Introduction to Political Economy 14.770
Problem Set 3 Solutions

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Question 1:

Consider the all-pay auction model of lobbying.

Suppose there are N lobbies competing to get the politician's support to have the legislation in their favor. Assume that the value of having legislation in one's favor is worth $\bar{x}$ for each lobby. Each lobby makes a contribution the politician before the legislation is decided, and the contribution is non-refundable. The lobbies don't observe other lobbies' contributions before the legislation passes. The politician passes the legislation in favor of the lobby which pays the highest contribution. (If there are multiple lobbies which pay the highest contribution, the politician decides randomly.) If a lobby pays x and gets the legislation in its favor, then its payoff is $\bar{x} - x$. If the legislation is not in one's favor, then the payoff is $-x$. For simplicity, normalize $\bar{x} = 1$.

1. *Assume $N = 2$. Does this game have any pure strategy Nash Equilibrium? Explain.*

   Claim 1. *This game does not admit any pure strategy Nash Equilibrium.*

   Proof. For expositional simplicity, let the set of lobbies be $\{i, j\}$ and, to get a contradiction, let $(s_i, s_j) \in S_i \times S_j$ be pure strategy Nash Equilibrium. We'll cover two cases:

   (a) $\mathbf{s_i = s_j}$. In this case, i's payoff is: $u_i(s_i, s_j) = \frac{1}{2} - s_i$. Yet, lobby i can deviate to playing $s'_i = s_i + \varepsilon$ for $\varepsilon \in (0, \frac{1}{2})$. This would give a payoff of $u_i(s'_i, s_j) = 1 - s'_i = 1 - s_i - \varepsilon > u_i(s_i, s_j)$ to lobby i, thus it has a strictly profitable deviation. Therefore, this cannot be a Nash Equilibrium.

(b) $\mathbf{s_i \neq s_j}$. Without loss of generality, assume $s_i > s_j$. In this case, i's payoff is: $u_i(s_i, s_j) = 1 - s_i$. Yet, lobby i can deviate to playing $s_i' \in (s_j, s_i)$ and get a payoff of $u_i(s_i', s_j) = 1 - s_i' = 1 - > u_i(s_i, s_j)$ to Player i, thus she has a strictly profitable deviation. Therefore, this cannot be a Nash Equilibrium.

□

The argument is easily generalizable to the $N > 2$ case.

2. *Assume $N = 2$. Find a symmetric mixed strategy equilibrium where both lobbies randomize over possible contributions according to a c.d.f. $F(x)$.*

Again, for expositional simplicity, the set of lobbies be $\mathcal{I} = \{i, j\}$. Let i mix according to the c.d.f. $F_i : \mathbb{R}^+ \to [0, 1]$, and similarly for j.

Claim 2. *The following is a mixed strategy equilibrium of this game: $F_i = F_j = F$, where*

$$F(x) = \begin{cases} 0, & \text{if } x < 0; \\ x, & \text{if } x \in [0, 1]; \\ 1, & \text{if } x > 1. \end{cases}$$

In other words, the strategy profile where each lobby independently mixes according to the uniform distribution over $[0, 1]$ *is a Nash Equilibrium.*

Proof. We need to show two things:

(a) Given $s_j \sim F_j$, i is indifferent between any pure strategy in $[0, 1]$.

(b) Given $s_j \sim F_j$, i does not have a strictly profitable deviation: anything outside $[0, 1]$ gives a (weakly) lower payoff than those in $[0, 1]$.

The symmetry of the game then ensures that this is a Nash Equilibrium.

