Introduction to Political Economy 14.770
Problem Set 6 Solutions

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Question 1:

Consider the Banerjee, Hanna, and Mullainathan model of corruption. Assume there are two types of agents in the economy, group 1 *with size* N_1 *and group* 2 *with size* N_2*. There is a good that the central government owns and the total measure of this good is 1. The social value of assigning an agent of group 1 the good is* H *and the social value of assigning the agents of group two the good is* L*, and assume that* $H > L$*. There is a misalignment between the social valuation and the private valuation. In particular, assume that the private valuation and the ability to pay for an agent in group 1 are* h *and* y_1 *respectively, and for group 2 are* l *and* y_2 *with* $h = y_1 < l = y_2$*. There is scarcity in the good to be allocated, in particular assume that* $N_1 = 1$ *and* $N_2 > 1$*. The agents' type is private information.*

Assume also that there is a detection procedure in which by testing t *units of time you can find the agent's type with probability* $\phi(t)$ *where* $\phi'(t) > 0$*. The cost of testing is zero for the bureaucrat, and* δ *per unit of testing for both the social planner and agents of both types.*

Finally, assume that bureaucrats vary according to their cost of being corrupt. In particular, if a given bureaucrat pays a fixed cost γ*, he can be corrupt and ignore whatever rules the government sets. Bureaucrats are distributed according to the cdf* $F(\gamma)$*.*

1. *Define allocative efficiency in this case.*

   Since we have that the social value of an agent of group 1 getting the good (H) is higher than the social value of an agent of group 2 getting the good (L) we have that allocative efficiency implies that only people in group 1 get the good.

2. *Suppose the government can set pairs (p_i, t_i) of prices and testing for each type. Let π_i be the probability that a member of group i gets the good (this is not related to the issue of testing and getting 'found out', but rather that the good may be rationed randomly given limited supply). Write down the IR (individual rationality/participation constraint) and IC (incentive compatibility) constraints for both types. Assume that if testing reveals an agent to be lying then that agent does not get the good.*

$$\pi_1 (h - p_1) - \delta t_1 \geq 0 \ \ (IR - 1)$$
$$\pi_2 (l - p_2) - \delta t_2 \geq 0 \ \ (IR - 2)$$
$$\pi_1 (h - p_1) - \delta t_1 \geq (1 - \phi(t_2)) \pi_2 (h - p_2) - \delta t_2 \ \ (IC - 1)$$
$$\pi_2 (l - p_2) - \delta t_2 \geq (1 - \phi(t_1)) \pi_1 (l - p_1) - \delta t_1 \ \ (IC - 2)$$

Note that in the IC we have implicitly used that if tested and found to be lying you don't get the good. This is the optimal implementation since this is the one that deters disguising behavior the most. All that adding extra positive elements to the left in the IC can do is making a more efficient implementation more difficult.

3. *Solve for the winner pay mechanism that the social planner would use to maximize allocative efficiency in this economy. (You can ignore the possibility of corrupt bureaucrats for now – assume that the rules are followed.)*

As discussed in the answer to part 1, for allocative efficiency we need $\pi_1 = 1 - \pi_2 = 1$. Then, all we have to care about is that

- people in 1 want to demand the good (IR-1)
- people in 2 do not want to impersonate people from 1 (IC-2), and
- people in 1 have enough funds to pay for the good (BC-1)

Then there are three constraints that have to be taken into account:

$$h - p_1 - \delta t_1 \geq 0 \ \ (IR - 1)$$

$$0 \geq (1 - \phi(t_1)) (l - p_1) - \delta t_1 \ \ (IC - 2)$$

$$p_1 \leq h \ \ (BC-1)$$

Clearly the price constraint (BC-1) will be slack to satisfy IR-1, so we can drop this. And since testing has a social cost of δ per unit, the social planner would like to minimize the testing, which means then that IC-2 will be binding. Otherwise, we could lower t_1 which would not affect IR-1 and BC-1, and improve welfare. In addition, IR-1 will always be binding since, otherwise, we could increase p_1, lower t_1 and improve social welfare. Hence, optimal p_1 and t_1 are given by

$$t_1^* : \ (l-h) = \delta \frac{\phi(t_1^*)\, t_1^*}{(1-\phi(t_1^*))}$$

$$p_1^* = h - \delta t_1^*$$

which satisfies $p_1 \leq h$ since $h - p_1 = \delta t_1^* \geq 0$.

4. *Solve for the winner pay mechanism that a corrupt bureaucrat will use in order to maximize his profits.*

   The corrupt bureaucrat wants to sell the entire measure one of goods to the highest bidder, which in this case will be the agents in group 2, who have a willingness and ability to pay of l. The bureaucrat can then maximize profits by charging $p_2 = l$, with no testing.

