

**Game Theory  
for  
Strategic Advantage**

**15.025**

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# Game Plan for Repeated Games

- Today: formalizing the intuition:  
**Cooperate if reward – punishment > temptation**
  - stick and carrot strategies
  - conditions for sustaining cooperation
- **Next week: make theory work for you**
  - Toyota & Johnson Controls, Inc. case
  - GE-Westinghouse case

The promise of future rewards (**carrots**) and the threat of future punishments (**sticks**) may provide incentives for good\* behavior today.

# Twice-Repeated Prisoners' Dilemma

- Simultaneous play in each period
- Maximize total payoff
- First-period outcome publicly observed

Pl. 1

Pl. 2

		Defect	Cooperate
Defect		(1, 1)	(5, 0)
Cooperate		(0, 5)	(4, 4)

- Use the past to coordinate future actions?
- Backwards induction: second period payoffs, roll back.
- NE in the second period?

# (Twice) Repeated Prisoners' Dilemma

PI. 2

PERIOD 1

PI. 1

	Defect	Cooperate
Defect	(1+1, 1+1)	(5+1, 0+1)
Cooperate	(0+1, 5+1)	(4+1, 4+1)

PERIOD 2

Prisoners'  
Dilemma

NE = (D, D)  
Payoffs: (1,1)

Prisoners'  
Dilemma

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Prisoners'  
Dilemma

NE = (D, D)  
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Prisoners'  
Dilemma

NE = (D, D)  
Payoffs: (1,1)

# (Twice) Repeated Prisoners' Dilemma

- Total payoffs matrix (from period 1's perspective)

Pl. 2

		Defect	Cooperate
<u>Pl. 1</u>	Defect	(2, 2)	(6, 1)
	Cooperate	(1, 6)	(5, 5)

## Unraveling from the back!

- Unique equilibrium in period 2
- First-period play cannot credibly affect the future
- True for all finitely-repeated PD (though weird)

# Any Hope of Cooperation?

- First Stage

Pl. 1

		<u>Pl. 2</u>	
		Defect	Cooperate
Pl. 1	Defect	(1, 1)	(5, 0)
	Cooperate	(0, 5)	(4, 4)

- Second Stage

Pl. 1

		<u>Pl. 2</u>	
		Trust	Don't
Pl. 1	Trust	(4, 4)	(0, 2)
	Don't	(2, 0)	(2, 2)

# PD + Stag Hunt

- *“Play **Cooperate** in Round 1. If your partner also chose **Cooperate**, play **Trust** in Stage 2. If your partner did not choose **Cooperate**, play **Don’t**.”*
- Key observation: how many Nash equilibria are there in the Stag Hunt have?
- Is the threat of not trusting credible?
- Can it be used to induce cooperation early on?

