in (0, 1)$.) Heuristically, when α is higher, $\theta = 0$ is more likely, so voters tend to vote for $x = 0$ when independent.
- An increase in γ makes it more difficult to satisfy this condition. Consequently, for sufficiently high values of γ there is always a BNE in which all uninformed independent voters vote for $x = 1$ with probability one. The intuition is most transparent when $\gamma \approx \frac{1}{2}$. In that case, almost everyone is surely a partisan – so an uninformed voter is not really "surprised" when there is a tie. Consequently, being pivotal is less informative as an event, and the uninformed independent voter is more likely to rely on her prior (i.e. vote for $x = 1$).
- The effect of an increase in q on this inequality is nonmonotonic. Note that when $q = \frac{1}{2}$ it's easiest to satisfy this condition, and when $q = 0$ or $q = 1$ this condition is never satisfied (left hand-side is equal to 1). Heuristically, when $q = 0$ there is no "information" in the model, so an uninformed independent voter realizes that her being pivotal does not contain any information – so she relies on her prior and vote for $x = 1$. When $q = 1$, she realizes that being pivotal means that (i) one of the other voters is a partisan, and (ii) the other one is either a partisan or an informed voter. In any case, she cannot distinguish these events in different states, so she relies

on her prior.

2. *Generalize the same condition to $N \geq 2$. Then show that there exists $\bar{N}(\alpha, \gamma, q)$ such that for $N > \bar{N}(\alpha, \gamma\ q)$, there can not be a Bayesian Nash Equilibrium where all uninformed independent voters vote for $x = 1$ with probability one.*

   I'll not go through all the steps here – they're almost one-by-one replications of what we did in Part 1. You should be able to see that the condition is

$$\left(\frac{\sigma_{0,0}\sigma_{1,0}}{\sigma_{0,1}\sigma_{1,1}}\right)^{n/2} > \frac{1-\alpha}{\alpha} \Leftrightarrow \left(\frac{(\frac{\gamma}{2}+(1-\gamma)q)(\frac{\gamma}{2}+(1-\gamma)(1-q))}{\frac{\gamma}{2}(\frac{\gamma}{2}+(1-\gamma))}\right)^{n/2} > \frac{1-\alpha}{\alpha}$$

   Repeating the same argument, $\frac{\sigma_{0,0}\sigma_{1,0}}{\sigma_{0,1}\sigma_{1,1}}$ term is greater than one, so the left hand-side explodes as $n \to \infty$. This implies that for sufficiently large n, this condition will surely hold. Once again, you can see:

   - For a larger α, it takes a small number of voters to break the equilibrium where an uninformed independent voter votes according to her prior.
   - For a larger γ, it takes a large number of voters to break the equilibrium.
   - The effect of q is nonmonotonic: the closer it is to $\frac{1}{2}$, the easier it is.

Question 3:

A policymaker chooses the level of a policy vector, $\mathbf{x}$, which affects the welfare of several interest groups and the general public. Each group i offers a non-negative payment schedule C_i to influence policy. The schedule C_i is a contract stipulating that if the policymaker sets the policy at $\mathbf{x}$, then group i will pay the policymaker $C_i(\mathbf{x})$. The utility of the policymaker is $G(\mathbf{x}) = a\sum_{i=0}^{n} W_i(\mathbf{x}) + \sum_{i=1}^{n} C_i(\mathbf{x})$, where $W_i(\mathbf{x})$ is the welfare of group i, and this formulation implicitly assumes that there are n groups that are organized and group $i = 0$ is unorganized and represents all other citizens. The utility of each group i is $U_i(\mathbf{x}, c_i) = W_i(\mathbf{x}) - C_i(\mathbf{x})$. Assume $W_0, W_1, ..., W_n$ are strictly concave, twice continuously differentiable functions.

The order of play is as follows: First, all groups simultaneously choose their payment schedules. Next, the policymaker observes the schedules and chooses $\mathbf{x}$. An equilibrium is defined as a subgame-perfect Nash equilibrium.