- To show the first point, we need to demonstrate that $\mathbb{E}_{s_j \sim F_j}[u_i(s_i, s_j)] = \mathbb{E}_{s_j \sim F_j}[u_i(s_i', s_j)]$ for each $s_i, s_i' \in [0, 1]$. Begin by remembering that:

$$\mathbb{E}_{s_j \sim F_j}[u_i(s_i, s_j)] = P_{s_j \sim F_j}\{s_j < s_i\} + \frac{1}{2} P_{s_j \sim F_j}\{s_j = s_i\} - s_i$$

But $P_{s_j \sim F_j}\{s_j = x\} = 0$ for each $x \in \mathbb{R}^+$, because the uniform distribution is atomless. Also, $P_{s_j \sim F_j}\{s_j < x\} = F_j(x) = x$ for each $x \in [0,1]$, so that it simplifies to:

$$\mathbb{E}_{s_j \sim F_j}[u_i(s_i, s_j)] = F_j(s_i) - s_i = s_i - s_i = 0$$

for each $s_i \in [0,1]$. It follows that $\mathbb{E}_{s_j \sim F_j}[u_i(s_i, s_j)] = \mathbb{E}_{s_j \sim F_j}[u_i(s_i', s_j)] = 0$ for each $s_i, s_i' \in [0,1]$.

- To show the second point, we need to demonstrate that $\mathbb{E}_{s_j \sim F_j}[u_i(s_i, s_j)] \leq \mathbb{E}_{s_j \sim F_j}[u_i(s_i', s_j)]$ for each $s_i \in S_i \setminus [0,1]$ and $s_i' \in [0,1]$. By the first point, we already know that $E_{s_j \sim F_j}[u_i(s_i', s_j)] = 0$ for each $s_i' \in [0,1]$, so we just need to show:
  $\mathbb{E}_{s_j \sim F_j}[u_i(s_i, s_j)] \leq 0$ for each $s_i > 1$.
  But this is easy to see, because given $s_j \sim F_j$, any $s_i > 1$ wins the auction for sure, so the payoff of lobby i is: $1 - s_i < 0$. The result follows.

□

3. *Now, consider a general case where N can be any integer. Find a symmetric mixed strategy equilibrium where each lobby (independently) randomizes over possible contributions according to a c.d.f. $F(x)$.*

   Let the set of lobbies be $\mathcal{I}$, with $|\mathcal{I}| = N$.

   Claim 3. *The following is a mixed strategy equilibrium of this game: $F_i = F$ for each $i \in \mathcal{I}$, where*

$$F(x) = \begin{cases} 0, & \text{if } x < 0; \\ x^{\frac{1}{N-1}}, & \text{if } x \in [0,1]; \\ 1, & \text{if } x > 1. \end{cases}$$

   Proof. Once again, we need to show two things. For lobby $i \in \mathcal{I}$,

   (a) Given $s_j \sim F_j$ for each $j \in \mathcal{I} \setminus \{i\}$, i is indifferent between any pure strategy in $[0,1]$.

   (b) Given $s_j \sim F_j$ for each $j \in \mathcal{I} \setminus \{i\}$, i does not have a strictly profitable deviation: anything outside $[0,1]$ gives a (weakly) lower payoff than those in $[0,1]$.

   The symmetry of the game then ensures that this is a Nash Equilibrium.

- To show the first point, we need to demonstrate that $\mathbb{E}_{s_{-i}\sim F_{-i}}[u_i(s_i, s_{-i})] = \mathbb{E}_{s_{-i}\sim F_{-i}}[u_i(s'_i, s_{-i})]$ for each $s_i, s'_i \in [0,1]$.
  Once again, since the hypothesized distribution is atomless, $P_{s_j\sim F_j}\{s_j = x\} = 0$ for each $x \in \mathbb{R}^+$ and $j \in \mathcal{I}\setminus\{i\}$. Also, the independent mixing ensures that $P_{s_{-i}\sim F_{-i}}\{s_j < x \textit{ for each } j \in \mathcal{I}\setminus\{i\}\} = \prod_{j\in\mathcal{I}\setminus\{i\}} P_{s_j\sim F_j}\{s_j < x\} = \prod_{j\in\mathcal{I}\setminus\{i\}} F_j(x)$ for each $x \in [0,1]$, so that it simplifies to:

  $$\mathbb{E}_{s_j\sim F_j}[u_i(s_i, s_j)] = \prod_{j\in\mathcal{I}\setminus\{i\}} F_j(s_i) - s_i = (F_j(s_i))^{N-1} - s_i = (s_i^{\frac{1}{N-1}})^{I-1} - s_i = s_i - s_i = 0$$

  for each $s_i \in [0,1]$. The first point follows.
- To show the second point, we need to demonstrate that $\mathbb{E}_{s_{-i}\sim F_{-i}}[u_i(s_i, s_{-i})] \leq \mathbb{E}_{s_{-i}\sim F_{-i}}[u_i(s'_i, s_{-i})]$ for each $s_i \in S_i \setminus [0,1]$ and $s'_i \in [0,1]$. By the first point, we already know that $E_{s_{-i}\sim F_{-i}}[u_i(s'_i, s_{-i})] = 0$ for each $s'_i \in [0,1]$, so we just need to show:
  $\mathbb{E}_{s_j\sim F_{-i}}[u_i(s_i, s_{-i})] \leq 0$ for each $s_i > 1$.
  But this is easy to see, because given $s_j \sim F_j$ for $j \in \mathcal{I}\setminus\{i\}$, any $s_i > 1$ wins the auction for sure, so the payoff of Player i is: $1 - s_i < 0$. The result follows.

□

4. *How do the equilibrium distributions change with N? Can you suggest an economic intuition on why the equilibrium changes in this way? Calculate the expected total contribution that politician receives. How does it change with N? Are more lobbies better or worse for the politician? What if the politician is risk averse/risk loving?*

   - The equilibrium distribution for an individual contribution when there are N lobbies First Order Stochastically Dominates the distribution when there are $N' > N$ lobbies. This intuitively means that higher individual contributions are less likely when there are more lobbies. The reason being: with more lobbies, each lobby's bid is more likely to be overbid by some other lobby. But since the payment is "certain" regardless of whether one wins or not, this means that there are smaller incentives to bid higher. The result is an equilibrium distribution with smaller contributions.

- The expected total contribution remains constant at 1, so a risk-neutral politician is indifferent between fewer or more lobbies. We answer *does* change when the politician has different attitudes towards risk, though. It seems like the equilibrium distribution for total contributions when there are N lobbies is a mean-preserving spread of the distribution when there are $N > N'$ lobbies. This means that a risk-loving politician prefers more lobbies, whereas a risk-averse politician prefers less lobbies. Heuristically, with more lobbies, there's a teeny-tiny probability of receiving a total contribution of N (1 from each lobby) – a risk-loving politician likes to take the risk of having this event even though it's not much likely.

Question 2:

Assume that there is a population of voters whose measure is normalized to 1 (indexed by $v \in [0, 1]$). Everyone has 1 unit of resources and have linear utility over goods.

There are 2 parties, and they make binding promises to voters concerning their policy conditional on winning the election. A party can:

- *Offer different taxes and transfers to different voters (it is possible to target resources to individuals), or,*
- *Offer to provide a public good (to all voters). The public good costs 1 unit of resources per head (i.e. requires taxing everyone fully), and generates a utility G for each voter.*

Each voter votes for the party who promises her the greatest utility. Parties maximize their expected vote share.

1. *Suppose $G > 2$. Show that the only equilibrium is one with both parties offering public goods.*

   Begin by observing that, by standard arguments, in any equilibrium both parties must receive a vote share of $\frac{1}{2}$ (otherwise the losing party can simply imitate the strategy of the winning party and secure a vote share of $\frac{1}{2}$).

