Sloan School of Management
Massachusetts Institute of Technology
Spring 2004, H1

# Qwest Bond Swap (Revised 2/13/04 for clarity) 

## Due Date

This assignment is due at or before the beginning of class on February 19.

## INSTRUCTIONS

Turn in one solution per team.
All questions can be answered with just pencil-and-paper. Of course, you may choose to use a software program to help you (like Excel), but in this case please (i) provide documentation of what you did in your solution and (ii) email your program, spreadsheet, etc. to the TA.

Sloan School of Management
Massachusetts Institute of Technology
15.040-Game Theory for Managers

Professor David McAdams

# Team Assignment \#1: Qwest's Bond Swap 

For this article, please see:
Norris, Floyd. "A Bond Swap Available Only To Big Players,"
The New York Times, 18 December 2002, C1.

## Setup

In this homework we will examine this bond swap from a game theory point of view. In our (extremely simplified) model, each bondholder has one $\$ 1000$ Qwest bond. We ignore interest paid on the bonds and focus only on principal repayment. (We also ignore discounting.) We simplify matters further by

[^0]assuming that Qwest has just one issue outstanding so that, prior to the tender offer, all bondholders have equal repayment priority. That is to say, if Qwest turns out to have X dollars in assets to pay back its bond obligations, then:

- Each bondholder gets back full face value of $\$ 1000$ as long as $X \geq \$ 1.5$ Billion. (And Qwest does not go bankrupt.)
- Each bondholder only gets back $\$ 1000{ }^{*}\left(\frac{X}{1.5 \text { Billion }}\right)$ if $\mathrm{X}<\$ 1.5$ Billion. (And Qwest goes bankrupt.)

Qwest's tender offer allows any bondholder to trade her \$1000 bond for a $\$ 500$ bond that has repayment priority over those who do not tender. ( $\$ 545$ from the article has been replaced by $\$ 500$ for numerical simplicity.) That is to say, suppose that after the tendering process the face-value of all outstanding newly-issued bonds is Y dollars and the face-value of all outstanding old bonds is $Z$ dollars. Since each new $\$ 500$ bond-holder had to turn in a $\$ 1000$ bond, we know that $2^{*} \mathrm{Y}+Z=\$ 1.5$ Billion.

- If $Y<X$, then all newly-issued bonds are repaid full face-value of $\$ 500$ and each old bond is partially repaid $\$ 1000{ }^{*} \frac{X-Y}{Z}$.
- If $\mathrm{Y}>\mathrm{X}$, then old bonds get nothing and newly-issued bonds are partially repaid $\$ 500{ }^{*} \frac{X}{Y}$.


## Known Assets

In this section, we assume that there is no uncertainty about $X$, the value of assets that Qwest will have available to pay off its bonds.

1. Suppose that $X=\$ 500$ Million. Show that tendering is a dominant strategy for each bond holder.
2. Suppose that $X=\$ 1$ Billion. Show that NOT tendering is a dominant strategy for each bond holder.

## Uncertain Assets

Qwest was not bankrupt yet at the time of the NY Times article's writing. And even if bankruptcy was viewed as being highly likely, there was certainly substantial uncertainty surrounding the exact value of Qwest's assets in the event of bankruptcy and hence uncertainty over how much bondholders would be paid back.

In this section, we treat the value of Qwest's assets, X , as a random variable. To get at the important ideas of the analysis without getting bogged down in complications, we will assume that X has a very simple distribution:

$$
\operatorname{Pr}(\mathrm{X}=\$ 500 \text { million })=1-\mathrm{p}, \operatorname{Pr}(\mathrm{X}=\$ 1 \text { billion })=\mathrm{p}
$$

The important features of this setup: Qwest is certain to go bankrupt if it can not get bondholders to tender but, if all bondholders tender, Qwest will escape bankruptcy with probability p. (If all tender, the face value of its debt will be only $\$ 750$ million.) Note that in questions 1 and 2 you analyzed the special cases in which $\mathrm{p}=0$ and $\mathrm{p}=1$, respectively.
3. For which $p$ is it a dominant strategy to tender? Hint: You will find that it is a dominant strategy to tender as long as $\mathrm{p} \leq \mathrm{p}^{*}$ for some $\mathrm{p}^{*}<1$.
4. Your answer to question $\mathbf{3}$ shows that, ironically, Qwest can only hope to avoid bankruptcy through this tender offer if its chances are sufficiently small of escaping bankruptcy after a successful tender offer! Briefly explain (preferably without math) why this makes sense.

Up to this point, we have taken the terms of the tender offer as given. Suppose now that you are CFO of Qwest and must decide the terms of this tender offer. The CEO urges you to make the most favorable tender offer possible that is certain to succeed. (You interpret this as meaning that tendering should be a dominant strategy for bondholders.)
5. To be concrete, suppose that $\mathrm{p}=1 / 2$. What tender offer do you make?


[^0]:    ${ }^{1}$ Floyd Norris, while certainly an excellent reporter of facts, is not a game theorist. His analysis and conclusions may or may not be correct.

