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ANDREW LO: OK. What I'd like to do today is to continue where we left off last time in talking about this risk-reward trade-off, which ultimately will allow us to be able to figure out how to calculate the proper discount rate for any project under the sun. Now, where we left off last time was this equation and the one after it. This equation we actually derived. I showed you how to get this equation from this particular bullet and the tangency line. And today I am going to give you names for them.

The bullet, as we've said before, is the frontier. It's the set of frontier portfolios. And the upper arc of that bullet is called the efficient frontier.

Now the tangency line has a special name too. That tangency line is known as the capital market line because it represents what all efficient capital markets should represent in terms of risk-reward trade-off. So if you are an efficient portfolio manager, you want to be on that line. OK?

So the capital market line, the equation for that tangency line, is given by this. The expected rate of return is equal to the risk-free rate plus some kind of risk premium, where the risk premium is given by a multiple of the market's risk premium, or the market excess return. And the multiple is simply the ratio of the riskiness of your efficient portfolio relative to the market portfolio, or the tangency portfolio. If it's twice as risky, you're going to get twice the risk premium. If it's half as risky, you're going to get half the risk premium.

And we said last time that, while this is helpful and interesting and even possibly useful, it's going to be of limited applicability because not everything is an efficient portfolio. What we mean by an efficient portfolio, the intuition for what an efficient portfolio is, is a portfolio where you cannot do better. By cannot do better, I mean you can't get less risk for that same level of return, or you can't get more expected return for that same level of risk. That's what we mean by an efficient portfolio. It's the best you can do.

Now, most investments are, frankly, not efficient. If you pick an arbitrary stock, like IBM, that's not an efficient portfolio. It doesn't mean it's no good. It doesn't mean you don't want to hold it. But it means that you would never want to hold just IBM because if you mixed IBM with other stuff, you can always do better.

By do better, again I'm going to reiterate, I mean you can have higher expected return for the same level of risk or lower risk for the same level of expected return. That's what I mean by do better. So you would never want to hold IBM just by itself because you can do better, right? You can do better in getting up to that Northwest quadrant from the IBM point.

But even though IBM is not efficient, you might still want to hold it. And more importantly, you might still want to know what the appropriate discount rate is for companies that are like IBM. That's what we're going to do next.

Where I left off last time was not the capital market line, but this equation, which I did not derive, but which I argued comes from the equilibrium argument that Bill Sharpe made 50 years ago. And this really relies on the fact that, if markets are in equilibrium, there is a relationship between risk and expected return for all securities, not just efficient portfolios. But any arbitrary security has to satisfy this equation if supply equals demand, if there's an equilibrium.

If everybody holds the tangency portfolio, and the tangency portfolio therefore is the market portfolio, in that situation, this equation has to hold. So where we left off was to try to interpret this equation. This equation is almost identical to the capital market line.

There's only one difference. The only difference is that the multiplier, the thing that multiplies the market risk premium, is not sigma p over sigma m. It is beta. And beta, we said, was the ratio of the covariance of an asset with the market divided by the variance of the market. It is a measure of that particular asset's riskiness.

It's not variance anymore, or standard deviation, it's something else. And so I want to spend this class talking about what that something else is and why it makes sense. First of all, let's make sure we understand the equation. I want to do a few special cases, and then I'm going to take apart this notion of beta as being the right measure of risk in all circumstances.

So the first thing I want to do is to look at some examples. Let's take an example where the beta is equal to 1. If the beta is equal to 1, what that's saying is that the covariance between the asset and the market divided by the variance of the market, that number is equal to 1.

If that's the case, then it turns out that the expected rate of return of this asset with a beta of 1 is going to be just equal to the market risk premium. Or the expected return is equal to the market's expected rate of return. When beta is equal to 1, the Rf's cancel out, and the

expected rate of return of the asset is equal to the expected rate of return on the market.

Now, what about if the beta is equal to 0? If the beta is equal to 0, then you get the risk-free rate. It's important to realize that if the beta of an asset is equal to 0, it doesn't mean that the asset has no volatility. In the past, when we've looked at mixing the risk-free asset, T-bills, with the market, we know that when the risk-free asset is included, you get that straight line. And it's because the risk-free asset has no covariance. It has no risk at all.

But now, with an asset that has a beta of 0, you're getting the risk-free return, even though an asset with a beta of 0 still may have some volatility. It's not the risk-free asset. It's any asset with a 0 beta.

The third observation that I want to make is when a beta is negative. With the beta that's negative, the expected rate of return is actually less than the risk-free rate. Now that's really odd. I've got an asset that is risky. But just because it has a negative beta, the expected rate of return is less than the risk-free rate. That means that you are willing to take a lower expected rate of return than the risk-free rate for an asset that has this weird property of negative beta.

And the question I want to answer today is, why is that? Why is it the case that you might be willing to take such a low rate of return? And actually, if the beta is negative enough, if this beta is negative enough, it could be the case that the expected rate of return is negative.

In other words, you might be willing to pay somebody for the privilege of bearing that risk. That seems completely counterintuitive. Why would you be willing to pay to take risk? You should be getting paid to take risk, right? That's the standard hypothesis that we come into the financial markets with.

So yeah, Leah?

AUDIENCE: [INAUDIBLE]

ANDREW LO: That's right.

AUDIENCE: [INAUDIBLE]

ANDREW LO: Exactly. That's exactly the intuition for all three of these cases. It turns out that the beta, remember, has this covariance term in there. And it is going to turn out that, if you can find an

asset that is negatively correlated to the tangency portfolio, that is going to be of tremendous value to you. It's very, very valuable.

Now, OK. Let me try to explain the logic behind it. And we're going to do a few examples. So let's go through the basic logic of what's going on.

The tangency portfolio plays a central role in that everybody in the world is going to want to have a portfolio that's a combination of the risk-free asset and the tangency portfolio. That's fact number one. Fact number two, the tangency portfolio is a portfolio that has the aggregate measure of the total amount of risk in the economy that cannot be diversified beyond that point.

In other words, in order to get lower risk than this particular portfolio, you have to decrease your expected rate of return. There's no way to get lower risk and keep that same level of expected return. You can't go this way. You have to go down this line. OK?

So if you're going to hold a portfolio of purely risky securities, then basically this is the best that you can do. This is the best trade-off that you can get in terms of risk-reward. So right away you know that this market portfolio plays a very special role.

