

*Probability theory presents a systematic
way to think about such issues*

Examples

- *Probability of:*
 - *A newborn will be male?*
 - *Getting HH in a coin toss?*
 - *Getting a particular face on a die?*
 - *In a bag with 1 red, 1 green and 3 blue balls. What is the probability to draw a red ball, a red or a blue ball?*

Compound Probabilities

Disjunctive probabilities linked by addition

A or B

Conjunctive probability

A and B

Disjunctive

- *In a bag with 1 red, 1 green and 3 blue balls. What is the likelihood that you draw a red **or** a green ball?*
-
- *In a bag with 1 red, 1 green and 3 blue balls. What is the likelihood that you draw a red **or** a blue ball?*

Conjunctive I

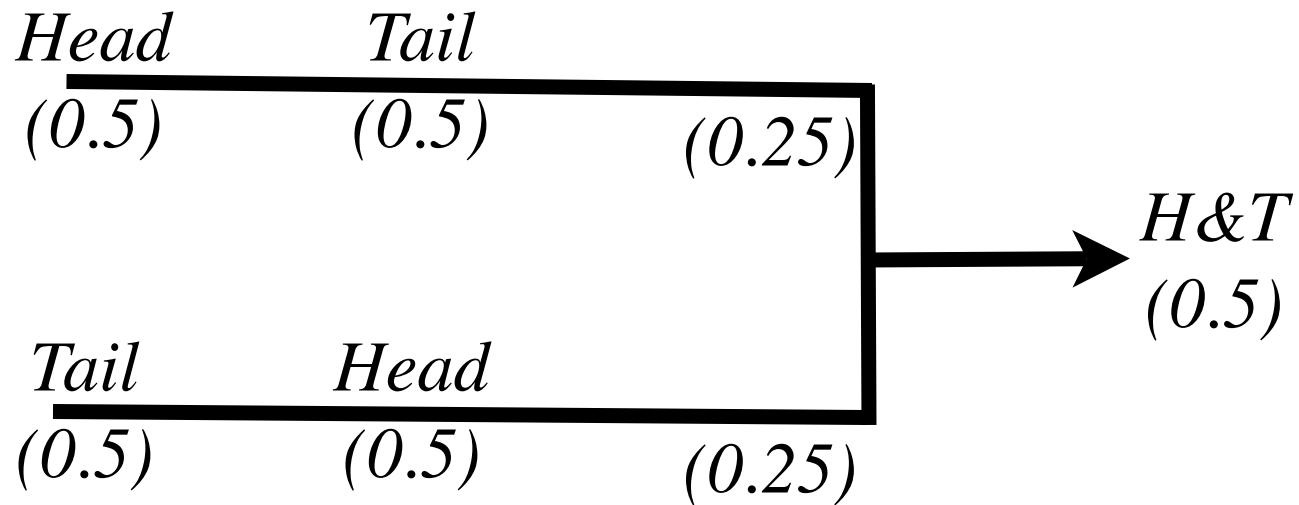
- *In 2 coin tosses, what's the likelihood of getting 2 heads?*

