#### Game Theory for Strategic Advantage

#### <u>15.025</u>

Alessandro Bonatti

#### What Have We Learned So Far?

- You must account for your own and your opponents' rationality / sophistication
- In some games, it is appropriate (useful!) to exploit the logic of rationalizability

 Having the right game in mind is a source of competitive advantage. (Recall Epson vs. HP)

#### **Class 3 Game Plan**

1. Building a Language: *ask lots of questions!* 

2. Nash Equilibrium: *the Good, the Bad...* 

3. Prototypical Games: PD, Coordination, Chicken

#### A Structured Approach

- 1) Game theory is a **toolkit for strategic analysis**
- 2) Specify a game: payoffs represent total utility
- 3) Use all available information to describe the game
- 4) But once we are in the game, *we are in the game*.
- 5) Base the **analysis** on the game's elements alone

# More formally ...

A game is a multi-player decision problem:

- players
- strategies
- payoff functions (utilities)

i = 1, 2, ..., n

 $a_i$  from "feasible set"  $A_i$  $u_1(a_1, a_2)$ ,  $u_2(a_1, a_2)$ 

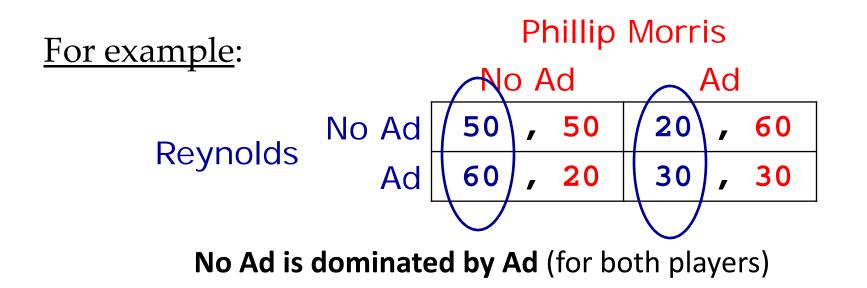
• typically:

 $A_{Row} = \{Top, Bottom\}$   $A_{Column} = \{Left, Right\}$ Payoff = <u>total utility</u> in "payoff matrix"

#### many more examples (like beauty contest)

# **Dominated Strategies**

<u>Definition</u>: Strategy *x* is *dominated* for player *i* if there exists another strategy *y* that guarantees a higher payoff to player *i*.



#### **Dominant Strategies**

A *dominant strategy* for player *i* **always** gives player *i* a **higher payoff** than any other strategy.

<u>For example</u>: placing yourself at the **median voter's** location (if all you care about is winning the election)

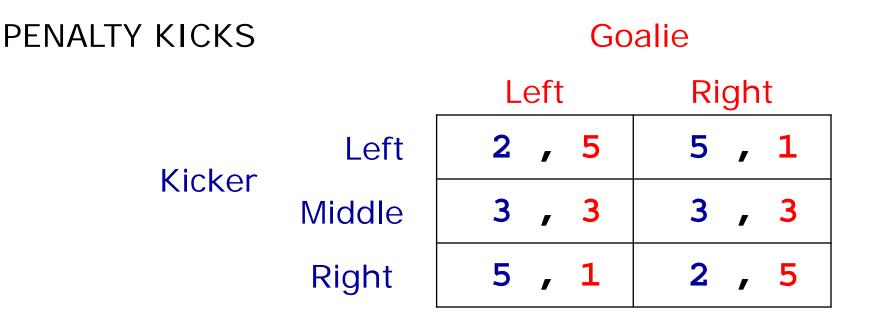
#### **Recall: Cigarette Ad Game**

# Phillip Morris No Ad Ad Reynolds Ad 50 , 50 20 , 60 Ad 60 , 20 30 , 30

Reynolds' best strategy is Ad <u>regardless</u> of what Philip Morris does

#### → Ad is a *"dominant strategy"*

#### **Dominance: a Risk-Free Concept**



#### Middle is not dominated for the Kicker! (but it's not a great idea either)

MIT Sloan 15.025 Spring 2014

#### Iterative Elimination of *Strictly* Dominated Strategies

Strategies that survive all rounds of elimination are called **rationalizable strategies** 

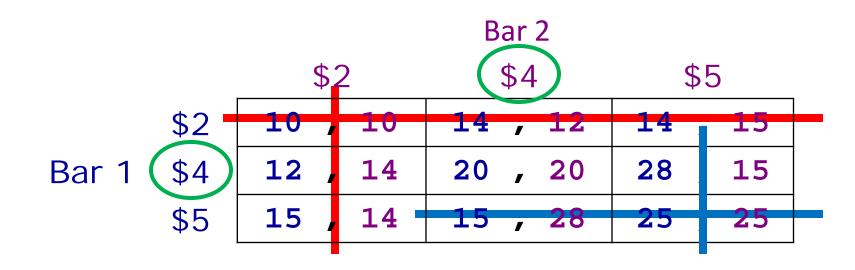
**Required assumptions:** 

- Know the game
- Rational player
- Rational opponents
- Knowledge of knowledge of ... of rationality