   We begin by asserting that a party cannot offer redistribution in equilibrium. Because the total resources in the economy is 1, when a party offers redistribution, it can offer a payoff of at least G to at most $\frac{1}{G}$ people. Consequently, at least $1 - \frac{1}{G}$ people receive a payoff less than G under any redistribution scheme. But then the other party secures a vote share of $1 - \frac{1}{G} > \frac{1}{2}$ by offering public good. By the argument in the previous paragraph, then, we cannot have an equilibrium when redistribution is offered with nonzero probability.

It is clear, on the other hand, both parties offering public good with probability 1 in equilibrium. In this equilibrium both parties get a vote share of $\frac{1}{2}$. If a party deviates to offering redistribution, it can offer a payoff of at least G to at most $\frac{1}{G}$ people. Since $G > 2$, $\frac{1}{G} < \frac{1}{2}$, implying that this cannot be a profitable deviation. We conclude that both parties offering public goods is an equilibrium. Combined with the discussion in the above paragraph, this is indeed the only equilibrium.

2. *Now suppose $G < 2$. Show that there is not an equilibrium in which a party offers the public good with probability one.*

   Suppose, to get a contradiction, that one of the parties offers the public good with probability one. As discussed above, each party must get $\frac{1}{2}$ of the votes in any equilibrium. But then the party not offering the public good can offer a redistribution scheme which yields $G + \epsilon < 2$ to $\frac{1}{G+\epsilon}$ voters. This would secure a vote share of $\frac{1}{G+\epsilon} > \frac{1}{2}$, implying that we have a strictly profitable deviation. We conclude that we cannot have an equilibrium in which a party offers the public good with probability one.

3. *Suppose $G < 2$. Show that there is not an equilibrium in which a party offers a transfer scheme in pure strategies, either. Conclude that there is no pure strategy equilibrium.*

   We know, by the previous part, that when $G < 2$ we must have redistribution being offered with nonzero probability. We now show that the redistribution cannot be in the form of a pure strategy – i.e. a party cannot offer a non-random redistribution scheme.

   Suppose, to get a contradiction, that one of the parties (say, Party i) offers a non-random redistribution scheme in equilibrium:

$$\Phi_i : [0,1] \to [0,\infty) \qquad \int_{v\in[0,1]} \Phi_i(v)dv = 1$$

   Notationwise, $\Phi_i(v)$ denotes the payoff of voter v under the offered redistribution scheme (i.e. it is the initial unit of resources v has plus/minus the transfers). Given Φ_i, the other party (say, Party j) can offer:

$$\Phi_j : [0,1] \to [0,\infty)$$

with

$$\Phi_j(v) = 0 \textit{ for } v \in V^*$$

where V^* is a set of measure ϵ of people people for whom $\Phi_i(v) > 0$, and

$$\Phi_j(v) = \Phi_i(v) + \frac{\int_{\tilde{v} \in V^*} \Phi_i(\tilde{v})}{1-\epsilon} \textit{ for } v \notin V^*$$

Heuristically, Party j can offer zero to an infinitesimally small set of people and redistribute their payoffs among the others, giving Party j a vote share of $1-\epsilon$. This would imply that Party j has a strictly profitable deviation. We conclude that we cannot have an equilibrium in which a party offers a transfer scheme in pure strategies. Consequently, when $G < 2$, there is no pure strategy equilibrium.

4. *Now, consider the case $G < 1$. Show that none of the parties offer public good in equilibrium. Find a symmetric mixed strategy equilibrium where each party offers each voter v a transfer drawn from a distribution with c.d.f. $F(.)$.*

   When $G < 1$, public good is not offered in equilibrium because it is *inefficient*: if one of the parties offers the public good (giving everyone a payoff of G), the other party would secure a win by offering no transfers (which would give everyone a payoff of 1). We conclude that both parties must offer redistribution and, following the previous part, the redistribution scheme must be random.

