Quiz For Lecture # 11

European Call Option using Black-Scholes/Merton

Consider a European call option on a stock when there are ex-dividend dates in two months and five months. The dividend on each ex-dividend date is expected to be \$ 0.50. The current share price is \$50 and the strike price is \$50. The stock price volatility is 20% per annum and the risk free rate is 8.329% per annum, the volatility is continuously compounded, the interest rate is a simple interest rate, the time to maturity is six months. Calculate the call option's price and delta using Black-Scholes/Merton.

(<u>Reminder</u>: Showing the *essential* steps along the way will enhance your chances of partial credit in case you make an error.)[1]

$$r = n \cdot \ln\left(1 + \frac{r_1}{n}\right) \qquad 0.08 = 1 \cdot \ln(1 + 0.08329/1)$$

$$d_y = 0.5 \cdot e^{-0.08 \cdot \frac{2}{12}} + 0.5 \cdot e^{-0.08 \cdot \frac{5}{12}} = 0.9770$$

$$c_0 = (S_0 - d_y) \cdot N(d_1) - Ke^{-r_f T} N(d_2)$$

where

$$d_1 = \frac{\ln\left(\frac{S_0 - d_y}{K}\right) + \left(r_f + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{T}$$

$S_0 = 50$, $K = 50$, $r_f = 0.08$, $\sigma = 0.2$, $T = 0.5$.

We can use the discounted dividend of 0.977 and deduct it from the spot price of the stock. Using the B-S/M formula on a dividend-paying stock,

$$d_1 = \frac{\ln\left(\frac{50 - 0.977}{50}\right) + \left(0.08 + \left(\frac{0.2^2}{2}\right) \cdot \frac{6}{12}\right)}{0.2 \cdot \sqrt{\frac{6}{12}}} = 0.2140 \quad \text{and} \quad d_2 = d_1 - 0.2\sqrt{0.5} =$$

0.0.0726

[1] This question is taken from a former midterm exam. It was worth 25% of the entire midterm.

This implies that: $N(d_1) = 0.58$, $N(d_2) = 0.53$

so that the call price c_0 from the B-S/M formula is:

$$c_0 = (50-0.977) \cdot 0.58 - 50 \cdot 0.53 \cdot e^{-0.08 \times 0.5} = 3.26.$$