It is really the representation of the aggregate risk in the stock market. And that's why it can serve as a kind of a benchmark for what the stock market is doing. We're going to come back to that benchmark idea in a few minutes.

So, if you have a security that is very highly correlated to that market return, then that's not going to help you in terms of your own diversification. If, on the other hand, you have a security that is negatively correlated with that market portfolio, that's going to help you a lot. And if it's going to help you a lot, you're willing to pay for it.

When you're willing to pay for it, what does that mean? You drive the price today high. And therefore, the expected rate of return, which is the return between today and next period, that becomes lower.

So an asset that helps you hedge what is essentially unhedgedgeable, in other words, the market portfolio, that can benefit you a great deal. As a result, it's going to be very hard to find negative beta securities or assets. But in any case, this relationship really tells you that, given a particular covariance, you can measure the expected rate of return that you ought to be getting.

And this relationship is so important that we give it a separate name. It's called the security market line, not the capital market line, but the security market line because this applies to every single security in your entire universe.

Yeah, Dennis?

AUDIENCE: You say it's hard to find something with a negative beta. But when you short something, does that mean you're getting a negative beta?

ANDREW LO: It is. You do. But the problem with shorting something where you get a negative beta, you also get a negative expected rate of return typically. So what you want to have is an asset that's got a positive beta-- or negative beta, but a positive expected rate of return. That's what's very rare.

But you can manufacture a negative beta security very easily by just shorting it. The problem is that when you short a stock, you're going to also get a very negative expected rate of return. And that doesn't help you in terms of producing an attractive investment opportunity.

Ken, question?

AUDIENCE: So, what's an example of a negative beta security?

ANDREW LO: Well, it's very hard to come by. But the closest thing that exists in markets today is stocks that are involved in gold production, gold mining stocks. That has a beta of around 0, but sometimes it's negative. Sometimes it's positive. But it's small.

So that's an example, but that's about the only example that we can come up with in the data that looks slightly negative. By and large, most of the betas in the data sets are positive. And they're actually clustered around 1. So the typical beta is in the neighborhood of 0.5 to 1.5.

All right. Now, before we start looking at the data-- I'm going to show you some data in a minute-- I want to take the security market line and apply it. So the security market line I did not derive. I want to make that clear.

And for those of you who are interested in the derivation, you can take a look at the appendix in the Brealey, Meyers, and Allen. They provide the derivation. It's a little bit messy, but with a little bit of matrix algebra you can work through it.

But the implications are extremely important, so I want all of you to know how to use it. So it may look pretty simple, but I'll show you a few applications that you might not have thought of. For example, suppose this is true for every single security. Well, if it's true for every single security, it turns out that this works for portfolios as well. And let me show you why.

Suppose you've got a portfolio that's a weighted average of the returns for the individual components securities. Then if I calculate the covariance between the portfolio return and the tangency portfolio, or the market, the covariance is going to look like this, which is actually just going to look like a weighted average of the covariances. This is a mathematical identity, from here down to here.

And therefore, when I divide both sides by the variance of the market, I get something pretty neat, which is that the beta of my portfolio-- where beta is defined as the covariance between the portfolio and the market divided by the variance-- the beta of the portfolio is just equal to the weighted average of the betas of my component securities.

That's really neat because what that says is that, when I want to measure the risk of a collection of securities, as long as I know the betas of each individual security, I can calculate a weighted average of those betas. And that is the beta of my portfolio. So if you think of beta as a measure of risk, this measure of risk is actually linear, unlike volatility, which is not linear.

The variance of a portfolio is not simply equal to the sum of the variances weighted by their portfolio weights. It's that complicated expression where you're adding up all of those cross products as well. So we get an enormous simplification with the security market line. It says that we can measure the risk of a portfolio using this concept called beta. And beta happens to be linear in the sense that, when you take a weighted average, the beta is equal to the weighted average of the individual asset betas.

So therefore, if you know that the betas are going to be a weighted average, then, in fact, the expected rate of return on the portfolio now is equal to the risk-free rate plus this weighted average beta times the market risk premium. Do you see the power of this? This now allows you to analyze the expected return on anything, any collection of assets. If you know what the betas are for the individual components, you know what the betas are for the whole thing.

So now you can calculate the appropriate rate of return for virtually anything. And this is not just limited to stocks. You can apply this to projects. For example, if you want to know what the expected rate of return is on an oil drilling project, well, measure the beta of oil drilling stocks,

use that beta, and that will be appropriate discount rate for that particular oil drilling project. It's a really remarkable result.

So we have an expression for the required rate of return, opportunity cost of capital, risk-adjusted discount rate, for all the various different kind of examples and cases that we looked at up until now. And the last point I want to make about this equation is, how do you actually take it out for a spin? How do you estimate the expected rate of return on the market and the risk-free rate?

Well, that comes from the data. That comes from the marketplace. We observe it in the marketplace. And we can actually see it.

OK. So let's do some examples, just to make sure that we all get this and know how to apply it. Using returns from 1990 to 2001, we estimate that Microsoft's beta during that period of time is 1.49. And if you do the same thing for Gillette, you get that Gillette's beta is 0.81.

Now, let's not even look at the next set of numbers for a moment, and just talk about those two numbers, 1.49 and 0.81. Does that make sense to you? Let's think about what that's saying.

1.49 says that the covariance between Microsoft and the market portfolio is actually a lot higher than the variance of the market itself. So let me ask you to think about whether or not adding Microsoft to your portfolio is going to make it less risky or more risky. And here's how I want you to think about it. Remember what we said about diversification. When you hold a collection of securities, what matters more, the variances or the covariances?

AUDIENCE: Covariances.

ANDREW LO: Right. Why are the covariances more important? What's a quick and dirty way of arguing that the covariances matter more?

AUDIENCE: Because there are n squared minus n.

ANDREW LO: Exactly. There are a heck of a lot more covariances than there are variances. You only got n variances to worry about, but you got n squared minus n covariances. And if they all line up in the same direction, you get the subprime crisis problem, right?

So covariances matter more than variances. Well, if that's the case, then when we look at a stock and think about bringing it into our portfolio, we want to ask the question, what is it doing

in terms of adding or subtracting covariances to our portfolio? And one way to measure whether or not it's adding or subtracting is to ask the question, what is the covariance between Microsoft and my stock holdings? Now what are your stock holdings?