1. *Show that if contribution schedules are continuously differentiable, then each group $i > 0$ making a positive payment in equilibrium will offer a payment schedule that must satisfy $\partial C_i(\mathbf{x}^*)/\partial x_j = \partial W_i(\mathbf{x}^*)/\partial x_j$, for each component x_j of $\mathbf{x}$. Interpret this condition. What happens if we do not make this continuous differentiability assumption? Is this assumption plausible?*

   The model we have here is exactly the model analyzed in the Grossman and Helpman model of lobbying – a model we discussed in Lectures 6 and 7. For the sake of brevity I'll not replicate the proofs here – you can check the lecture notes for a refresher.

   A SPE of this model is a set of payment schedules $\{\hat{C}_i(\cdot)\}_{i=1}^n$ for each organized lobby and a policy $\mathbf{x}^*$ such that:

   (a) For each $i \in \{1, \ldots, n\}$ and for each $\mathbf{x}$:

   $$\hat{C}_i(\mathbf{x}) \in [0, W_i(\mathbf{x})]$$

   (b) $\mathbf{x}^*$ is given by:

   $$\mathbf{x}^* \in \arg\max_{\mathbf{x}} \sum_{i=1}^{n} \hat{C}_i(\mathbf{x}) + a \sum_{i=0}^{n} W_i(\mathbf{x})$$

   (c) For each $i \in \{1, \ldots, n\}$:

   $$\mathbf{x}^* \in \arg\max_{\mathbf{x}} W_i(\mathbf{x}) - \hat{C}_i(\mathbf{x}) + \sum_{i=1}^{n} \hat{C}_i(\mathbf{x}) + a \sum_{i=0}^{n} W_i(\mathbf{x})$$

   (d) For each $i \in \{1, \ldots, n\}$, there exists $\mathbf{x}_i$ such that:

   $$\mathbf{x}_i \in \arg\max_{\mathbf{x}} \sum_{i=1}^{n} \hat{C}_i(\mathbf{x}) + a \sum_{i=0}^{n} W_i(\mathbf{x})$$

   and $\hat{C}_i(\mathbf{x}_i) = 0$.

   Take a policy dimension j. Using condition (b), and assuming that $\hat{C}_i(\cdot)$'s are differentiable, the optimal policy $\mathbf{x}^*$ satisfies the first-order condition

   $$\sum_{i=1}^{n} \frac{\partial \hat{C}_i(\mathbf{x}^*)}{\partial x_j} + a \sum_{i=0}^{n} \frac{\partial W_i(\mathbf{x}^*)}{\partial x_j} = 0 \tag{1}$$

Take some group $i > 0$. Suppose group i makes a positive payment in equilibrium, so we have an interior solution. Using condition (c) for group i and once again assuming differentiability, $\mathbf{x}^*$ satisfies the first-order condition

$$\frac{\partial W_i(\mathbf{x}^*)}{\partial x_j} - \frac{\partial \hat{C}_i(\mathbf{x}^*)}{\partial x_j} + \sum_{i=1}^{n} \frac{\partial \hat{C}_i(\mathbf{x}^*)}{\partial x_j} + a \sum_{i=0}^{n} \frac{\partial W_i(\mathbf{x}^*)}{\partial x_j} = 0 \qquad (2)$$

Plugging Equation (1) into Equation (2), we have:

$$\frac{\partial W_i(\mathbf{x}^*)}{\partial x_j} - \frac{\partial \hat{C}_i(\mathbf{x}^*)}{\partial x_j} = 0 \qquad (3)$$

For each group $i > 0$ making a positive payment and each policy dimension x_j. The interpretation is that that the contribution schedules are *locally truthful*, i.e. the marginal contribution in return for a policy is equal to the marginal increase the welfare of the lobby.

Note the the assumption of continuous differentiability of $\hat{C}_i(\cdot)$ plays an important role in this derivation. Without this assumption, many (discontinuous) contribution schedules may be consistent with equilibrium. In particular, there is nothing to prevent a lobby from announcing that it makes a positive payment only when $\mathbf{x} = \mathbf{x}^*$, and zero otherwise.

The plausibility of this assumption really depends on your take. In my view it seems somewhat implausible in many relevant settings. The enforceability of a possible nonlinear contract over the set of policies and and payments does not seem much realistic.