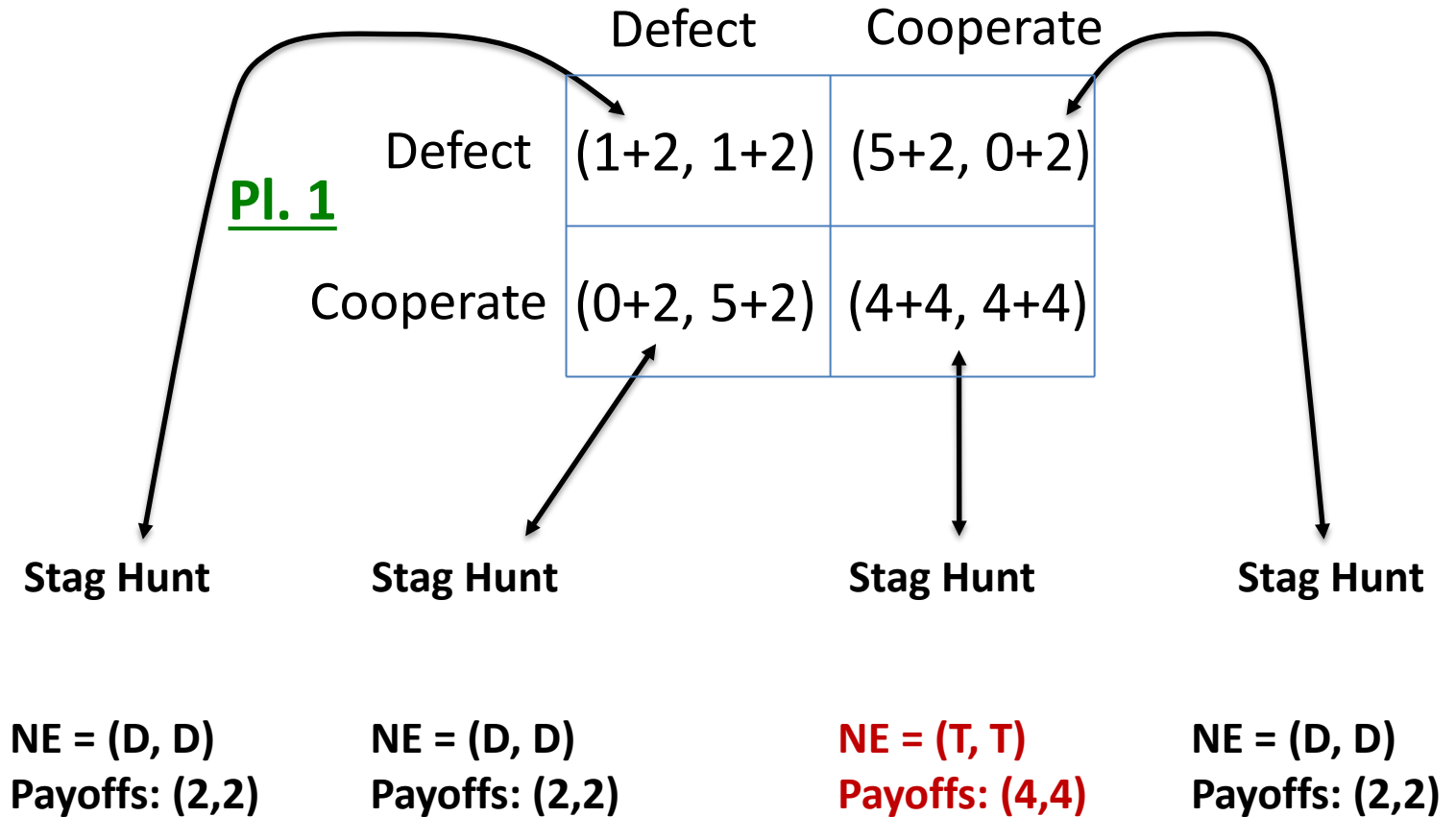


# PD + Stag Hunt

Pl. 2

PERIOD 1

PERIOD 2



# PD + Stag Hunt

Pl. 2

PERIOD 1

Pl. 1

	Defect	Cooperate
Defect	(3, 3)	(7, 2)
Cooperate	(2, 7)	(8, 8)

PERIOD 2

Stag Hunt

Stag Hunt

Stag Hunt

Stag Hunt

NE = (D, D)  
Payoffs: (2,2)

NE = (D, D)  
Payoffs: (2,2)

NE = (T, T)  
Payoffs: (4,4)

NE = (D, D)  
Payoffs: (2,2)

# Some Lessons

1. **History-independent play → guaranteed defect**
2. Future play must be variable (condition on the past)
3. Mutual defection (and distrust) may still be an equilibrium

## **Strategy may require playing a “bad” NE in Stage 2**

- Problem: *renegotiation / moral hazard / bailouts*
- Trade-off: *ex-ante efficiency vs. ex-post efficiency*

# Infinitely Repeated PD

- End-game effects were crucial
- What if no end game (or I don't know it)?
- Consider **infinite repetition** of this game

Pl. 2

		Defect	Cooperate
<u>Pl. 1</u>	Defect	(1, 1)	(5, 0)
	Cooperate	(0, 5)	(4, 4)

**How many possible strategies are there?**

# Strategies in Infinitely Repeated Games

Grim-trigger:

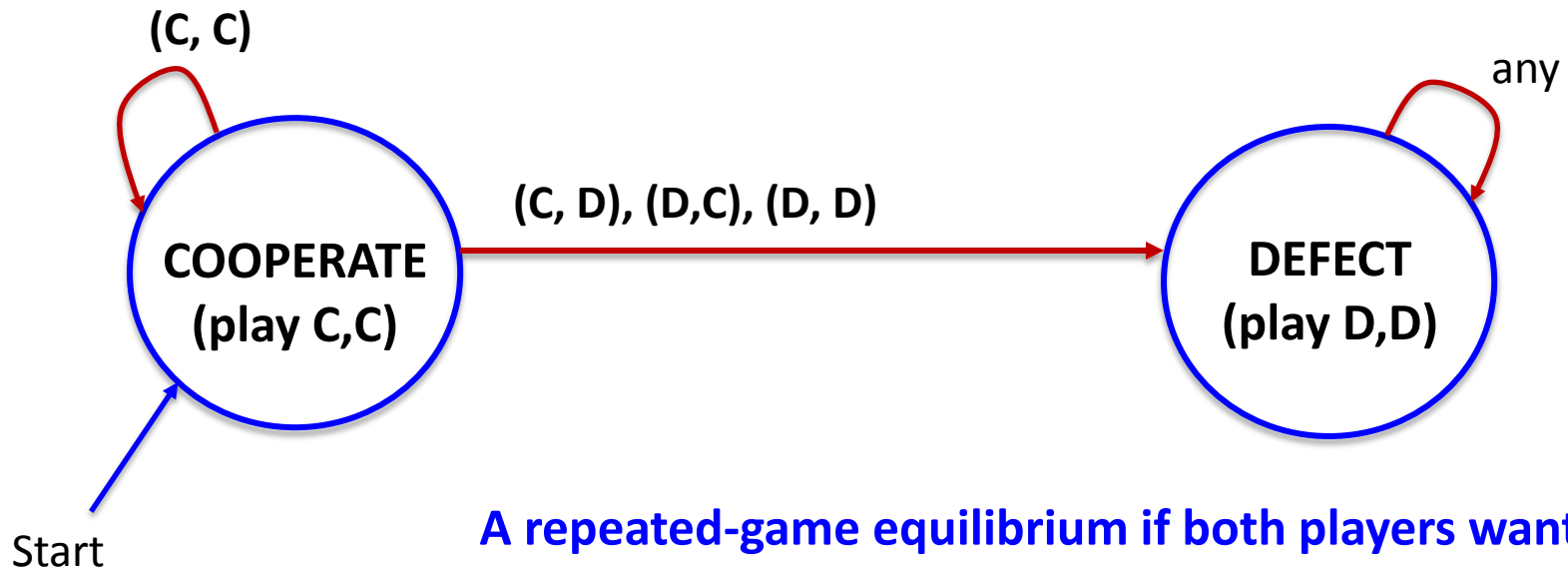
- Play Cooperate in the 1<sup>st</sup> period
- Play Cooperate if no-one has ever Defected
- Play Defect otherwise

Tit-for-tat:

- Play Cooperate in the 1<sup>st</sup> period
- Play Cooperate if your opponent Cooperated in previous period
- Play Defect otherwise