If everybody is a rational investor-- by rational I mean you like return and you don't like risk-- then you know you're going to be holding the risk-free rate and the market portfolio. You're going to be on that capital market line. So if you're a rational investor, the only stock holding you have is that mutual fund, m, the tangency portfolio.

So therefore, the most important thing in your mind is, when you think about buying a new stock and putting into your portfolio, is this going to be highly correlated with my market portfolio? Well, that's what beta measures. Beta is a relative measure that says, OK, the total variance that you're holding in risky securities, that's sigma m squared. That's the variance of the market portfolio.

How does Microsoft compare to that in terms of what it will contribute, in terms of its covariance with your holding? So you're holding one mutual fund, and you're thinking about adding Microsoft. The only covariance that you should care about is the covariance between Microsoft and what you're holding.

Well, that's what beta measures. If the number is greater than 1, what it's saying is that, when you bring Microsoft into your portfolio, you're going to be increasing the variance because the covariance, which is what we care about, is greater than the variance of what you're holding. If, on the other hand, the beta is less than 1, then, presumably, that's helping you because that's lowering the variance relative to what you're holding. But helping or hurting, that only can be answered directly if you explain what you're getting in terms of the expected rate of return.

So looking at beta by itself is not enough. Beta is a measure of risk. It measures this covariance divided by the variance, or covariance per unit variance in the marketplace. But you want to know what the expected rate of return is as well. That's what the security market line gives you.

OK. Now let's get back to the example. Microsoft is a lot more risky than the market. It's about 49% more risky according to this measure. On the other hand, Gillette is actually less risky than the market.

Now do you guys buy that? Does that pass the smell test? Does that make sense? Why? What's the intuition for that? Courtney?

AUDIENCE: Well, people don't necessarily need computers. And the technology is variable, but Gillette sells razor products and deodorant, which is kind of a staple in a lot of people's--

ANDREW LO: Exactly. That's right. If you make the argument that, from 1990 to 2001, if there are economic downturns, what's the first to go, razorblades or Windows? Thankfully Windows.

[LAUGHTER]

Nowadays, I don't know the answer to that actually. Because nowadays, we depend so much on the internet, that actually it could be different. So I haven't updated this analysis to see what the beta is from 2001 to 2008, but it could be different. So now we may have unshaven geeks that, you know, during downturns-- and maybe it's flipped around.

Eduard?

AUDIENCE: Could you give us an intuition? Because beta allows us to compute the appropriate return for a certain risk.

ANDREW LO: Yes.

AUDIENCE: But what is the intuition of, how far away am I from the efficiency portfolio? So, how bad is this portfolio compared to the efficient one?

ANDREW LO: Yeah. So, that's a good question. Let me go back to this equation and take a look at it, and try to provide even more intuition for this before we go on.

So the idea behind this equation is that it tells you that this is the rate of return that you should have if you have a certain beta. Now you can actually measure the deviation from that very simply by asking the question. For a portfolio manager or an investment project that yields an expected rate of return that's different from this, that difference is actually what we call alpha. And alpha could be positive or negative.

So when you say, how far away are you from efficiency? This gives you a direct measure of how far away you are. It's basically the difference between the expected rate of return you have versus what you're supposed to have given the beta of the security.

But let me add one more thing to that, which is that beta is a measure of a particular kind of risk that a particular security has. And the kind of risk, as I said before, is this covariance between the rate of return on a particular asset and the rate of return on the market portfolio.

This kind of risk is not the total risk of a particular security. In fact, it is called the systematic risk. The systematic risk is the portion of the risk that is related to the market portfolio. So how far away you are from efficiency really depends upon how much risk you have that is not necessarily systematic risk.

Now, I don't expect you to understand all of it yet because I need to develop a little bit more machinery. But I'm going to get back to that intuition in just a few minutes, OK? So I'm going to give you a better answer to your question than what I just did because I'll explain the difference between systematic risk and idiosyncratic risk. And I think then it'll make this completely transparent. So give me another 15 minutes.

Yeah?

AUDIENCE: So you mentioned that gold stocks can reduce the volatility. They may have--

ANDREW LO: Historically it has.

AUDIENCE: Yeah. So there are some currencies that are indexed to gold prices. So let's say you're company x. If you are listed in the Dow Jones versus this country, whose currency is indexed to gold, do you expect the company that's listed in the gold index currency to have a lower beta?

ANDREW LO: Well, it could. But on the other hand, the question is, what are they doing to try to hedge that currency exposure? In other words, if they end up hedging all of that exposure, then it doesn't matter anymore, right? So it depends.

But the idea is that, if it has exposure to a 0 beta asset, then you will find that the ultimate fluctuations are going to have less correlation to the market portfolio.

Slomi?

AUDIENCE: Maybe Microsoft has a bigger beta because during this period, the 10 years, Microsoft was a [? growth ?] company, and [INAUDIBLE]. So this is the reason why it has a bigger beta.

ANDREW LO: You know, that's possible. But remember that when we estimate the beta, we're estimating it

using monthly returns. And so we're measuring the covariance on a month-to-month basis, not just the trend. We're measuring fluctuations around that trend.

So the trend alone won't necessarily explain all of it. It has to also be the fluctuations are actually going both up and down, higher than the variability of the market portfolio. Let me continue on with the example. And we're going to come back. I'm going to show how to estimate this, and then you'll develop more intuition from it.

So this makes sense from the smell test in the sense that Microsoft, at least during that period of time, was not necessarily something that you would expect would do well in good times and bad. But something like razor blades and shaving cream we need to use regardless. So that sort of tells us that these betas look about right.

OK. If you agree with the betas, then it turns out that we can actually calculate the required rate of return for each of these two stocks. So if you assume that the risk-free rate is 5%, which is what it was about in that period-- not today obviously, but back in that period, so about 5%. And if, historically, the risk premium, as I told you last time, is about 6%, then when you do the calculations using the security market line, you get a very sharp answer to the question, what are the appropriate discount rates or costs of capital for these two companies?

The answer, for Gillette, it's about 9.86%, for Microsoft, 13.94%. So now if you're sitting in these companies and you're asking the question, we're going to expand our operations, but in order to do that, we have to do an NPV calculation to see whether it's worthwhile. We have to compute the expected net present value of expansion.

And in order to do that, we've got estimates of what our cash flows are going to be for our expansion, but we don't know what the cost of capital is. Well here's the answer for you. You've actually got hard numbers to plug in to your NPV calculations now.