2. *Show that the equilibrium policy maximizes a weighted sum of aggregate welfare and the sum of the groups' welfares. What are the weights?*

   Once again plugging back Equation (3) for all $i > 0$ into Equation (1), we have:

$$\sum_{i=1}^{n} \frac{\partial W_i(\mathbf{x}^*)}{\partial x_j} + a \sum_{i=0}^{n} \frac{\partial W_i(\mathbf{x}^*)}{\partial x_j} = 0 \qquad (4)$$

   and rearranging gives:

$$a\frac{\partial W_0(\mathbf{x}^*)}{\partial x_j} + (1+a)\sum_{i=1}^{n}\frac{\partial W_i(\mathbf{x}^*)}{\partial x_j} = 0$$

for each policy dimension j. But this is the solution to the following problem (note the role played by concavity of W_i's here):

$$\arg\max_{\mathbf{x}} aW_0(\mathbf{x}) + (1+a)\sum_{i=1}^{n} W_i(\mathbf{x})$$

As argued in the question, this is a weighted welfare maximization problem, where the unorganized group receives weight a and each organized group receives weight $1+a$. Equivalently, one can interpret this as a weighted sum of aggregate welfare $\sum_{i=0}^{n} W_i(\mathbf{x})$ and organized groups' welfare $\sum_{i=1}^{n} W_i(\mathbf{x})$, where the former receives weight a and the latter receives weight 1.

3. *Suppose* $\mathbf{x} = (x_1, x_2)$*, and suppose* W_0 *can be written as* $W_0(\mathbf{x}) = \beta W_0^1(x_1) + (1-\beta)W_0^2(x_2)$*. Also, suppose there are two lobby groups, one that cares only about* x_1 *and one that cares only about* x_2*. Suppose the first policy dimension becomes relatively more salient to the public, in the sense that* β *increases. What happens to* x_1 *and* x_2*, and to the equilibrium contributions made by each group?*

   This is a particular case of the above analysis where $\mathbf{x} = (x_1, x_2)$ and

$$\begin{aligned} W_0(x_1, x_2) &= \beta W_0^1(x_1) + (1-\beta)W_0^2(x_2) \\ W_1(x_1, x_2) &= W_1(x_1) \\ W_2(x_1, x_2) &= W_2(x_2) \end{aligned}$$

   Using the result derived in Part 2, the optimal policy is the solution to the following problem:

$$\arg\max_{x_1, x_2} a(\beta W_0^1(x_1) + (1-\beta)W_0^2(x_2)) + (1+a)W_1(x_1) + (1+a)W_2(x_2)$$

   The first-order conditions are:

$$\begin{aligned} a\beta W_0^{1\prime}(x_1^*) + (1+a)W_1'(x_1^*) &= 0 \\ a(1-\beta)W_0^{2\prime}(x_2^*) + (1+a)W_2'(x_2^*) &= 0 \end{aligned}$$

In order to proceed from here, I'll assume specific functional forms – qualitative insights would remain the same. Let's assume that

$$\begin{aligned} W_0^1(x_1) &= 1 - (x_1)^2 \\ W_0^2(x_2) &= 1 - (x_2)^2 \\ W_1(x_1) &= 1 - (1 - x_1)^2 \\ W_2(x_2) &= 1 - (1 - x_2)^2 \end{aligned}$$

That is, the disorganized group's most preferred policy in both dimensions is zero, the organized groups' most preferred policies in their respective dimensions are one, and the losses are quadratic. It is easy to derive that the equilibrium policy in this case is:

$$(x_1^*, x_2^*) = (\frac{1+a}{1+a+a\beta}, \frac{1+a}{1+a+a(1-\beta)})$$

That is, the equilibrium policies are between zero and one, and they depend on the relative weights of the groups as well as the salience of the policy dimensions. An increase in β decreases x_1^* and increases x_2^* That is, the equilibrium policy in the more salient dimension gets closer to the unorganized group's bliss point and moves further away from the organized group's bliss point. The opposite effect is observed in the less salient dimension.