# Matrices, Trees, Machines

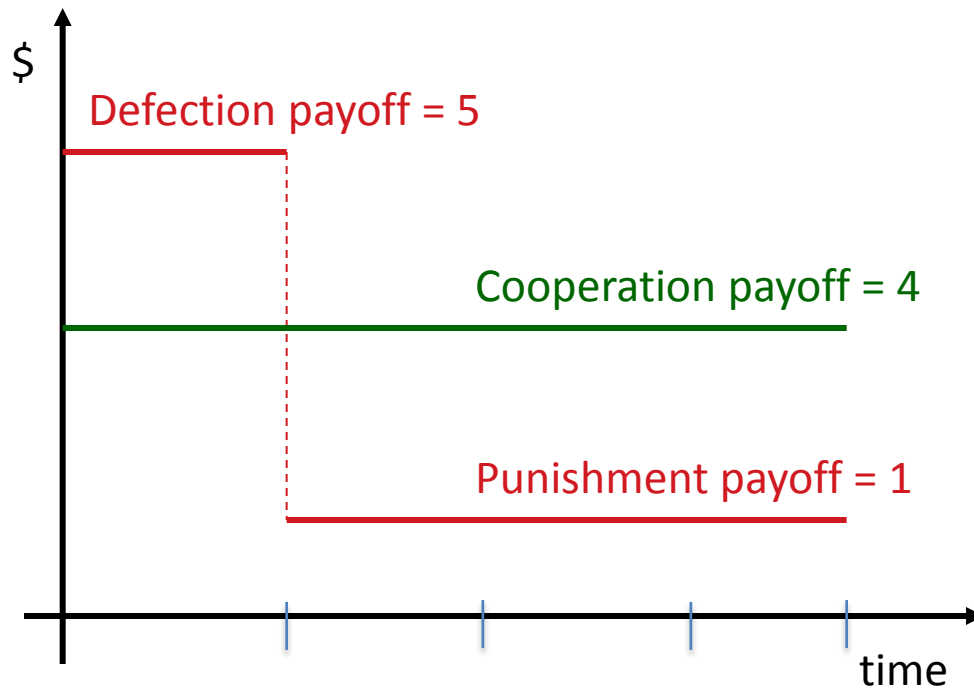
- Αυτοματων
- Best tool to represent repeated-game strategies
- Grim-trigger strategy



**A repeated-game equilibrium if both players want to follow the recommendation in each state**

# Payoff Comparison

Thinking about defecting...  
Which TOTAL payoff do you prefer?



		<u>Pl. 2</u>	
		Defect	Cooperate
<u>Pl. 1</u>	Defect	(1, 1)	(5, 0)
	Cooperate	(0, 5)	(4, 4)

4 forever vs. 5 once, 1 forever?

Depends on the interest rate!

# Putting Weight on Future

- Time preference / opportunity cost
- Probability of breakdown / reset / resample
- In all these cases, future payoffs matter less
- Interest rate  $r \rightarrow$   
 $1/(1+r) =$  **weight on tomorrow's payoff**



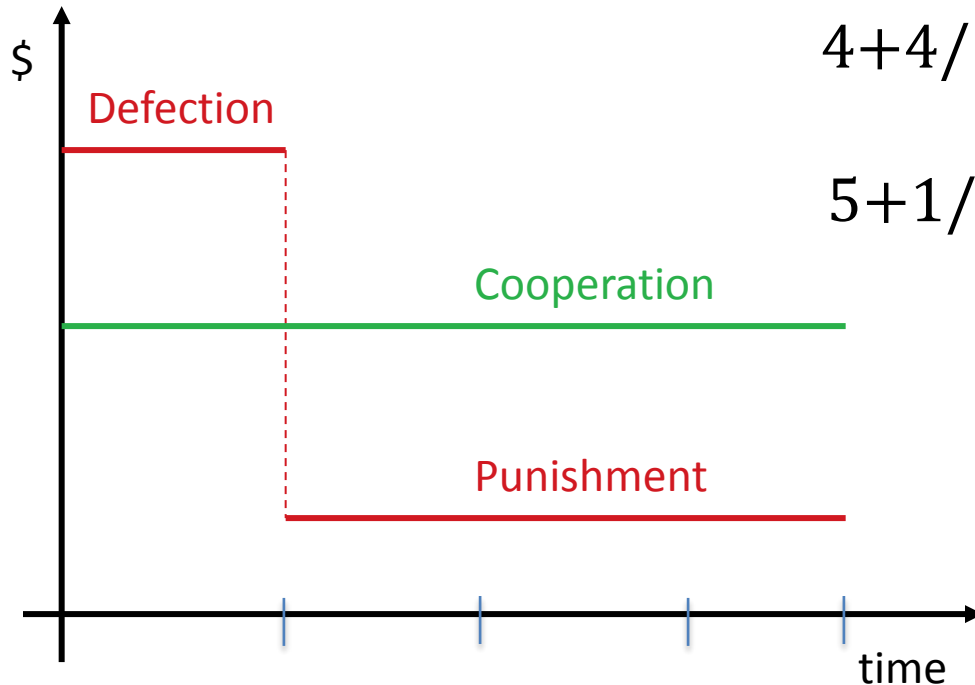
# Payoff Comparison

Which total payoff do you prefer?

