

15.072

Homework Assignment 4

Given: April 5, 2006

Due: April 12, 2006

Problem 1 Consider a $G/M/1$ queueing system where the distribution of the interarrival times is a mixture of two exponential distributions with parameters $\lambda_1 = 1$ and $\lambda_2 = 2$ and probabilities $p_1 = .3, p_2 = 1 - p_1$. The service rate is assumed to be $\mu = 2$. Determine the probability distribution of the waiting time in steady state.

Problem 2 * Consider a $G/M/1$ queueing system with interarrival times A and service rate μ . Prove that the expected steady state number of customers in the system observed by an arriving customer diverges to infinity in the limit as μ approaches $1/\mathbb{E}[A]$.

HINT: express the expected number in the system in terms of σ and prove that σ approaches unity as $\mu \rightarrow 1/\mathbb{E}[A]$. Use the second order Taylor expansion to establish this fact.

Problem 3 Suppose two call centers A and B with m_A and m_B agents respectively, serve a demand from the same pool. The demand is Poisson with rate λ . The service rate in the two centers is identical μ . Upon arrival of a call the scheduler needs to route the call to one of the two centers. It is too costly to make the routing decision based on the number of calls in progress. As a result the decision needs to be done in an oblivious way. Only the values of λ, μ, m_A, m_B are known, and with some probability p ($1 - p$) each call is routed to call center A (call center B), independently from everything else.

A. Propose a routing scheme $p = p(\lambda, m_A, m_B, \mu)$ which minimizes the steady state cost given as $\mathbb{P}(W_A > 0) + \mathbb{P}(W_B > 0)$, where W_A, W_B are waiting times in the call centers A, B , respectively.

B. Suppose $\lambda = 6400, \mu = 10, m_A = 300, m_B = 400$. Find approximately the optimal p up to 2 decimal points.

HINT: Standard normal table can be found on the internet ...

Problem 4 Consider k identical call centers A which are in a Halfin-Whitt regime. Each of them is characterized by parameters $m, \mu, \lambda = m\mu - \beta\mu\sqrt{m}$, where, as usual, m, μ, λ stand for the number of agents, service rate and the arrival rate. Suppose we merge these k call centers into one call center kA . Prove that the probability of waiting $\mathbb{P}(W > 0)$ in the merged call center converges to zero geometrically fast as a function of k . Namely, $\mathbb{P}(W > 0) \leq \delta^k$ for some $\delta < 1$. Obtain a bound on δ in terms of the parameter β of the original call center.
