

**15.066j System Optimization and Analysis**  
**Summer 2003**  
**Professor Stephen C. Graves**

**Group Problem Set 1**

**This problem set is due at the end of recitation on 19<sup>th</sup> June.**

*Please work in groups but hand in only ONE copy for each group. The problem set can be handwritten, but please make sure what you hand in is NEAT and CLEAR. Please also attach an Excel spreadsheet printout if you are required to solve the LP using the Excel Solver.*

**Problem 1:** During each 4-hour period of the day, the Cambridge Police Department needs at least the number of officers shown in the table below. Officers can be hired to work either 8 consecutive hours or 12 consecutive hours. The officers are paid \$4/hr for each of the first 8 hours a day they work and \$6/hr for each of the next 4 hours they work in a day. The hiring requirements and the pay are the same everyday. Formulate an LP to minimize the cost of meeting Cambridge’s daily police requirements. You don’t need to solve this (the mathematical formulation is enough).

<b>Time Period</b>	<b>Number of Officers required</b>
12 midnight – 4am	37
4am-8am	17
8am-12 noon	15
12 noon – 4pm	25
4pm-8pm	28
8pm – 12 midnight	40

**Table 1: Data for Problem 2**

**Problem 2:** Manufacto produces two types of products: A and B. The company has agreed to deliver the products on the schedule shown in Table 2. The company has two assembly lines, 1 and 2, with available production hour as shown in Table 3. The production rate for each assembly line and product combination, in terms of hours per product, is shown in Table 4. For example, it takes 0.15 hours to manufacture one unit of product A on line 1. It costs \$5 per hour of line time to produce any product. The inventory carrying cost per month per product is \$0.2 per unit (charged on each month’s inventory). At present, there are 500 units of A and 750 units of B in inventory. Manufacto would like at least 1000 units of each product in inventory at the end of April. Determine the product schedule that minimizes the total cost incurred in meeting the company’s demand on time. Formulate an LP and solve it in Excel.

	<b>A</b>	<b>B</b>
<b>March 31</b>	5000	2000
<b>April 30</b>	8000	4000

**Table 2: Demand**

	<b>A</b>	<b>B</b>
<b>March 31</b>	5000	2000
<b>April 30</b>	8000	4000

**Table 3: Available production hours**

	<b>Line 1</b>	<b>Line 2</b>
<b>Product A</b>	0.15	0.16
<b>Product B</b>	0.12	0.14

**Table 4: Production rates**

**Problem 3:** Sunco Oil manufactures three types of gasoline (gas 1, gas 2, gas 3). Each type is produced by blending three types of crude oil (crude 1, crude 2, crude 3). The sale price per barrel of gasoline and the purchase price per barrel of crude oil are given in Table 5. Sunco can purchase up to 5000 barrels of each type of crude oil daily. The 3 types of gasoline differ in their octane rating and sulfur content. The crude oil blended to form gas 1 must have an average octane rating of at least 10 and contain at most 1% sulphur. The crude oil blended to form gas 2 must have an average octane rating of at least 8 and contain at most 2% sulphur. The crude oil blended to form gas 3 must have an average octane rating of at least 6 and contain at most 1% sulphur. The octane rating and the sulphur content are given in Table 6. It costs \$4 to transform one barrel of oil into one barrel of gasoline, and Sunco's refinery can produce up to 14,000 barrels of gasoline daily. The daily demand for Sunco's gas 1 is 1,300 barrels, gas 2 is 2,200 and gas 3 is 1000. Formulate an LP and solve it. Assume that Sunco cannot sell more than its demand.

	<b>Sale Price per Barrel</b>		<b>Purchase Price per Barrel</b>
<b>Gas 1</b>	\$70	<b>Crude 1</b>	\$45
<b>Gas 2</b>	\$60	<b>Crude 2</b>	\$35
<b>Gas 3</b>	\$50	<b>Crude 3</b>	\$25

**Table 5: Sale and purchase prices**

	<b>Octane Rating</b>	<b>Sulfur Content</b>
<b>Crude 1</b>	12	0.5%
<b>Crude 2</b>	6	2%
<b>Crude 3</b>	8	3%

**Table 6: Octane rating and Sulfur content**

**Problem 4:** Exercise 4 on pg. 2.28 in the textbook.

**Problem 5:** Exercise 5 on pg. 2.29 in the textbook. (Ignore the shipping/ inspection cost)

**Problem 6:** Exercise 6 on pg. 2.29 in the textbook.