## Agenda

## \&Practice Exam to help you prepare for Tuesday <br> \&Questions

## Practice Exam: Problem 1

$\mathscr{H}$ Autopower Europe trying to ship motors from the ports to the plants.
\&What are the basic variables in this optimal solution?
\&Sensitivity Report

## Basic Variables

HHow many should there be?
©Answer: 6
$\triangle$ Why: There are 7 constraints (3 ports, 4 plants), but one is redundant
\&What must the 6 basic variables be?
$\triangle$ The 6 with positive values
$\triangle$ Careful, sometimes basic variables have value 0, but they always have Reduced Cost of 0 . That's how reduced cost is defined.

## Question 2

## \&Is this the only optimal solution? \&Sensitivity Report

## Unique Optimum?

\&No.
\&How do we know: The reduced cost of Amsterdam to Leipzig is zero. We can move flow onto this edge without increasing the objective.
\&Careful, if some of the basic variables had been zero, we may not have been able to move any flow onto this edge.

## Question 3:

$\mathscr{H}$ One carrier would like to win business between Antwerp and Tilburg. At what price per unit would Autopower be willing to use this route?
\&Sensitivity Report

## Reduced Cost

$\mathscr{H}$ Current Cost per unit is $\$ 110$
\&Reduced Cost is $\$ 118$
$\mathscr{H}$ Each unit we send costs $\$ 110$ but decreases the objective value by $\$ 118$.
$\mathscr{H}$ The carrier would have to pay us $\$ 8$ for each unit we sent on this edge before it would be attractive!

## Question 4

\&If we could shift a small amount of production from one plant to another, all else being equal, what plant would it be best to move production from and what plant should we move the production to?
\&Sensitivity Report

## Shadow Price

${ }_{H}$ Move it from the plant with the highest shadow price and to the plant with the lowest shadow price.
$\mathscr{L}$ So, from Leipzig to Liege.
$\mathscr{E}$ The net effect should be to reduce the objective by 120-41 = \$79 for each unit we move.

## Modeling

$\&$ Demands for Autopower's products outstrip its available supply of motors. In fact, demand in Liege has risen to 300 . We estimate the cost of backordering (failing to meet current demand) at each plant to be:
\&Plant
\&Leipzig
\&Liege
\&Nancy
\&Tilburg

Cost per motor backordered \$50
\$70
\$30
\$100
15.057 Spring 02

## Backordering

## Autopower Transportation Model

## Unit Cost

| From/To | Leipzig |  | Nancy |  | Liege |  | Tilburg |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Amsterdam | \$ | 120.0 | \$ | 130.0 | \$ | 41.0 | \$ | 59.5 |
| Antwerp | \$ | 61.0 | \$ | 40.0 | \$ | 100.0 | \$ | 110.0 |
| The Hague | \$ | 102.5 | \$ | 90.0 | \$ | 122.0 | \$ | 42.0 |
| Backorder | \$ | 50.0 | \$ | 30.0 | \$ | 70.0 | \$ | 100.0 |

## Shipments

| From/To | Leipzig | Nancy | Liege | Tilburg | Total | Available |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Amsterdam | - | - | 300 | 200 |  | 500 | 500 |
| Antwerp | - | 700 | - | - |  | 700 | 700 |
| The Hague | 300 | 200 | - | 300 |  | 800 | 800 |
| Backorder | 100 | - | - | - |  | 100 | 100 |

## AMPL Model

\&set PORT;
\&set PLANT;
param supply \{PORT\};
param demand \{PLANT\}; param cost \{PORT, PLANT\}; param BackOrderCost\{PLANT\};

## Model Cont'd

var Trans \{PORT, PLANT\} >=0;
var BackOrder\{PLANT\} >=0; minimize Total_Cost:
sum\{f in PORT, t in PLANT\} cost[f,t]*Trans[f,t] +
sum\{t in PLANT\} BackOrderCost[t]*BackOrder[t]; s.t. Supply \{p in PORT\}:
sum\{t in PLANT\} Trans[p, t] <= supply[p];
s.t. MaxBackOrder:
sum\{t in PLANT\} BackOrder[t]
<= sum\{t in PLANT\}demand[t] - sum\{p in PORT\} supply[p];
s.t. Demand \{t in PLANT\}:
$\operatorname{sum}\{\mathrm{p}$ in PORT $\} \operatorname{Trans}[\mathrm{p}, \mathrm{t}]]_{5.057}+$ Backing o2 $\operatorname{rder}[\mathrm{t}]>=$ demand $[\mathrm{t}] ;$

## Modeling Time

\&Suppose the Singapore Electric Generator Company must send its completed generators to Australia for testing before they can be sold. This process takes an entire month so that, for example, generators made in January are not available for sale until February. Extend the Singapore Electric Generator Model accommodate this delay.

## Modeling Time

| Singapore Electric Generator Production |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Unit Costs Dec |  |  | Jan | Feb |  | Mar |  | Apr. |  | May |
| Production | \$ | 29.00 | \$ 28.00 | \$ | 27.00 | \$ | 27.80 |  | 29.00 |  |
| Inventory | \$ | 0.30 | \$ 0.30 | \$ | 0.30 | \$ | 0.30 | \$ | 0.30 |  |
| Production Qty |  | 50 | 29 |  | 62 |  | 38 |  | 0 |  |
| Production Limits |  |  | 60 |  | 62 |  | 64 |  | 66 |  |
| Beginning Inventory |  | 7 | 15 |  | 7 |  | 0 |  | 28 |  |
| Delivery Reqmts |  |  | 58 |  | 36 |  | 34 |  | 59 | Minimum |
| Ending Inventory |  |  | 7 |  | - |  | 28 |  | 7 | 7 |
| Production Cost |  | 1,450.00 | \$ 812.00 | \$ | 1,674.00 | \$ | 1,056.40 | \$ | - |  |
| Inventory Cost | \$ | 3.30 | \$ 3.30 | \$ | 1.05 | \$ | 4.20 | \$ | 5.25 | Total |
| Total Cost |  | 1,453.30 | \$ 815.30 | \$ | 1,675.05 | \$ | 1,060.60 | \$ | 5.25 | \$ 5,009.50 |

## Short Answer

Give an example of an optimal, but not basic solution to a network flow problem.

## Optimal, but not basic



## Basic Variables

\&How many basic variables will there be in a basic feasible solution to a Transportation Problem like Autopower's, but with 4 ports and 5 plants?
\& 8
\&Why? 9 Constraints, but 1 is redundant.

## Exam Structure

\&Distribute Word document and Excel Spreadsheets
\&Return Word document and Excel spreadsheets, etc.
\&Keep time to 2 hrs.
\&Ayla and Lincoln in class during class time.