Order of elimination does <u>not</u> matter

# **Example: Tourists & Natives**

- Two bars can charge a price per drink of \$2, \$4, or \$5
  - 6,000 tourists pick a bar randomly
  - 4,000 natives select bar with lowest price
- Example: Both charge \$2
   − each gets 5,000 customers → payoff = \$10,000
- Example: Bar 1 charges \$4, Bar 2 charges \$5
  - Bar 1 gets 3,000+4,000=7,000 customers ( → \$28,000)
  - Bar 2 gets 3,000 customers (→ \$15,000)

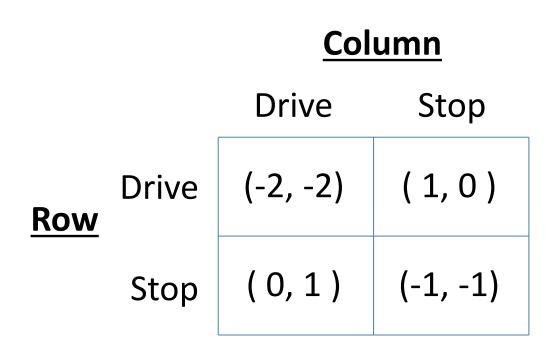


For each Bar, \$2 is dominated by both \$4 and \$5

# In the reduced game (with only 4 and 5), \$5 is dominated by \$4

#### (\$4,\$4) is the only rationalizable strategy profile

#### **Traffic Game**



#### **Rationalizable Outcomes = all four!!**

#### **Beliefs and Best Responses**

<u>Definition</u>: Player *i*'s *belief* about the strategy that *i*'s opponents will play is a *probability distribution* over their actions.

<u>Definition:</u> Strategy *x* for player *i* is a *best response* if *x* maximizes *i*'s *expected payoff*, given *i*'s beliefs.

<u>Example</u>: In the traffic game, "drive" is a best response if player 1 believes player 2 "stops" with probability >50%.

#### Best Responses: more advanced examples

- "Guess 0.75\*Average" → best response = 75% of your estimate of your opponents' average.
- Hide and Seek → best response = hide where you think your opponents are *least likely to search*, seek where you think they are *most likely to hide*.
- Product Positioning → best response = locate to the left/right of your competitor's expected position\*\*\*

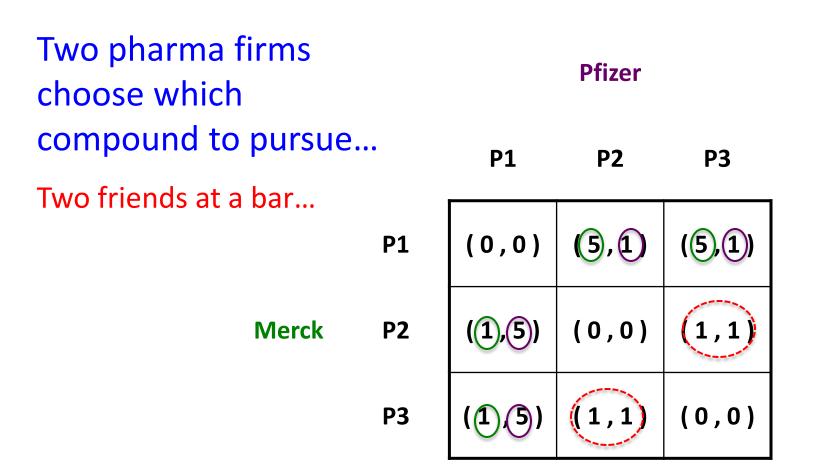
# Nash Equilibrium

<u>Definition</u>: A profile of strategies (i.e., one for each player) is a *Nash Equilibrium* if each player's strategy is a *best response* to the other players' strategies.

#### Examples:

- both firms locating their product at the center of the line;
- everyone choosing the number 1;
- player 1 driving and player 2 stopping.

#### In the movie





#### (\$4,\$4) is the only **rationalizable** strategy profile

#### (\$4,\$4) is also the unique **nash equilibrium**

Iterated elimination of dominated strategies might

yield a Nash equilibrium and cannot eliminate one.

Prof. Alessandro Bonatti

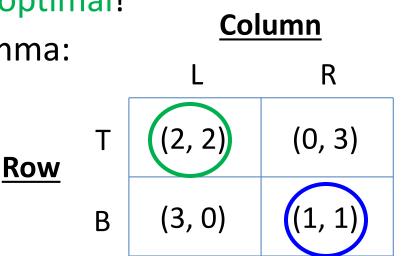
MIT Sloan 15.025 Spring 2014

# Nash Equilibrium: the Good

- It always exists (John Nash, 1950)
- Easy to find
  - For us
  - For firms (given enough time)
- It is "stable"
- A tool for out-of-sample predictions
- A criterion for investment decisions (next class)
  - What if demand ↗? ↘?
  - What if one firm cuts its costs?