It is difficult to pin down the change in contributions – it is because we know that $\frac{\partial W_i(\mathbf{x})}{\partial x_j} = \frac{\partial \hat{C}_i(\mathbf{x})}{\partial x_j}$ holds at $\mathbf{x} = \mathbf{x}^*$, but not necessarily for all $\mathbf{x}$, and the change in β also changes the contribution schedules possibly. One thing we can say is that the welfare level of first organized group decreases. It seems like the contribution schedule $\hat{C}_i(\mathbf{x}) = W_i(\mathbf{x})$ is one of the SPEs – in that case, in equilibrium, the contribution of first lobby decreases and the contribution of second lobby increases; but I'm not sure how robust this insight is.

Question 4:

Consider the following regressions. In each case, explain the reasoning and criticize it. Feel free to elaborate as much as you like, in particular, giving suggestions of how you would improve on the empirical strategy.

Of course there is not a particular answer we're looking for in this question. Below are the discussions provided by an earlier TA, just for the sake of reference.

A) *A researcher wants to find out whether greater ethnic fragmentation leads to worse political decisions. For this reason, she runs a regression of the fraction of local government revenues in U.S. cities spent for education on an index of ethnic diversity in the city.*

The most basic problem with this regression is that it has no clear welfare implications. In particular, suppose we find a negative coefficient, and interpret this as evidence in favor of the hypothesis. Does this imply that the optimal share of local government revenues spent on education is 100 per cent, with none going to rubbish collection, police or other services? Presumably not, and so there is no clear way of interpreting any of the results. Another problem with interpretation is the normalization of spending as a share of government revenues. For example, perhaps ethnically diverse neighborhoods do not distort their overall spending on education, but raise insufficient taxes.

Standard endogeneity problems also arise. If people have an exogenous taste for homogeneity, perhaps diverse neighborhoods are also poor neighborhoods, and poor neighborhoods make worse political decisions.

B) *A researcher wants to find out whether common (British) law leads to better political outcomes. For this reason, he runs a regression of an index of corruption on a dummy for having common law rather than French civil law or German legal code.*

The key problem with interpreting this regression is that a country's legal system is essentially perfectly correlated with its colonial power. So the regression has very little explanatory power as against any other difference between former British colonies and former colonies of other powers. To the extent that these variables are not perfectly collinear, presumably they may well differ for endogenous reasons. For instance, if common law is so much better than civil law, perhaps the best organized countries were the ones that adopted common law.

C) *Another researcher wants to answer the same question, and he runs a regression of an index for corruption on a dummy for having common law, and instruments this using a dummy for having been a British colony.*

This researcher is trying to address the endogeneity problem raised above. But the first problem remains or is even exacerbated - now we are really assessing the effect of having been a British colony, and this tells us little in itself about common vs civil law. Moreover, the suggested instrument is clearly not valid, as there could be many channels through which being a British colony in the past affects current performance.

D) *A researcher wants to investigate the relationship between democracy and inequality, so he runs a regression of various measures of democracy on measures of inequality.*

This is a classic case of endogeneity bias. We have seen plenty of examples of how greater inequality may lead to less democratic institutions, as well as how greater democracy may lead to less inequality. So without an instrument there is no causal interpretation of this regression.

E) *A researcher wants to investigate whether political instability in a country's neighbors has a negative effect on economic performance. So he runs a regression of log income on a variety of controls, an index of political instability in the country, and the average of the index of political instability among the country's neighbors.*

This regression is laden with endogeneity problems. To start with, political instability within a country and log income are jointly determined. But if any variable on the right-hand side of a regression is endogenous, none of the coefficients in the regression can be interpreted causally, even if the other variables are exogenous. Note that the potential joint determination of own instability and neighbors' instability need not itself be a problem.

F) *A researcher wants to investigate the relationship between inequality and growth, so he runs a regression of growth on initial inequality using cross-sectional data. He also runs a panel regression of growth in a five-year period on inequality during the five-year period, as well as country fixed effects and time effects.*

All of the problems of reverse causality that we have already discussed apply to the cross-sectional regression. The interpretation of the panel regression is not too clear. First, having a panel does not itself solve the sorts of reverse causality problems we have considered. Second, even if we were convinced of the identification strategy, many of the mechanisms linking inequality and growth act through channels like effects on institutions, which seem unlikely to be operative over a five year period.

G) *A researcher wishes to show that Downsian policy convergence fails, so runs the regression of economic policy and various economic outcomes on the identity of the party that is elected at the local level.*