Now, there are a bunch of assumptions that we've made. So we're going to have to go back and justify those assumptions each and every time you use this technology. It's not physics. This is not mathematics. You're applying a set of theories and approximations to a much, much more complex reality.

Every time you apply it, you've got to go back and ask the question, does it make sense? Do these assumptions hold? And if so, great, go ahead and use it. If not, you've got to go back and rederive some of these analytics.

OK, so the security market line is now a line that describes the expected return, or required rate of return, on an asset or a project as a function of the riskiness, where the riskiness is now measured by beta, not by sigma. It's not variance or standard deviation that measures the appropriate risk for most projects.

Most projects, the way you measure their risk is not by sigma. It turns out that the way you measure their risk, for the purposes of calculating the required rate of return, you measure it by beta. OK?

That's a very deep insight. It changes the way we think about risk and expected rate of return. It's not to say that risk, in terms of volatility, doesn't matter. Of course it does.

That is the basis of this entire framework. We started out by saying that people don't like sigma, and they do like mu. They don't like variants. They do like expected rate of return. That still holds.

But in doing so, when we derive all of these implications, what we find is that, as an investor, you don't get rewarded for taking larger and larger amounts of volatility necessarily. You do get rewarded for taking larger and larger amounts of beta. That's the sense in which beta is a better measure of risk. For the purposes of computing the required rate of return, beta is the right measurement, not sigma.

The only cases where sigma is the right measure of risk-- when I say right, I mean where increases in sigma must imply increases in the required rate of return-- is when? When is sigma the right measure of risk for the purposes of computing the required rate of return? For what kind of securities or portfolios?

Yeah?

AUDIENCE: Is it for efficient portfolios?

ANDREW LO: Exactly. For efficient portfolios only. Efficient portfolios meaning these guys, meaning these guys. Anything on this line, then sigma is the right measure. Sigma p over sigma m, that is the right measure for any kind of portfolio that is efficient.

But we know that the typical security, the typical project, the typical division, is not efficient. Efficient, again, meaning you can get the-- you can't get any better expected rate of return for the same risk, or you can't get a lower amount of risk for the same expected rate of return.

So for all of the inefficient securities, portfolios, or projects, you've got to use this relationship, and this relationship tells you beta is what matters, not sigma. Sigma is not the same as beta, except if you happen to be an efficient portfolio.

OK. Question-- do you have a question? No? OK.

All right. So here's an example of the security market line at work. And the slope of this security market line is, of course, the expected rate of return on the market minus the risk-free rate. And the idea behind the security market line is that, no matter what your beta is, you've got a required rate of return that's determined by this slope.

And now, to answer Eduard's question about deviations and how far away you are from efficiency if you deviate from this line, then the vertical distance is the alpha of your portfolio or project or investment opportunity. If markets are working exactly the way they should, then you're on this line. You're always on this line.

And if you're Warren Buffett, you're off of this line. You've got a very large, positive alpha. So if you've got skill, if you can forecast markets, then you will do better than this. But what this framework tells you is that even if you cannot forecast markets, even if you don't know what's going to happen next year to stock prices, you should still do as well as what this line suggests that you can do. OK? On average, this line should be achievable by everybody that understands the basics of portfolio theory.

Now, as I said, the performance evaluation approach to using the security market line is just a measure of the vertical distances. And it can lead to some interesting results. For example, here are three managers. All three of these managers have a 15% expected rate of return. But they have different betas.

And so the question is, if you had money to put into these managers, which would you choose? Well, clearly you would choose manager A because the manager is only supposed to have a 6% rate of return, but, in fact, he's offering 15 for that level of risk. Manager B is just basically doing what you would expect the manager should be doing. And manager C is actually underperforming. Given the risk that manager C is exposing you to, manager C should be doing much better than he is.

And by the way, notice that I've said that the same-- all three managers have the same

volatility, 20%. You can have the same volatility but have different betas.

Betas and volatilities do not necessarily go hand in hand. There is actually a relationship between beta and volatility. We'll talk about that in a little while. But that relationship is not nearly as straightforward as you might think.

Ingrid?

AUDIENCE: Can you estimate a future return of a mutual fund by [INAUDIBLE]. I mean, I understand it's the best you can do, but how realistic is it?

ANDREW LO: Well, so it depends on who you are. If you are a typical index fund manager, you would argue that mutual funds are basically going to provide you with a relatively stable expected rate of return over time. So in other words, it'll fluctuate up and down because it's got some variance, but there is a baseline expected rate of return that a mutual fund offers you.

And that's what people are buying. When you put your money in an emerging market equity mutual fund, you're going to have a higher return on average, on average, than if you put your money in a S&P 500 index fund. Why are you going to have a higher rate of return on average? Because you're going be bearing more risk on average.

The only way to convince you to put your money in an emerging market fund is if it does have that higher expected rate of return on average. So what you're basing these kinds of calculations on is not that I can forecast what mutual funds are going to do next year, but rather, mutual funds offer expected rate of returns that are stable over time.

So what happened last year and the year before and the year before that, when you average it all together, it's about what you're going to get over the next five years. That's it. That's the argument.

AUDIENCE: [INAUDIBLE]

ANDREW LO: Yes. We don't know-- right, exactly. So there is a very large idiosyncratic component that fluctuates year by year, and who knows what that could be. OK. Yes?

AUDIENCE: Last time, when you were describing more [INAUDIBLE].

ANDREW LO: Yes.

AUDIENCE: You had mentioned that for more risk people will expect more return, but not necessarily proportional to the-- so you end up having a line that just goes up exponentially, right? Because you had given the analogy that, OK, now to get investors to be in more risk, to take on more risk, you have to offer a lot more than just [INAUDIBLE].

ANDREW LO: Well, you may. But the equilibrium theory that we've argued has to hold actually says that the risk-reward trade-off is, in fact, linear for efficient portfolios. If it's not for efficient portfolios, then who knows? But the theory of the capital asset pricing model, or the security market line, which is what we derive-- what we what we talked about today, this says that, in fact, it is linear.

So in other words, this result, as I told you, we didn't derive it, but it's a major result that shows that the relationship between risk and expected return, where risk is measured by beta now, is linear. This is a linear relationship. So this is a major advance that we didn't expect.