Pl. 1

Pl. 2

		Defect	Cooperate
Pl. 1	Defect	(1, 1)	(5, 0)
	Cooperate	(0, 5)	(4, 4)



$$4 + 4/(1+r) + 4/(1+r)^2 + \dots = 4 + 4/r$$

vs.

$$5 + 1/(1+r) + 1/(1+r)^2 + \dots = 5 + 1/r$$

# Infinitely Repeated PD

Value of relationship after cooperation

– Value of relationship after cheating

> Gain if cheat

reward – punishment > temptation

tomorrow

today

- In our example, is it the case that

$4 + 4/r > 5 + 1/r$ ? Yes, if  $3/r > 1$ , or

$r < 3$

- Trigger strategies work if the future matters “enough”
- Converse: fix  $r$ , how much temptation can you tolerate?

In repeated games (as in all games),

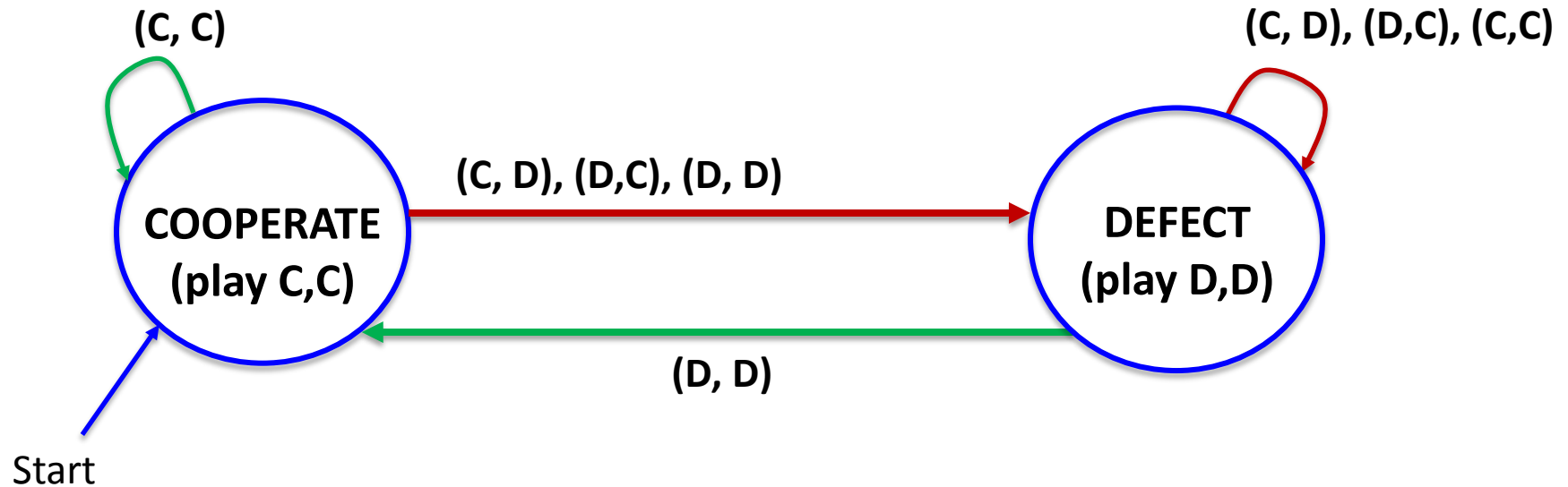
look forward & think back

The *shadow of the future*

helps sustain cooperation

# Less Severe Punishments

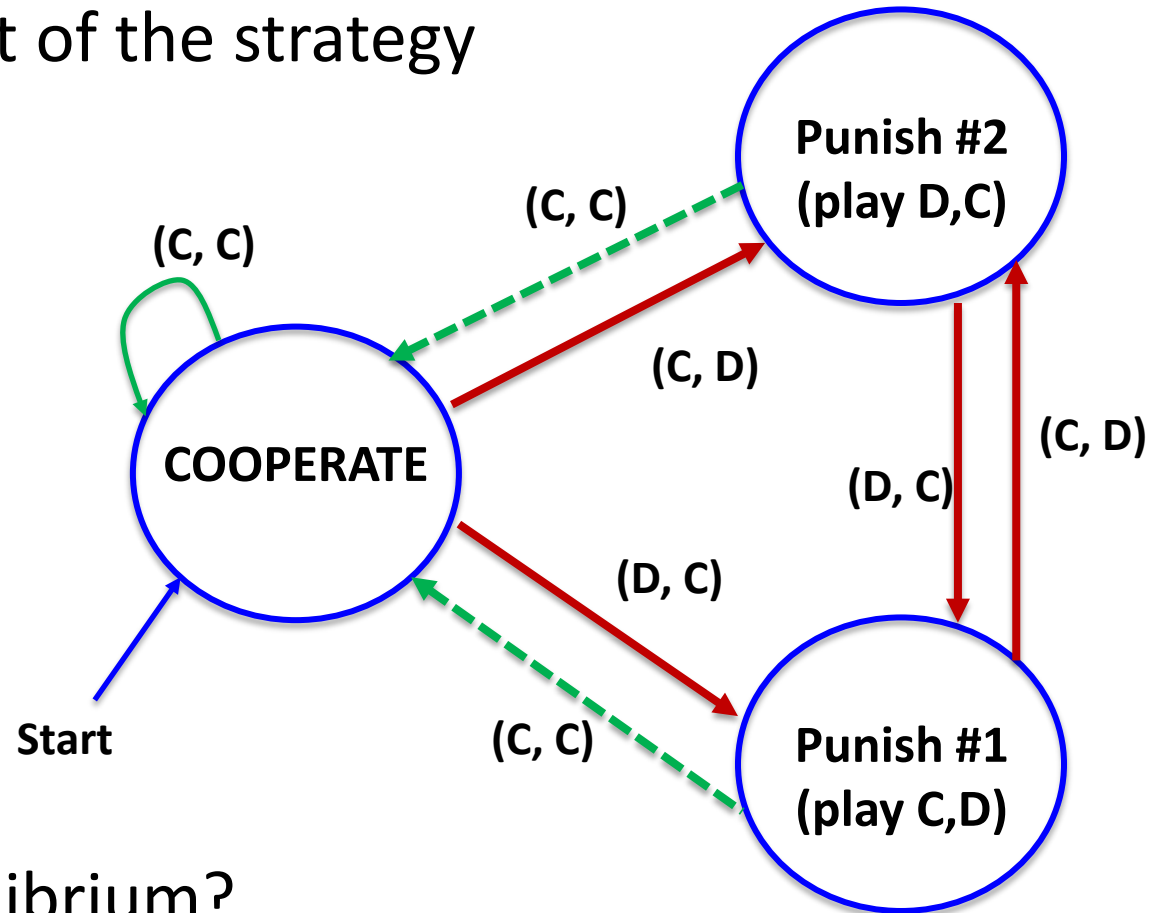
- One-period punishment



- Reward – Punishment =  $(4-1)/(1+r)$
- One-period temptation = 1
- Is this an equilibrium? Need  $(4-1) > 1+r \iff r < 2$
- Softer punishment  $\rightarrow$  harder to sustain cooperation

# Tit-for-Tat

- Draw only part of the strategy



- Is this an equilibrium?

# Main Takeaway

Threats, rewards and punishments  
must be **credible**

# What Makes Cooperation Easy / Hard ?

- Transparency +
- Similar players +
- Growing relationships +
  
- Transitory fluctuations -
- Permanent shocks -
- Number of players -

# Toyota & Johnson Controls Inc

- Why isn't Toyota vertically integrating the design and production of its car seats?
- What protects the small suppliers from Toyota's bargaining power, in the absence of a written contract? How can the parties trust each other?
- If demand is strong, and the need for a second assembly line comes up, should Toyota give the business of both assembly lines to JCI?



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