T	T
T	H
H	T
H	H

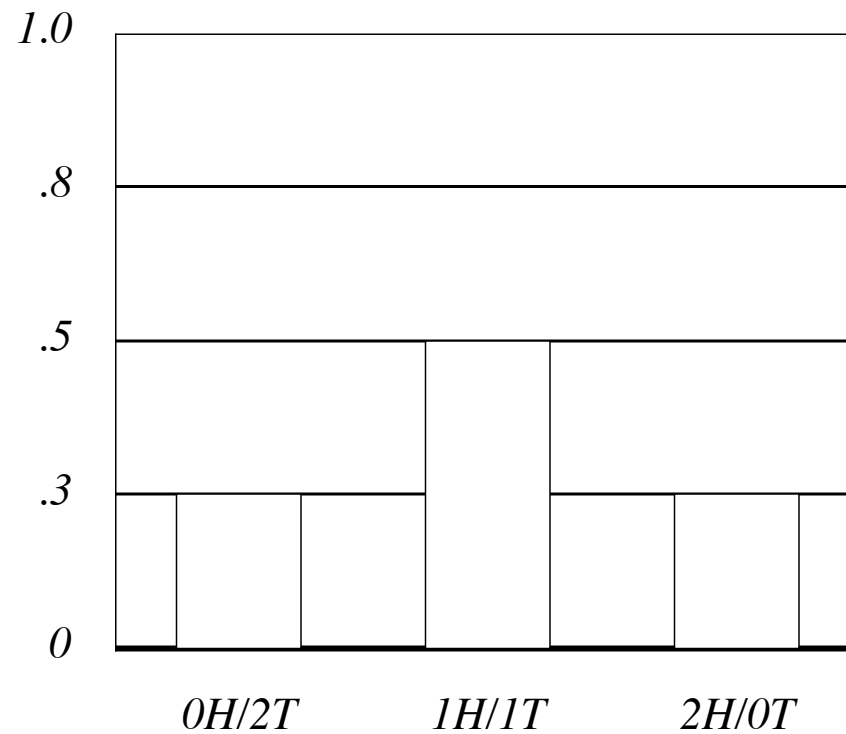
Conjunctive II

- *In a bag with 1 red, 1 green and 3 blue balls. What is the likelihood that in 2 drawings you get 2 red balls?*
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- *In a bag with 1 red, 1 green and 3 blue balls. What is the likelihood that 2 drawings you get 1 red and 1 blue ball?*