And, in fact, so what I showed before was the preferences. So in other words, we talked last time about the situation where, suppose that you're an investor, and I start you off at a point like this. And I ask the question, if I want you to tell me where you're going to be such that you're just as well off. You have the same level of utility.

That curve, that indifference curve, is going to look something like this. That's going to be curved. That's what you're talking about. But that's the behavior of one individual.

The point about the CAPM is that if you aggregate all of the individuals together and ask the question, what does the expected rate of return and volatility or expected rate of return and beta look like? How are they related? In fact, it's magical that it actually is linear.

So it's exactly the fact that we didn't expect linearity. Given that there are diminishing marginal returns to risk and reward, you wouldn't expect linearity. But, in fact, it drops out. I mean, this drops out of this tangency portfolio argument, right?

Nothing up my sleeve, this was an argument that we all did together. And we derived this curve from first principles. So this is really an astounding result, but it's even more astonishing that you get this result for all securities, not just for efficient portfolios. OK, other questions?

All right. Let's do another example. So here I want to show you how you can use the security market line, also called the CAPM, C-A-P-M, for Capital Asset Pricing Model.

I want to show you how you can use it to do performance attribution. This is the data from a real live hedge fund manager, who will go nameless since I don't want to get sued by him for any reason. Hedge fund managers are both very wealthy, typically, and also very litigious, so you want to be careful when you talk about them in public.

Hedge fund XYZ had an average annualized return of 12.5% and a return standard deviation of 5.5% from January '85 to December 2002, and the estimated beta over this period was minus 0.028. Now, somebody asked about a negative beta. Well here's an example of a negative beta asset. Positive expected rate of return, negative beta.

So if it's got a positive expected return and a negative beta, you know something can't exactly be right because that doesn't sound like it makes sense in terms of the CAPM framework. And, in fact, it doesn't. It doesn't make sense. Well, let's do the math.

The expected the rate of return is equal to Rf plus beta times the market risk premium. Market risk premium of 6%, risk-free rate of 5%, plug that in with the beta of minus 0.028, and you get that this manager should have earned 4.83% per year. That's what the manager should have earned. In fact, the manager earned a rate of return of 12.% per year.

So that's an alpha. If you define the alpha as what the manager did earn minus what the manager should have earned, you've got an alpha of 771 basis points per year. That's a humongous alpha. Very, very big amount of excess performance.

This is why people are excited about hedge funds. Now, we're not going to talk in great detail about it in this course because it goes beyond the scope of Finance 401. But in an investments course, the next level of sophistication would be to take a look at this and say, OK, is this alpha really alpha, or is it due to other factors, other risks that we're not measuring?

Right now the only risk we're measuring is this tangency portfolio risk, this beta risk. But maybe there are multiple betas out there. We're not going to talk about it in this course, but in 433 you will discuss it. And it will turn out that hedge funds actually do have multiple betas.

So you shouldn't go out and put all your money in hedge funds right away because this extra performance, some of it is due to true genius and insight and unique skill. But part of it is also due to the fact that you're bearing risks that you had no idea you're bearing. And so you've got to be careful about getting on the bandwagon and saying, yeah, give me some hedge funds. I want some of this alpha.

Anon?

AUDIENCE: [INAUDIBLE] But how do we know this is good for the investor? Because they could have invested in the asset [INAUDIBLE].

ANDREW LO: So the question is, how do we know that this is actually good for the investor? Because they could have gotten some returns from the S&P. The way we know that is because we're measuring the expected rate of return relative to the S&P.

So in other words, the way I got this number, this is the excess return on the S&P. That's what the market risk premium is. So in fact, given the beta of this manager, it should have only given you 4.83% return relative to what the S&P would have given you, which is a 6% excess rate of return. And, in fact, what we see is that this manager produced a 12% rate of return, or 7% above and beyond what it was supposed to have done.

So this takes that into account. What it doesn't take into account is how much liquidity risk the hedge fund manager is taking, how much currency risk, how much commodity risk, and a bunch of other risks that are not represented by the tangency portfolio.

Megan?

AUDIENCE: [? When you hear about ?] looking out for beta dressed up as alpha, it's really because there are multiple sources of beta that aren't getting wrapped into that.

ANDREW LO: Exactly. That's exactly right. Recently, a lot of institutional investors have become skeptical of hedge funds because they say, hedge funds alpha is really dressed up data. In other words, hedge fund managers are taking risks that are not captured by this very simple framework.

And so when you run these kind of regressions and do this analysis, you're getting tremendous alpha, but in fact, it's not all alpha. There's other kinds of betas in there. And so there's a whole literature that has developed about multiple, what are called exotic, betas or alternative betas. Again, not part of the scope of introductory finance, but it is something that's covered in investments.

Just another illustration of what this hedge fund has done. Take a look at the growth of $1 invested in the hedge fund over the last 20 years, and you'll see that the blue line is the hedge fund. The red line is the S&P 500. So to your point, Anon, the S&P 500 gave you a wild ride. And so for a while you were doing better than the hedge fund, but, in fact, now this hedge fund

has done quite a bit better.

Now this ends in 2002. I'll give you a little bit of an update. I don't have it here in the graph, but you can use your imagination. It turns out that up to 2007, the blue line is way ahead of the red line.

That's actually changed in 2008. This hedge fund has done very badly this year. Of course, the S&P has done even worse. So the gap is not as wide as it used to be. There's still a gap, but the gap is actually narrowed a bit.

Yeah?

AUDIENCE: [INAUDIBLE] The risk-free rate is changing. The market rate is changing, so--

ANDREW LO: No. In fact, everything is changing. So if you want to take seriously change, you've basically got to figure out how the risk-free rate is changing, the expected rate of return is changing, and the betas actually are also changing. But what I'm trying to do with a simple illustration is use a long period and say, over that entire period, let's average across all of these changes.

Justin?

AUDIENCE: [INAUDIBLE] If you look at the graph and it seems like the differences of the graph were just higher with the market.

ANDREW LO: It is, but you don't adjust for the beta. That's the key, right? The S&P has a beta of 1. This guy, this hedge fund manager, has a beta of 0 or slightly negative. That's the difference.

That's why looking at volatility can be misleading. If you look at volatility, you'd say, well, you know, obviously the S&P has done better. But keep in mind that look how smooth the blue line is. And the lesson of the CAPM is that investors pay for smoothness, but only a certain kind of smoothness.