5. *Suppose that the social planner sets the rule (p_1^*, t_1^*) as the one that maximizes allocative efficiency. If the bureaucrat is not corrupt, he keeps all prices paid under this rule; but, if he pays the corruption cost γ, he can set his own rule and keep all profits. What is the level of γ that makes a bureaucrat indifferent between being corrupt or not?*

   If the bureaucrat is corrupt he gets $l - \gamma$. If he is not corrupt he gets $p_1^* = h - \delta t_1^* = l - \delta \frac{\phi(t_1^*) t_1^*}{(1-\phi(t_1^*))} - \delta t_1^* = l - \frac{\delta t_1^*}{(1-\phi(t_1^*))}$. It follows that the indifferent bureaucrat has

$$\gamma_I = \frac{\delta t_1^*}{1-\phi(t_1^*)}$$

   Note that, because $\phi' > 0$, the larger the t_1^*, the larger the γ_I, and the more likely that a given bureaucrat is corrupt. Mathematically,

$$\frac{\partial \gamma_I}{\partial t_1^*} = \delta \frac{(1-\phi(t_1^*)) + t_1^* \phi'(t_1^*)}{(1-\phi(t_1^*))^2} > 0$$

   which shows that the more testing in equilibrium, the larger the proportion of bureaucrats that want to be corrupt.

6. *Assume citizens are randomly matched to bureaucrats, whose costs γ are distributed $F(\gamma)$. What is the average level of testing in the economy amongst those in group 1? What is the fraction of corrupt bureaucrats in the economy? How do average testing and corruption vary as the misalignment level varies $(l-h)$?*

   The fraction of corrupt bureaucrats in the economy that will do no testing is $F\left(\frac{\delta t_1^*}{(1-\phi(t_1^*))}\right)$ and the fraction of honest bureaucrats that will do t_1^* testing is then $\left(1-F\left(\frac{\delta t_1^*}{(1-\phi(t_1^*))}\right)\right)$. The average level of testing in the economy is $t_1^*\left(1-F\left(\frac{\delta t_1^*}{(1-\phi(t_1^*))}\right)\right)$.

   Since $(l-h)=\delta\frac{\phi(t_1^*)t_1^*}{(1-\phi(t_1^*))}$, the larger the misalignment level $(l-h)$ the larger t_1^* because $\phi'>0$. In turn, as seen above, the larger t_1^*, the larger the γ_I, which implies a larger share of corrupt bureaucrats. Mathematically,

$$\frac{\partial t_1^*}{\partial(l-h)}=\frac{(1-\phi(t_1^*))^2}{\delta(\phi(t_1^*)(1-\phi(t_1^*))+\phi'(t_1^*)t_1^*)}>0$$

$$\frac{\partial\gamma_I}{\partial(l-h)}=\frac{\partial\gamma_I}{\partial t_1^*}\frac{\partial t_1^*}{\partial(l-h)}>0$$

   The result is intuitive: as misalignment increases the price charged by the honest bureaucrat decreases and hence corruption becomes more profitable. Average testing could increase or decrease with misalignment since t_1^* goes up but the number of tested individuals goes down.

7. *Next, consider the case where there is no social misalignment, i.e., $h\geq y_1>y_2\geq l$. Also assume $N_1<1$. What are the socially efficient prices and testing levels in this case?*

   In this case allocative efficiency has $\pi_1=1$ and $\pi_2=(1-N_1)/N_2$. Since testing is socially inefficient we would like to separate by charging prices. Charging $p_1=l+\epsilon$ (and $p_2=l$), people in group 2 would not be able to disguise themselves as people of group 1 because they do not want to pay more than l. People in group 1 would be willing to pay an ϵ more to secure the good and hence would have no incentive to disguise themselves as people of group 2. To check this

$$(IC-1):\ h-p_1\geq\left(\frac{1-N_1}{N_2}\right)(h-p_2)\Rightarrow h-l-\epsilon\geq\left(\frac{1-N_1}{N_2}\right)(h-l)$$

which is trivially satisfied for ϵ small enough. IC-2 is satisfied by the fact that people in group 2 do not want to pay more than l since l is their private valuation.

$$(IR-1):\ h - p_1 \geq 0 \Rightarrow h - l - \epsilon \geq 0$$

which is trivially satisfied for ϵ small enough since $h > l$.

$$(IR-2):\ \left(\frac{1-N_1}{N_2}\right)(l-p_2) \geq 0 \Rightarrow 0 \geq 0$$

$$(BC):\ h \geq p_1 \Rightarrow h \geq l+\epsilon, l \geq p_2 \Rightarrow l \geq l$$

which is trivially satisfied for ϵ small enough since $h > l$.

Question 2:

In class we discussed the idea that there could be multiple equilibria in corruption based on the idea that the probability of detection decreases as more people in the economy are corrupt. However, there are many other theories that could generate multiple equilibria in corruption levels. Here are two examples:

- *Ability to bribe the enforcers - you can be corrupt if the police are corrupt, and the police are corrupt because the people that keep them honest are corrupt.*
- *Chance other party is honest - in any given transaction you don't know whether the other side is honest or corrupt. So the probability you are honest depends on your belief that the other party in the transaction is honest too.*

Pick one of these two stories - or some other story (not the one discussed in class) and:

1. *Write down a simple model that encapsulates that theory and generates multiple equilibria.*
2. *Discuss comparative statics of with respect to at least one parameter of your model that determine of when multiple equilibria are more or less likely to obtain in your model.*
3. *Discuss what your model implies for effective anti-corruption policy.*

Open question. Please let me know if you want to pursue your model further, discuss about your model & receive feedback!