   As suggested in the question, we're looking for a symmetric mixed strategy equilibrium where each voter v receives an offer drawn from cdf $F(.)$ (note that there is no v subscript) and both parties use the same strategy ($F_i(.) = F_j(.) = F(.)$). Here, once again, the offer x given to a voter v represents to be the net payoff (which contains the initial unit of endowment and transfers). Consequently, any policy $F(.)$ offered by any party must satisfy the resource constraint:

$$\int_0^\infty x f(x) dx = 1$$

   We continue with the following claim.

Claim 4. *Suppose $G < 1$. The following is a mixed strategy equilibrium of this game: $F_i = F_j = F$, where*

$$F(x) = \begin{cases} 0, & \text{if } x < 0; \\ \frac{x}{2}, & \text{if } x \in [0, 2]; \\ 1, & \text{if } x > 2. \end{cases}$$

In other words, the strategy profile where each party independently mixes according to the uniform distribution over $[0, 2]$ *is a Nash Equilibrium.*

It is pretty straightforward to verify that this is an equilibrium, and the argument is similar to the one made in Question 1. (This is also the *unique* symmetric mixed strategy equilibrium, but it is beyond the scope of this question – see Myerson (1993) for a proof.)

To verify that this is an equilibrium, assume that Party i uniformly mixes in $[0, 2]$ and consider j's best response. If Party j offers a payoff $x \in \mathbb{R}^+$ to a voter v, it gets v's vote with probability $F(x)$. Consequently, if j adopts a distribution $F_j(.)$, its expected vote share is:

$$\begin{aligned} \int_0^x F(x) f_j(x) dx &\leq \int_0^x \frac{x}{2} f_j(x) dx \\ &= \frac{\int_0^x x f_j(x) dx}{2} = \frac{1}{2} \end{aligned}$$

Where the first inequality follows because $F(x) \leq \frac{x}{2}$ for all $x \in \mathbb{R}^+$, and the last equality follows due to the resource constraint. But this implies that Party j cannot win more than $\frac{1}{2}$ of the votes using any strategy when i is uniformly mixing in $[0, 2]$. Clearly, using $F_j(.) = F(.)$ (mimicking i's strategy) yields an expected vote share of $\frac{1}{2}$; therefore, it is a best response and both parties using uniform distribution over $[0, 2]$ is an equilibrium.

5. *Now, consider the case $1 < G < 2$. Show that the public good must be provided with positive probability in equilibrium. Find a symmetric mixed strategy equilibrium where each party offers the public good with probability β, offers transfers with probability $1 - \beta$, and if it offers transfers, each voter v is offered a transfer drawn from a distribution with c.d.f. $F(.)$.*

We begin by showing that the public good must be provided with positive probability in equilibrium. Suppose, to the contrary, that both parties offer redistribution with probability one. In any symmetric equilibrium, the strategies specified in the previous part must be used, so both parties must be using the uniform distribution over $[0, 2]$ and getting half of the share. Nevertheless, each party has the alternative of offering the public good and securing $\frac{G}{2}$ of the votes when the other party is using the aforementioned randomization. When $G > 1$, this is a strictly profitable deviation, therefore implying that we cannot have such an equilibrium.

Claim 5. *Suppose $1 < G < 2$. The following is a mixed strategy equilibrium of this game: each party offers the public good with probability*

$$\beta_i = \beta_j = G - 1$$

and, conditional on offering redistribution, both parties use $F_i(.) = F_j(.) = F(.)$, where

$$F(x) = \begin{cases} 0, & \text{if } x < 0; \\ \frac{x}{2G}, & \text{if } x \in [0, 2-G]; \\ \frac{2-G}{2G}, & \text{if } x \in [2-G, G]; \\ \frac{x+2-2G}{2G}, & \text{if } x \in [G, 2]; \\ 1, & \text{if } x > 2. \end{cases}$$

In other words, each party uses a distribution with a hole in $[2-G, G]$ and otherwise mixes uniformly.

For a proof of this (and for the more general case with N parties), I encourage you to read Lizzeri and Persico (2005). (The whole point of this question was to induce you into reading the paper and make sure that you follow the argument!)