In other words, smoothness means low volatility, right? The smoothest line, of course, is T-bills. That's a straight line. That will go sort of like this.

And investors are not going to pay a lot for that because that doesn't really help them generate expected rate of return. If you've got expected rate of return and smoothness together, you get a really big, big alpha. And that's exactly what we see here, an alpha of 771 basis points.

Now, here I talk about these multiple sources of systematic risk. I don't want to focus on that for this course because, as I said, it's going to be much more complicated and requires more machinery. But the basic intuition is the same. Instead of just one source of systematic risk, you may have multiple sources.

And so therefore, you're going to have multiple risk premia as opposed to just one. But for now, let's not focus on that. And we're going to focus our attention just on this simple equation, and make sure we understand it and know how to apply it.

Now, I want to go-- I want to give you one more intuition for why it is that beta is the appropriate measure of risk, and not sigma, for arbitrary inefficient portfolios.

And the idea is actually pretty simple. When you think of an investment like Microsoft or like Gillette, you can think of the risk of that portfolio as being-- the risk of that investment in those individual securities as having two components. So when you think of the volatility of Gillette, you can think of the volatility of Gillette coming from two sources.

One source is the aggregate risk that affects all companies. And the second source of risk is the risk unique to Gillette, the fact that they've got a particular manufacturing plant in a particular location of the country, the fact that they've got a specific set of managers that are either good or bad, the fact that they are subject to a very specific set of requirements for producing their blades, who knows, but very specific to that company.

When you think about that kind of risk, let me ask you a question from a purely business perspective. You're the investor. I'm a representative of Gillette. I'm trying to sell you my company's stock. I want you to invest in my company.

And therefore, I have to pay you to take the risk of Gillette. If I tell you that Gillette has these two pieces of risk-- so I'm the representative from Gillette and I tell you that our company is subject to economy-wide fluctuations that will help or hurt our business and unique fluctuations that are specific and special to Gillette-- which of these two risks are you going to be more concerned about from your investment portfolio perspective?

Rami?

AUDIENCE: I think you'd be more concerned about-- as a portfolio as a whole, you look at the economy, but I think, for this specific purpose, [INAUDIBLE] you obviously will pick Gillette-specific--

ANDREW LO: OK. But I want-- you hurried through the first point, and I want you to expand on that a little bit. You said that if you're worried about your portfolio, then obviously the economy-wide risk.

AUDIENCE: Absolutely. If you're worried about the economy-wide risk, and for example, over the next two, three years, you don't think the economy is going to recover, then you're going to just avoid that type of investment as a whole.

ANDREW LO: OK. OK, fine. But on the other hand, you also said something else, which is that if you're comparing between two stocks, then what you're focusing on is the risks that are unique to Gillette.

AUDIENCE: You also might look at how much the economic downturn would affect a company like Gillette.

ANDREW LO: Right.

AUDIENCE: So if the economic downturn, as you said before, affects Microsoft greater than it would Gillette, and you suspect something is going to happen, you'd go for Gillette. If, on the other hand, you didn't suspect that, you might go for--

ANDREW LO: OK. But let's now talk about the negotiations between you and me. I'm a representative of Gillette. I'm trying to get you to invest with us. And I've got two sources of risk that you might be concerned about, market risk or Gillette-specific risk.

From your portfolio perspective, since you just care about maximizing the value of your portfolio, you're not worried about Gillette in particular. You're not management. You're an investor. I'm management. I'm worried about Gillette.

I couldn't care less about your portfolio, I'm sorry to say. What I care about is my company, but what you care about is your portfolio. From your portfolio perspective, what are you going to care more about, Gillette-specific risk or the macroeconomic risk that I represent to your portfolio?

AUDIENCE: When I'm talking to you, I care more about your specific risks.

ANDREW LO: Well, I'm asking a question though about your portfolio. What you care about is your portfolio. I understand that when you're talking to me, you're going to be asking me about my company-specific risk to try to get a handle on it. But is that what you ultimately are going to be concerned about?

AUDIENCE: I would be concerned about, obviously, the macro portion of it and how you fit into that. But I'd look at your beta and see--

ANDREW LO: You don't know the CAPM, though. So now you're cheating because you now know the CAPM. But suppose you didn't. What I'm trying to get at is the intuition, a businessman's intuition, for what you would care about more in terms of what I do to your portfolio.

Yeah, Sema?

AUDIENCE: Didn't you say, two lectures ago, that the idiosyncratic risk is [? their survival? ?]

ANDREW LO: Yes.

AUDIENCE: So, you care more about systematic risk--

ANDREW LO: Why?

AUDIENCE: Because only one [INAUDIBLE] in my portfolio.

ANDREW LO: That's exactly right. The idea is that if it's Gillette-specific risk, then by definition, if you're holding a well-diversified portfolio, then you're not going to care about that because that's going to average out to nothing. Now, of course, we have to think about the case where you're not holding a diversified portfolio, but let me get back to that in a minute.

I'm assuming that all of you are good business folks, which means that you're going to be holding a diversified portfolio. You're not going to concentrate all your bets on one particular kind of investment, right?

So if you are already holding a very well-diversified portfolio, then when you interview me as a potential investment opportunity, do you really care about the idiosyncratic risk? Because that risk is going to be diversified away. What you care about from a portfolio perspective is, how much am I going to be contributing to your overall risk?

And now that you know the CAPM, you understand the logic of it. It says, you care about my beta because my beta is a measure of the amount of risk I'm going to be adding to your portfolio that you cannot get rid of. How do you know you cannot get rid of it?

Well, by definition it's the market portfolio. Everybody's holding it. Nobody wants to get rid of it

completely. You're on the capital market's line. That's the best you can do.

So this notion of firm-specific risk versus economy-wide risk, that distinction is a really important one. And I'll show you the mathematics of it in a minute, but I want to give you the intuition.

As a result, think about this conversation happening not just for Gillette, but for every company in the economy. If it's the case that portfolio managers that are buying stocks only care about the systematic risk, about the market risk, about the risk that they cannot get rid of, and then you have to reward them for that, then what that means is that you don't have to reward them for idiosyncratic risk. Why? Because that's not risk that you are forced to bear.

There's nothing that says you have to bear idiosyncratic risk. How do you get rid of idiosyncratic risk if you don't want to bear it? Diversify. Exactly. Just buy 10 stocks instead of one, and then you're diversified.