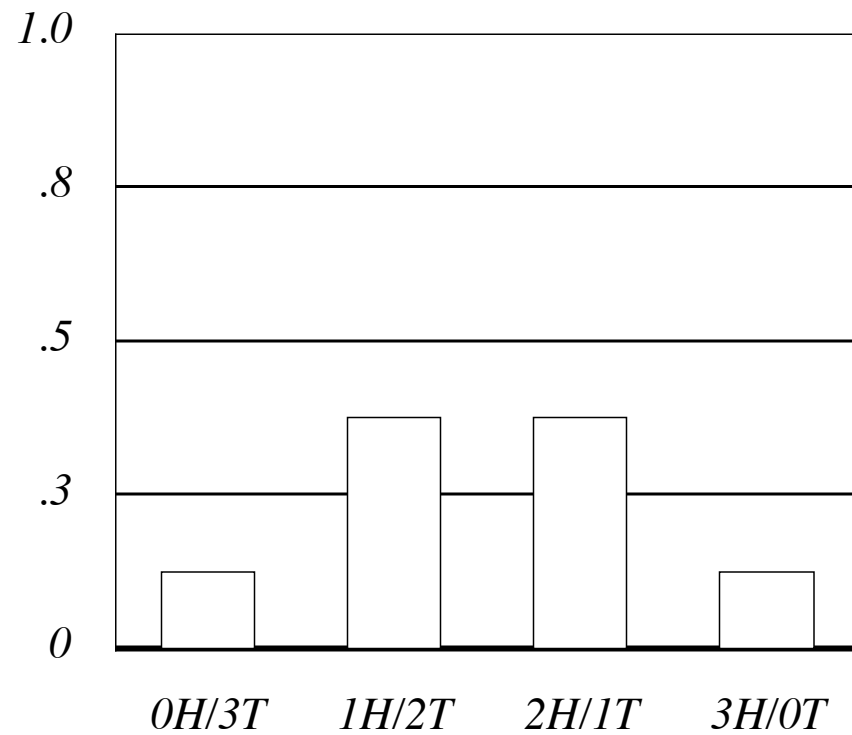
Probability pathway



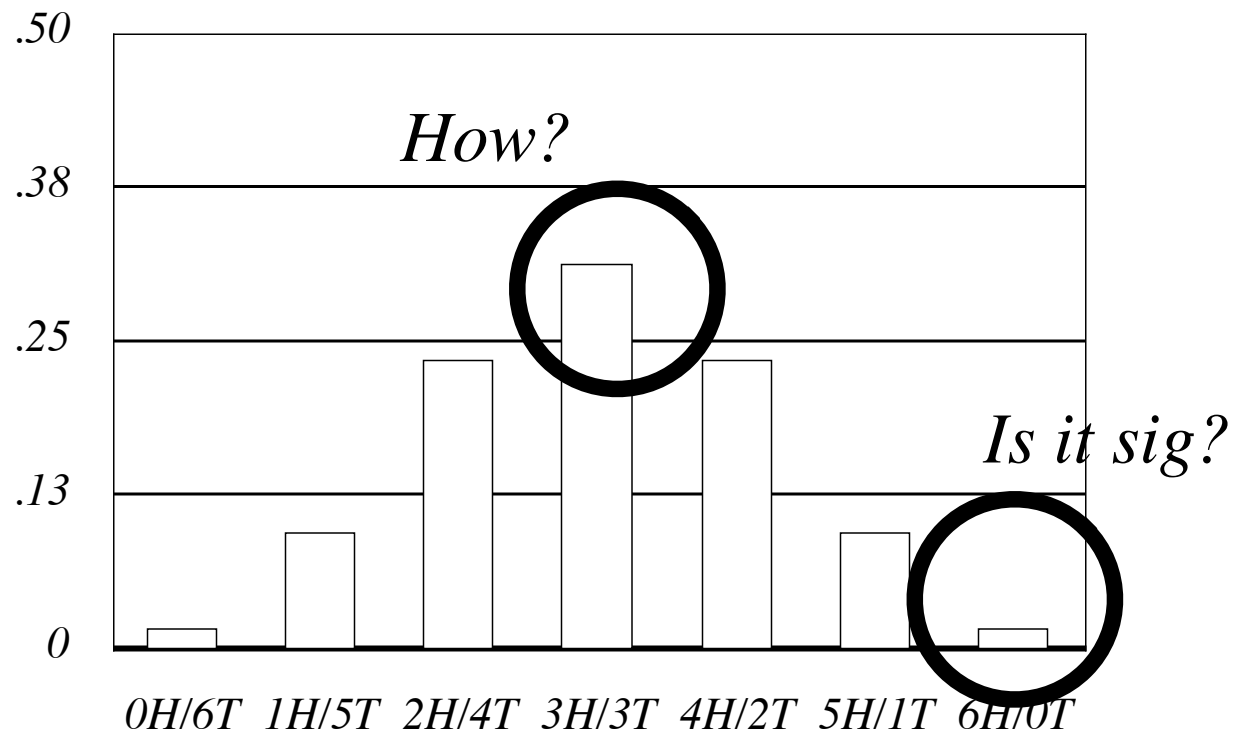
Frequency of 2 coin toss



Frequency of 3 coin toss



Frequency of 6 coin toss



Binomial Distribution II

- *A dramatic disease causes death in 60% of the cases. Only 40% people survive.*
- *What is the likelihood that (at least) 7 out of 10 people survive?*
-
- *A bit more complex ...*

Calculating binomial probabilities

- $P(r \text{ out of } n) = \text{number of possible ways } X$
Probability of each way
- $\text{Number of possible ways} = n!/[r!(n-r)!]$
- **$P(7 \text{ out of } 10) =$**
 - $0.4^{(r)} \times 0.6^{(n-r)} = 0.4^7 \times 0.6^3 =$
 0.00035389
 - $10!/7!3! = 120$
 - $P(7 \text{ out of } 10) = 0.0425$

7 or more out of 10?

- *7 out of 10 + 8 out of 10 + 9 out of 10 + 10 out of 10*
- *When we calculate the probability of an outcome (7), most times we calculate the probability of the outcome or a more extreme outcome*

Binomial summary

- *By using the binomial distribution we can directly calculate the probability of an event (or a more extreme event).*
- *We can compare this probability to 0.05 and decide if it is significant or not.*

Calculating distributions:

- *6 coins tossed:*
 - $n = 6; p = .5; q = .5$
 - *Theoretical mean (μ) = $np = 6 * .5 = 3$*
 - *central tendency of distribution*
 - *Variance (σ^2) = $n * p * q$*
 - *Standard deviation = σ*
 - $z = (r - \mu) / \sigma$

Z scores

$$z = (r - \mu) / \sigma$$

Probability of 6 Heads:

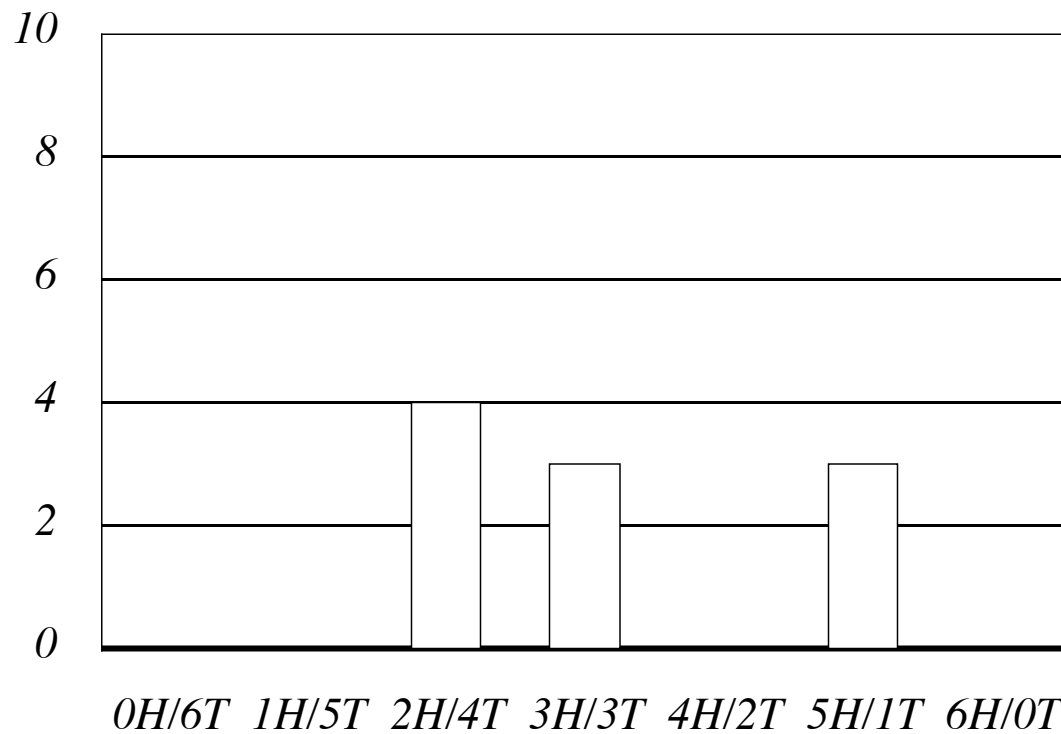
$$z = (6 - 3) / 1.225 = 2.45$$

Look for $z = 2.45$ in the *Normal distribution table*

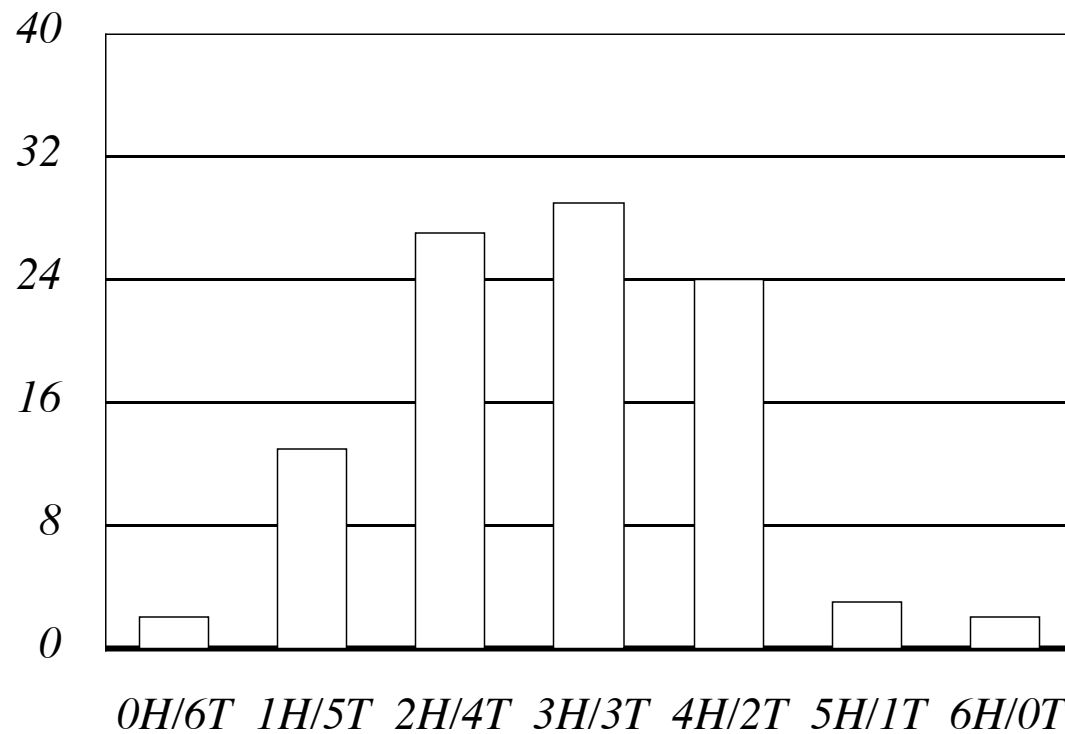
Central limit theorem

- *Approximations of normal distributions with increased number of observations*