6. *For the case $1 < G < 2$, what is the probability that the public good is offered in equilibrium? Comment on what features of this model lead to the inefficiency result.*

   As derived in the previous part, the probability that a party offers the public good is $G - 1$. This is increasing in G –good news, since public goods are efficient in this region and the probability of public good being offered is increasing in its efficiency. Still, there is a certain probability $2 - G > 0$ of offering redistribution, which is inefficient from a utilitarian welfare perspective.

There are obviously many reasons why we end up in inefficiency result – this is meant to be an open ended question. Arguably the most important feature is the possibility of *targeted transfers*, which, as we've seen in class, leads to inefficiencies more often than not.

Question 3:

Consider the following model: There are two periods $t \in \{1, 2\}$. The discount factor is $\delta \in (0, 1]$.

A politician has a (persistent) type $i \in \{c, nc\}$, where c is corrupt and nc is noncorrupt. Each politician's type is drawn independently from an distribution with $Pr\{i = nc\} = \pi \in (0, 1)$.

In each period $t \in \{1, 2\}$, there is a state of the world $s_t \in \{0, 1\}$, privately observed by the politician. Each period, the state of the world is drawn independently from a distribution with $Pr\{s_t = 1\} = \frac{1}{2}$.

In each period $t \in \{1, 2\}$, the elected politician of type i observes the state s_t and picks a policy $e_t(s_t, i) \in \{0, 1\}$. The citizens have a payoff of

$$u_t(s_t, e_t) = \begin{cases} V, & \text{if } e_t = s_t \\ 0, & \text{if } e_t \neq s_t \end{cases}$$

Each period, a non-corrupt politician receives a payoff of

$$u_t^{nc}(s_t, e_t) = u_t(s_t, e_t) + \mathbf{1}_{\{in office at period t\}} W$$

Where $W > 0$ is the "ego rents" from being in the office in period t.

A corrupt politician's per period payoff is:

$$u_t^c(s_t, e_t) = \begin{cases} \mathbf{1}_{\{in office at period t\}} W, & \text{if } e_t = 0 \\ \mathbf{1}_{\{in office at period t\}} (r_t + W) & \text{if } e_t = 1 \end{cases}$$

where r_t is the "private benefit" from setting $e = 1$. Each period, r_t is drawn independently from a distribution $G(r)$ with mean μ and support $[0, R]$. The timing of the game is as follows:

i. *An incumbent politician is in the office. The incumbent's type is drawn, and she privately observes her type.*

ii. *s_1 is drawn and observed by the politician.*

iii. *If the incumbent is corrupt, r_1 is drawn and observed by the politician.*

iv. *The incumbent chooses e_1, and it is observed by the citizens.*

v. *Citizens decide whether to keep the incumbent or elect a new politician. If they elect a new politician, her type is drawn randomly from the same distribution.*

vi. *Citizens observe their payoffs from period 1.*

vii. *In the second period, s_2 is drawn and observed by the elected politician, (if she is corrupt) r_2 is drawn and observed by the politician, and the elected politician chooses e_2. Payoffs are realized.*

1. *What does this timing imply for the role of retrospective voting in this model? Is this timing a realistic assumption?*

   Retrospective voting means that voters simply look at their payoffs and condition their voting decisions on their payoffs. The timing of this game effectively rules out any possibility of retrospective voting, because it assumes that the payoffs are observed **after** the election. This is a crucial assumption for this model – without this timing, the results would not hold.

   Is it realistic? It depends. In many contexts, politicians make long-term decisions whose impacts are not yet observed in the election period, so this may be a good fit for those contexts. It may also be the case that the voters observe only a *noisy* signal of their payoffs (as in the "populism" model we covered in class) – this model is an extreme version where the noise is just too much and eats up all the informativeness of the signal.