20 stocks is better than 10. And mathematically, after 50 stocks, you're basically diversified. You're done.

So nobody should be holding two or three stocks. Or if they do, they are bearing risk that they need not bear. And they may want to do it for other reasons.

For example, as a manager of Gillette, I believe in the company. I want to demonstrate to my shareholders that I'm tied to the company, so I'm going to hold a lot of my wealth in Gillette stock. That's not well diversified. I'm holding a lot of Gillette-specific risk.

That's not a smart thing to do from an investment point of view, but that is a smart thing to do from a management point of view because I'm tying my fate to the fate of the company. It's definitely not a smart thing from an investments perspective.

If you're in a financial services sector, you should not be buying financial services stocks. If you're in the pharmaceutical sector, you should not be buying biotech stocks. And yet we do that for reasons other than portfolio management.

But given that I'm teaching you about portfolio management, I'm not going to focus on those other reasons. If this were an organizational studies course, you'd be getting a different perspective. And you should get a different perspective. But for the purposes of building financial wealth, what you want to do is to focus on how much the systematic component is

contributing to your risk, because the idiosyncratic component you don't have to bear.

Yeah?

AUDIENCE: So like an employee purchase plan, where, if you're in financial services, you're working for a mutual fund company, and they offer you 10%, 15% you put your salary in, and you can buy stock--

ANDREW LO: Yes.

AUDIENCE: Would you recommend not doing that?

ANDREW LO: Well, I recommend not doing it from the financial perspective, but I may recommend doing it from the management, or managerial incentives, perspective. The reason that companies do that is very simple. They're trying to suck you in. They're trying to get you to be more intimately tied to the company so you'll act like an owner of the company, as opposed to an employee.

And if you act like an owner, you will engage in behavior that is much more productive for building the company's wealth, rather than as an employee. But from your personal perspective, you're bearing risk that you don't need to. So you know, the analogy that I give-- I've given before-- it may work for some of you. It may not. Let me explain.

Anybody know how much window washers in midtown Manhattan get paid on an annual basis? You know what I'm talking about? These are the folks that climb up on these two-foot catwalks that are 40 stories high, and they wash the windows of these skyscrapers. That's a pretty risky job.

Anybody know what their annual salary is, when you annualize it? I actually decided to find out one day. I was kind of curious about that because, you know, there's a trade-off of risk and return.

And that's really risky. You know, there was one day when I was staying at a hotel. I think it was the Millennium, and I was on the 30th floor, and it was a windy winter day. And, sure enough, there was somebody there pulling up the thing, cleaning the window, and looked happy as can be. You know, no problem.

And I was thinking, boy, this guy's taking a lot of risk, you know? And I hope he's getting paid

for it. And these salaries are determined by supply and demand. What do you think it would be? Anybody have a guess?

Yeah?

AUDIENCE: About $70,000.

ANDREW LO: $70,000, OK. Anybody else? Justin?

AUDIENCE: $125,000.

ANDREW LO: $125,000?

[LAUGHTER]

That's higher than some NBA starting salaries.

[LAUGHTER]

OK. Leah?

AUDIENCE: $30,000.

ANDREW LO: $30,000.

AUDIENCE: Do they take interns?

ANDREW LO: Insurance? Interns. Interns. I don't know about that.

[LAUGHTER]

Well, so when I looked last time, which is about four years ago, it turns out that the typical window washer for these skyscrapers gets paid about $60,000 a year, annual salary. $60,000. Now, you know, I don't know whether you think that's a lot or a little. But, seems to me that that compensation reflects the kind of risk that we're talking about. And you know, you have no educational requirements, no degrees, no certifications. You just show up and, you know, up you go.

Now let me ask you a question. Suppose that a window washer comes to the job who happens to really enjoy dancing while he washes windows. In particular, he dances that, you know, the Irish jig or whatever.

[LAUGHTER]

You know what I'm talking about? You know, that dancing, the very-- and he just likes to do that while he's washing windows on the 40th floor. You agree that that's more risky, right? Do you think that that particular individual gets paid more than $60,000 a year?

Why? He's taking more risk. Why not? Why isn't he getting paid more?

Exactly. He doesn't have to take that risk. That's not part of the job. He can choose to take that risk, but he's not going to get compensated for it because it's not necessary. And there are 100,000 people behind him waiting in line to get that job that won't necessarily need to take that risk.

Yeah?

AUDIENCE: I thought of it differently. If you had a long, short portfolio, would it be the other way around? Because you wouldn't care about the market risk. You can hedge that out. You're thinking only about specific risk. So like in that case, I would short the guy who likes to jig. And I would invest in the guy who didn't.

ANDREW LO: Well, that depends on whether or not doing the Irish jig actually makes you wash windows better or worse, in other words, where there's an alpha to that risk. It may be the case that dancing the Irish jig actually helps you scrape off dirt that much more effectively. In which case, you may not want to short him because then he will earn a premium in certain markets. Those with lots of pigeon poop on the windows.

[LAUGHTER]

So we can get into this analogy more deeply than we should.

[LAUGHTER]

But the point is that, when you think about the CAPM, all it's saying is that you get what you pay for. And you pay for what you get. In other words, if there is a certain amount of risk in a particular investment that is risk that nobody can get rid of easily-- in other words, you have to bear it-- then you have to pay for it. Because otherwise people aren't going to do it.

However, if there's risk in a particular company that you don't have to pay for, that you don't

have to take, then you don't have to pay for it. That's all that the CAPM is saying. Beta is a measure of that hard little pellet of risk that you can't get rid of. And the variance is the measure of the entire risk in a particular portfolio.

The only case where variance and beta are the same is for what kind of portfolio? An efficient portfolio. What is an efficient portfolio? It's one that is already maximally diversified. By adding more securities, you are not going to do any better than that straight line.

All of these portfolios have been completely diversified. How do I know that? Because you're at the tangency portfolio. There is nowhere to go in the Northwest region off of that line.

So for all of these securities, the sigma and the beta are literally numerically identical because there is no more extra risk in the portfolio. It's been diversified away. But for Gillette, for Microsoft, for IBM, General Motors, and Motorola, each of these contain both beta risk and non-beta risk.

The non-beta risk is that Irish jig that you don't have to do while you're washing windows. And you're not going to get paid for it, I'm sorry to say. So the relationship that you want to focus on is the capital asset pricing model's security market line.