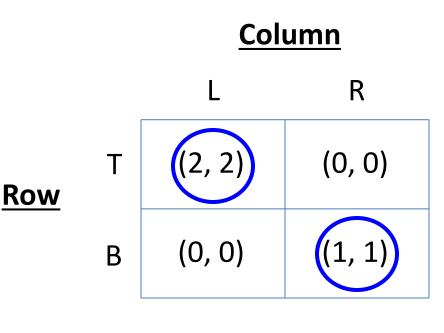
# Nash Equilibrium: the Bad

- Equilibrium does not mean optimal!
- Think of the prisoners' dilemma:
- Unique Nash equilibrium, but "Pareto-inefficient"



- Many interesting games have >1 Nash Equilibrium!
- Stability not-so-great anymore!

#### Nash Equilibrium: the Many



#### **Coordination Game**

- No dominated strategy for either player
- Two Pareto-ranked Nash equilibria
- Could have pathdependence!

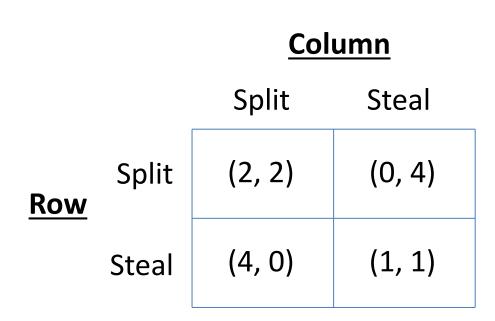
# **Selecting Nash Equilibrium**

• How to "steer the game"?

Commitment tactics! (Cigarettes, ice-cream vendors)

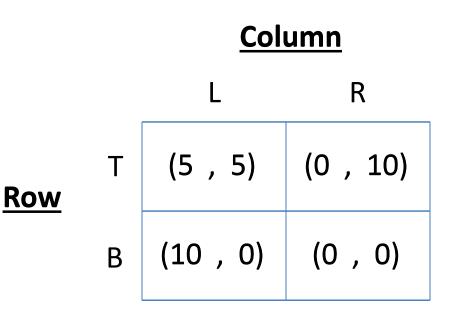
• Before then.... Do we have the **right game?** 

#### **The Last Chocolate**



- Dominant strategy for each player
- Seemingly selfevident solution to the game

#### **Tiny Details Matter**

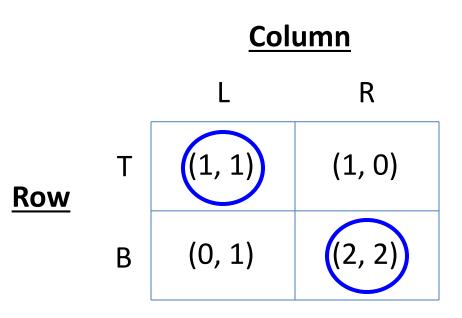


- Is B a dominant strategy for Row?
- Is R a dominant strategy for Column?
- Find all Nash Equilibria

Youtube **→** "<u>Best Split or Steal Ever</u>"

# Stag Hunt

(J.-J. Rousseau, 1754)



- Hunting stag vs. rabbit
- No dominated strategy for either player
- Two Pareto-ranked Nash equilibria
- The role of <u>trust</u>

#### **Example: Technology Adoption**

One upstream and one downstream firm

- Currently old inventory-management system → profit = \$1M each
- Each can independently invest \$1M in upgrading to a **new** system
- If only one upgrades, no one benefits
- If both upgrade, they exploit synergies worth \$3M (gross)
   What's the game?
   Old
- Dominated strategies? No
- What's the Prediction?
- Two Nash Equilibria:
- (Old,Old) and (New,New)
- This is a coordination game that may explain some interesting real world phenomena (mergers to coordinate on synergies)

Old

New

Downstream

(1.1)

(0,1)

New

(1,0)

(2,2)

# Takeaways: Building a Language

- 1. Dominant Strategy: performs better than all other strategies, regardless of opponents' behavior
- 2. Dominated Strategy: an alternative strategy always performs better, regardless of opponents' behavior
- 3. Rationalizable Strategies: survive the iterated elimination of dominated strategies
- 4. Best-Responses

#### 5. Nash Equilibrium

# **Building a New Approach**

- 1) Once in the game...
- 2) Assumptions -> Conclusions
- 3) Do we like the conclusions?

VES

Can we trust the assumptions?

Were the assumptions wrong? Or did we just learn something?

NO

- Who is the opponent?

- What is the *actual* game?

15.025 Game Theory for Strategic Advantage Spring 2015

For information about citing these materials or our Terms of Use, visit: http://ocw.mit.edu/terms.