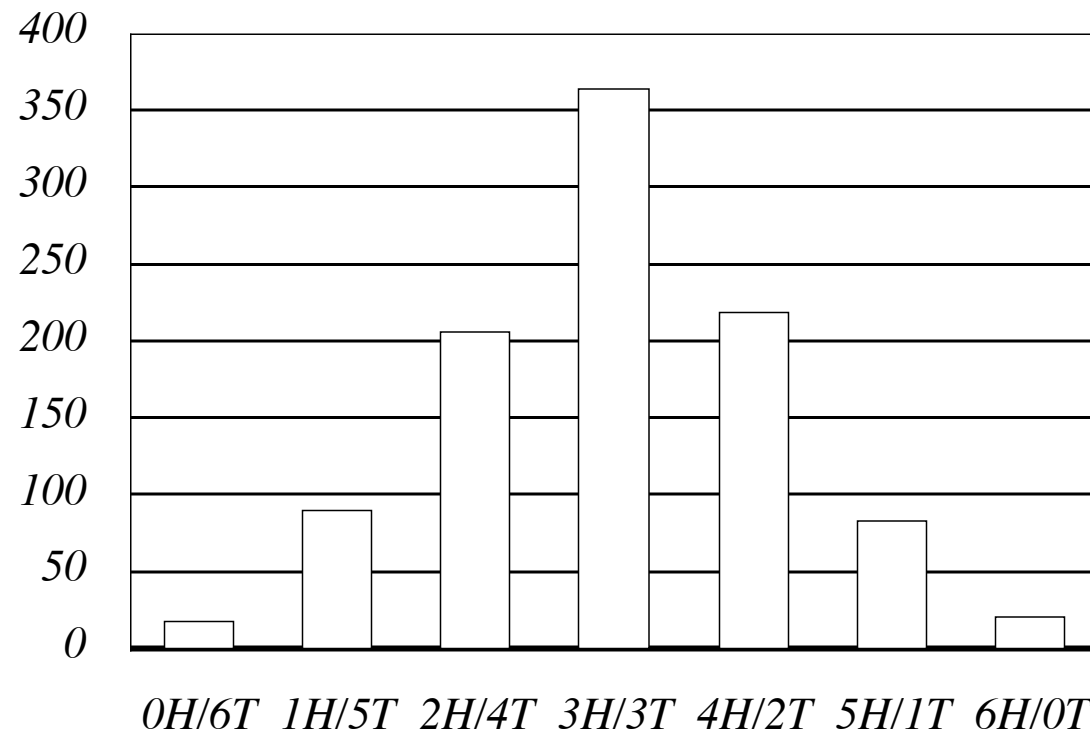
Frequency of 6 coin toss (10)



Frequency of 6 coin toss (100)



Frequency of 6 coin toss (1,000)



Summary

- *Probability*
 - *This is the base of statistics...*
- *Binomial:*
 - *A way to calculate probabilities of events directly*
- *Central limit theorem*