Measure the stuff that you're going to get paid for. And this is what you're going to get paid for it.

Yeah, Ingrid?

AUDIENCE: When you go back into the sort of the real world and include transaction costs, how do they enter here?

ANDREW LO: Well, so there have been versions of the CAPM with transactions cost. And it turns out that it doesn't change things too much. If you impose transactions cost on all securities, and say that there's a percentage cost for going in and out, you can derive a net of fee transactions costs.

It won't affect the pricing necessarily. What it will affect is the dynamics, the trading. What it will mean is that you will rebalance your portfolios less frequently than you otherwise might. But there is still going to be a relationship between the net of fee transactions costs of these securities.

So you can actually look at the transactions cost as a way to deduct the expected rate of

return from each of these securities. So transactions cost is not a big issue, but there are other issues that I'll come to that will be a problem for this.

AUDIENCE: Looking at this, [INAUDIBLE] one of the best, efficient portfolio that I can have is basically the index. If I want to allocate my money globally, should I buy indexes according to the market capital of the market?

ANDREW LO: Yeah.

AUDIENCE: This is the most efficient portfolio I would--

ANDREW LO: Right, so that's a great question. And we actually have a separate course on international finance that deals just with those kind of issues because they're so tricky. But I'll give you the short answer.

According to the theory, this tangency portfolio is not just for the US stock market. It should be for the world stock market, everything. So this tangency portfolio should not be the S&P 500 or the Russell 2000, it should be the MSCI, or EAFE, index that has all of the assets in the world weighted relative to their market cap in that particular currency of the investor that you happen to be.

So if you're a US investor, it'll be in dollars. If you're a Brazilian investor, it'll be in real. It'll be in whatever currency you trade in. But that presupposes that there's capital market integration throughout the world.

So in other words, if I call this the world portfolio, implicitly I'm assuming that you're free to trade stocks in any part of the world freely. There are no barriers to trading. And we know that that's not the case. There are barriers, in fact.

So what that means is that the CAPM, applied to international stocks, is an approximation that may actually be worse than applying it country by country and then looking to see whether there are any distances or discrepancies across those different countries.

But people have come up with international versions of the CAPM. And they don't work very well. At least, they didn't as of 10 years ago. Within the last 10 years, a lot has changed. So it could be that capital market integration has made the world CAPM look a lot better in terms of the data.

OK. Other questions?

All right. So now I'm going to talk about implementing it. And we're going to deal with all of the messy issues that I tried to put off a lecture ago.

How do you take this thing out for a spin? Well, one thing you could do is to try to test the CAPM to see if it works. And one way to test it is to ask the question, if we assume that all securities are priced according to this equation, then another way to write the equation that doesn't rely on expected returns, but relies on realized returns, is to write it as a regression equation, as you did in DMD.

The regression equation is the return, the actual realized return on security i, is given by the risk-free rate plus beta times the realized return on the market minus the risk-free rate plus epsilon. Epsilon is the error term, the disturbance, the residual, that is giving you the fluctuations around the expected value.

So when we remove the expected values from this equation, we have to stick in this epsilon term that sort of bounces around. By the way, when you look at this equation, you now have an explicit representation for systematic risk and idiosyncratic risk.

For a given return, it's comprised of three pieces, the risk-free rate, beta times the market return that bounces around, and this epsilon is the Irish jig. That is the idiosyncratic bouncing around that you don't get any reward for. How do you know you don't get any reward for it? Because on average, the expected value of this is equal to 0.

How do I know that? By definition that's how I got from this equation down here. If you take the expected value of this equation, the only way that the expected value of this gives you this equation on the top is if that epsilon has a 0 mean. So you don't get paid for bearing epsilon risk. It's there. And for some stocks it's huge. But you don't get paid for it because you don't have to bear it.

And the reason you don't have to bear it is, if you take 50 of these stocks and stick them in a portfolio, the epsilons average out to 0. How do I know that? Well, this relies on a piece of mathematics that's known as the law of large numbers. You may have heard that term used in casual conversation, but it's actually a real theorem.

What it says is that when you have large, large numbers of fluctuations that are not correlated with each other-- and by definition, idiosyncratic risks for Gillette and Microsoft and other

companies are not correlated because they're idiosyncratic. They're unique to those firms. That when you get a large number of these uncorrelated fluctuations, that in the limit, they actually go to 0. You can disregard them.

So the law of large numbers is what tells you that this idiosyncratic risk is not going to be something you will get paid for.

So this is the CAPM relationship using actual data. And if we stick in an alpha term to represent deviations from the CAPM, and I subtract the risk-free rate from both sides, just to have everything in excess returns, then the CAPM reduces to the hypothesis that the alpha-- across all stocks, all managers, all projects, the alpha is equal to 0. That's what the CAPM says.

And if you want to formulate it strictly in terms of total rates of return, it says that the alpha has to be equal to the risk-free rate times 1 minus beta. This is a different alpha than this alpha. This alpha represents the excess rate of return. OK, so let's do it. Let's see whether or not it's true.

Let's take a bunch of stocks, subtract the risk-free rate from the stock returns, run a regression of that stock's return on a constant and the market excess return, and let's see whether the intercept is equal to 0. Well, if you do this for two stocks, Biogen and Motorola-- I've done this from 1988 to I think 2006.

When you run that regression, here's what you get. For Biogen, the beta is 1.43, the intercept 1.61%, and the standard error is 1.1%. And then Rf times 1 minus beta is minus 2.1%. This should be equal to the alpha in that previous equation that I gave you.

So in other words, the alpha that we've estimated for Biogen using this CAPM regression is 3.7%, or on a monthly basis 45% alpha. Biogen is an incredibly good buy according to the CAPM, if you believe the CAPM. All right.

Now that's Biogen. What about Motorola? During the same period, when you estimate Motorola, it's got a beta of 1.42. And we're estimating an alpha not quite as big, but 23.5% percent on an annualized basis. That's still pretty big.

So if you run this regression and analyze it, this is what you would conclude, that these two stocks are wonderful buys. What we're going to talk about next time is whether this

interpretation really makes sense, or whether we've got some missing factors, or whether we're measuring things improperly.

We're going to need to do a bit more work. But we're very close to being able to figure out exactly how all of these pricing models work in tandem with the kinds of risk budgeting calculations that we're going to need to do for the rest of the course. So I'll see you on Monday.