2. *Find a Perfect Bayesian Nash Equilibrium of the game where*

   - *A non-corrupt incumbent picks $e_1 = 0$ regardless of s_1,*
   - *A corrupt incumbent picks $e_1 = 1$ only if r_1 is sufficiently high, and,*
   - *An incumbent is re-elected only if $e_1 = 0$.*

   This should be a fairly standard analysis by now, so I'll be brief. If you need a step-by-step derivation, you can check Section 3.4.3 of Besley (2006) or Recitation 4 notes (which cover a fairly similar model with different timing).

We'll check several optimality conditions for different agents in the economy, as well as the Bayesian updating whenever relevant.

- Assume that an incumbent is re-elected if and only if $e_1 = 0$ (the optimality of this behavior by voters will be verified later).
  - A **corrupt** incumbent with rent r_1 will find it optimal to pick $e_1 = 0$ if and only if
  $$r_1 \leq \delta(W + \mu) \tag{1}$$
  where the left-hand side is the "cost" of picking $e_1 = 0$ and the right-hand side is the gains from picking $e_1 = 1$ (and remaining in the office).
  - When state is $s_1 = 0$, a **non-corrupt** incumbent does not face any trade-off: she does the best for the voters and get re-elected anyway. She always picks $e_1 = 0$.
  - When state is $s_1 = 1$, a **non-corrupt** incumbent faces a trade-off, though. She finds it optimal to pick $e_1 = 0$ if and only if
  $$V + \delta V(\pi + (1-\pi)\frac{1}{2}) \leq \delta(W + V) \tag{2}$$
  where the left-hand side is the expected payoff of picking $e_1 = 1$ (thus getting a payoff of V today, getting kicked out of the office and getting V in the next period only if the replacing politician chooses the correct action) and the right-hand side is the gain from picking $e_1 = 1$ (foregoing the payoff for today, but getting ego rents and a payoff of V tomorrow).
- Assume that a a non-corrupt incumbent always chooses $e_1 = 0$ and a corrupt incumbent chooses $e_1 = 0$ iff $r_1 \leq \delta(W + \mu)$; therefore, the probability of a corrupt incumbent choosing $e_1 = 0$ is: $G(\delta(W + \mu))$. (The conditions for optimality of such behavior are already discussed.)
  - Bayes' rule suggests that upon seeing $e_1 = 1$, the voters immediately realize that the incumbent is corrupt:
  $$Pr\{i = nc | e_1 = 1\} = 0$$
  - Bayes' rule also implies that the beliefs about the incumbent's type upon seeing $e_1 = 0$ is given by:
  $$Pr\{i = nc | e_1 = 0\} = \frac{\pi}{\pi + (1-\pi)G(\delta(W + \mu))} \tag{3}$$

- Taking the incumbents' strategies as given, Bayesian voters replace the incumbent if and only if the expected payoff from replacing the incumbent is larger than keeping her:

  $$Pr\{i = nc|e_1\} \geq \pi$$

  where the left hand-side is the probability that the incumbent will work in voters' interest in the second period, and the right hand-side is the probability that the replacing politician will favor the voters.

  - Since $Pr\{i = nc|e_1 = 1\} = 0$ under incumbents' strategies, this inequality is never satisfied, so replacing an incumbent who picks $e_1 = 1$ is optimal.
  - Since $Pr\{i = nc|e_1 = 0\} = \frac{\pi}{\pi+(1-\pi)G(\delta(W+\mu))}$ under incumbents' strategies, this inequality is always satisfied, so re-electing an incumbent who picks $e_1 = 0$ is optimal.

3. *When does a corrupt incumbent choose $e_1 = 0$? What is the ex ante probability of this event? How does it depend on W, μ and δ?*

   We've provided a partial discussion for this above. Under the proposed equilibrium, a corrupt incumbent chooses $e_1 = 0$ iff

   $$r_1 \leq \delta(W + \mu)$$

   The probability of this event is:

   $$G(\delta(W + \mu))$$

   This probability is increasing in δ and W – if a corrupt incumbent values future more or if she cares about being in office more, then she foregoes the current rents more easily.

