

15.063 Communicating with Data Summer 2003

Homework Assignment #1

Issued: Lecture 3. **Due: Lecture 7, before Lecture.**

1. Decision Trees

Extensive logging has exposed a hillside in San Carlos to the possibility of a mudslide. Reforestation is underway, but it will be a year before the new vegetation will be mature enough to remove the danger. If a slide occurs in the interim, human injuries will be avoided because mud moves slowly. The damage from such a slide would be limited to the road that passes beneath the hill. Construction of a retaining wall on the uphill side of the road has been suggested as a possible step to prevent this damage.

The Mayor of San Carlos is puzzled by the uncertainty concerning the issue. He has consulted with an expert who states that there is only one chance in 100 that a slide will occur within the next year. The expert adds that roughly 5% of all such slides break through retaining walls like the one proposed. The retaining wall would cost \$40,000 to build. The road would cost about \$1,000,000 to repair if damaged by a mudslide. The expert points out that she can better assess the likelihood of a slide occurring in the next year if she conducts a geological test of the igneous rock layer below the hillside. Like any test, this one is imperfect. A positive test outcome would indicate a higher chance of a slide than a negative test outcome. The test has been conducted at sites at which slides eventually occurred and at sites at which slides did not subsequently occur. The information from these previous tests can be summarized as follows. Positive test outcomes had been reported on 90% of the sites at which slides subsequently occurred. Negative test results had been reported at 85% of the sites at which slides did not subsequently occur.

Assignment: As an aide to the mayor of San Carlos, what action would you recommend be taken? Please explain your results.

Note 1: assume that the cost of the test is minimal.

Note 2: you may use *Treeplan* or construct the decision tree yourself.

2. Laws of Probability

Exercise 2.3 from the book, *Data, Models, and Decisions: The Fundamentals of Management Science* by Dimitris Bertsimas and Robert M. Freund, Southwestern College Publishing, 2000.

Please explain your results.

3. Conditional Probability

To the best of our knowledge, with probability 0.8 Al is guilty of the crime for which he is about to be tried. Bo and Ci, each of whom knows whether or not Al is guilty, have been called to testify. Bo is a friend of Al's and will tell the truth if Al is innocent, but will lie with probability 0.2 if Al is guilty. Ci hates everybody but the judge and will tell the truth if Al is guilty but will lie with probability 0.3 if Al is innocent. Given this model:

- a. Determine the probability that the witnesses give conflicting testimony.
- b. Which witness is more likely to commit perjury?
- c. What is the conditional probability that Al is innocent, given that Bo and Ci gave conflicting testimony?
- d. Are the events "Bo tells a lie" and "Ci tells a lie" independent? Are these events conditionally independent to an observer who knows whether or not Al is guilty?

4. Gambling

A gambler has in his pocket a fair coin and a two-headed coin.

- a. He selects one of the coins at random, and when he flips it, it shows heads. What is the probability that it is the fair coin?
- b. Suppose that he flips the same coin a second time and again it shows heads. What is now the probability that it is the fair coin?
- c. Suppose that he flips the same coin a third time and it shows tails. What is now the probability that it is the fair coin?