   It's hard to say something definitive about μ, because we're also changing $G(.)$ once we vary μ. In general, if the corrupt politician expects higher rents in the future, she foregoes the current rents more easily.

4. *What is the condition on non-corrupt incumbent's period one incentives to sustain such an equilibrium? How does it depend on V, W, δ and π? Discuss.*

   The condition is given in equation (2), which I'll rewrite as:

   $$\frac{V}{W + V} \leq \frac{\delta}{1 + \delta\frac{1+\pi}{2}}$$

One thing is clear from this equation: we need $W > 0$ for this inequality to hold – the non-corrupt incumbent should also care about being in the office. In general, the more she cares about being in the office (higher W), the easier it is to have an equilibrium where the non-corrupt politician panders to the voters to stay in the office.

- This inequality is more difficult to satisfy with larger V. Heuristically, when the disutility of taking the wrong action is higher, the non-corrupt politician is more less easily convinced to pander to the voters by choosing the "wrong" action.
- This inequality is easier to satisfy when δ is higher: the more politician cares about the future, the more she cares about staying, so she panders to the voters.
- This inequality is more difficult to satisfy when π is higher: when a non-corrupt politician expects the replacement to be non-corrupt with higher probability, she doesn't worry much about being replaced, so she takes the "correct" action even though she knows she'll be replaced.

Question 4:

Consider the the alternating-offers bargaining model of by Rubinstein (1982), which we covered in Lecture 11. We'll denote Player 1's share as $x_1 \in [0,1]$ *and Player 2's share as* $x_2 \in [0,1]$*, so that* $x_1 + x_2 = 1$.

(Warm-Up). *First, consider the* ultimatum bargaining *game. Player 1 moves first and offers* $x_1 \in [0,1]$*. After observing the offer, Player 2 either accepts (Y) or rejects (N). If Player 2 accepts, the payoffs are* $(x_1, 1 - x_1)$*. If she rejects, the game ends with payoffs* $(0,0)$*. Find the backward induction equilibria of this game. (For simplicity, you can assume that a player accepts an offer when she is indifferent between accepting and rejecting.)*

1. *Now, take it one step further and assume there are two periods in which players can make offers. Once again, Player 1 begins by offering* $x_1 \in [0,1]$ *and Player 2 either accepts (Y) of rejects (N). If Player 2 accepts, the payoffs are* $(x_1, 1 - x_1)$*. If Player 2 rejects, then Player 2 moves to offer* $x_2 \in [0,1]$*. In this case, Player 1 responds by either accepting (Y) or rejecting (N). If Player 1 accepts, the payoffs are* $(\delta(1 - x_2), \delta x_2)$*, where* $\delta \in (0,1)$*. If Player 1 rejects, then the game*

ends with payoffs $(0,0)$*. Find the backward induction equilibria of this game.*

2. *Now, generalize the result to* $T \geq 2$ *periods. Player 1 makes offers in odd periods and Player 2 makes offers in even periods. Receiving a share of* x_i *in period* t *gives a payoff of* $\delta^{t-1} x_i$ *for player* $i \in \{1,2\}$*. Assuming* T *is even, find the payoff vectors in subgame perfect equilibrium.*

3. *What is the payoff vector if* T *is odd?*

4. *Comparing the results in parts 3 and 4, you should be able to observe the phenomena called* last-mover advantage *and* first-mover advantage*. Can you observe how they are reinforced/weakened as* $T \to \infty$ *and* $\delta \to 1$*? Can you offer an economic intuition on why the changes occur that way?*

See the solutions in the supplementary document. This used to be a question for a class on game theory, so the solutions are more detailed. I didn't expect you to be as rigorous as the solutions suggested. That being said, being more rigorous is always a dominant strategy!