

15.433 INVESTMENTS

Assignment 3: Futures

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1. A futures contract is available on a company that pays an annual dividend of \$5 and whose stock is currently priced at \$200. Each futures contract calls for delivery of 1,000 shares of stock in one year, daily marking to market, an initial margin of 10% and a maintenance margin of 5%. The current Treasury bill rate is 8%. (4 points)
- (a) Given the above information, what should the price of one futures contract be? (0.5 points)
- (b) If the company stock decreases by 7%, what will be, if any, the change in the futures price? (0.5 points)
- (c) As a result of the company stock decrease, will an investor that has a long position in one futures contract of this company realize a gain or a loss? Why? What will be the amount of this gain or loss? (1 point)
- (d) What must the initial balance in the margin account be? Following the stock decrease, what will be, if any, the change in the margin account? Will the investor need to top up the margin account? If yes, by how much and why? (1 point)
- (e) Given the company stock decrease, what is the percentage return on the investor's position? Is it higher, equal or lower than the 7% company stock decrease? Why? (1 point)

Answers:

(a) $F = 200 * (1 + 0.08 - (5/200)) = \211

(b) $F = 200 * (1 - 0.07) * (1 + 0.08 - (5/200)) = \196.23

- (c) An investor with a long position in one futures contract agrees to buy 1,000 stocks of the company in one year at \$211. Therefore this investor will benefit only if the futures price increases. In this case, the futures price has decreased and the investor will therefore realize a loss of:

$$(196.23 - 211) * 1,000 = -\$14,770 \quad (1)$$

- (d) The change in the margin account will be minus \$14,770. The initial required margin was $1,000 * \$211 * 10\% = \$21,100$ so now the balance will be $\$20,000 - \$14,770 = \$6,330$. This balance ($\$6,330 / \$211,000 = 3\%$) is below the required 5% ($\$10,550 / \$211,000 = 5\%$) maintenance margin. The investor will need to top up the margin account by at least $\$10,550 - \$6,330 = \$4,220$, else, the broker will close out enough of the investor's long position to meet the required maintenance margin.
- (e) The percentage return on the investor's position will be $-\$14,770 / \$21,100 = -70\%$ which is 10 times higher than the actual decrease in the stock price. The 10-to-1 ratio of percentage changes reflects the leverage inherent in the futures contract position.

2. You have recently been appointed CFO of a large multinational corporation based in Frankfurt. The CEO has asked you to raise 8,000,000 Euros one year from now to fund a strategic investment. The company has to raise the capital either through a U.K. (in £) or a Germany (in Euros) based bank. The annual interest rates on U.K. and German bills are 8% and 6% respectively. The current exchange rate is 1.5 Euro per 1 £ and the futures price for delivery one year from now is 1.46 Euros per 1 £. (4 points)

- (a) Where would you lend and where would you borrow? (0.5 points)
- (b) You realize that given the above data, you can benefit from an arbitrage opportunity that would allow you to raise the necessary capital at no cost for the company. Describe in a table like the one below the necessary actions (and associated cash flows in Euros) you need to take now and in one year in order to earn 8,000,000 Euros in one year at no cost. (2 points)

Action Now	Cash Flow in Euros	Action in 1 Year	Cash Flow in Euros*
Total		Total	

- (c) Assuming the interest rates and the current exchange rate given above are correct. What should the futures price of 1 be in Euros according to the interest rate parity theorem? (0.5 points)
- (d) Discuss why wouldn't all investors decide to invest their money in the U.K., even though the interest rate in the U.K. (8%) is higher than in Germany (6%). (1 point)

Answers:

- (a) Given the interest rates the best approach would be to borrow the capital in Germany, where the interest rate is lower and lend it in the U.K. where the interest rate is higher.
- (b) This is possible by simultaneously lending and borrowing. We can come up with an investment structure that requires an initial outlay of 0 Euros and yet will a benefit of approximately 8,000,000 Euros in the future.

Action Now	Cash Flow in Euros	Action in 1 Year	Cash Flow in Euros*
You lend XM in the U.K.	$-X*1.5M$	The amount you lent is repaid to you and you exchange it into	$1.08*(XM)*E1$
You borrow $X*1.5M$ Euros in Germany	$+X*1.5M$	You repay the amount you borrowed	$-1.06*(X*1.5M)$
You sell forward $1.08*XM$ in Euros		Unwind	$1,08*XM*(1.46-E1)$
Total	0	Total	8.0 Euros

$$1.08*(XM)*E1-1.06*(X*1.5M)+ 1,08*XM*(1.46-E1)=8.0M$$

$$-1.06*(X*1.5M)+ 1,08*XM*1.46=8.0M$$

$$-1.06*(X*1.5M)+ 1,08*XM*1.46=8.0M$$

$$X=£606,061 @£606.1M$$

* Where E1 is the exchange rate between Euros and in 1 year.

The transaction would be as follows:

Action Now	Cash Flow in Euros	Action in 1 Year	Cash Flow in Euros *
You lend £606.1M in the U.K.	-909.15M	The amount you lent is repaid to you and you exchange it into	$1.08*(606.61M)*E1$
You borrow 909.15M Euros in Germany	+909.15M	You repay the amount you borrowed	$-1.06*(909.15M)$
You sell forward $1.08*£606.1M$ in Euros		Unwind	$1,08*£476.2M*(1.46-E1)$
Total	0	Total	8.0M

(c) The futures price would be:

$$F_0 = E_0 * [(1+r_{GERMANY}) / (1+r_{U.K.})]^T = 3 \text{Euros} / 1 \text{£} * [(1.06) / (1.08)]^1 = 2.94 \text{DM} / 1 \quad (2)$$

(d) They may not because the Euros may be appreciating relative to the £. It is true that investments in Germany will grow less than investments in the U.K. but it is also through that as time passes 1DM is worth more £. This appreciation of the Euros with respect to the £ can be seen by the fact that E0 is 1.5 Euros/1 £ while F0 is 1.47 Euros /1 £. The future appreciation of the Euros with respect to the £ compensates for the difference in interest rates between the two countries.

3. You are required to make a payment of 1,000,000 euros in each of the next four years. Your revenues are in dollars and the current dollar-euro exchange rate is \$.90 per euro. You believe that the dollar will depreciate and would like to lock in the four payments of 1,000,000 euros each at today's exchange rate. In order to do so you decide to enter into a swap agreement. Assume that the U.S. and euro yield curves are flat at 6% and 8% respectively. (2 points)

(a) What will be the swap rate on this agreement to exchange currency over a four-year period? (2 points)

Note: Remember that a swap can be viewed as a portfolio of forward transactions, but instead of each transaction being priced independently, one unique forward price is applied to all transactions.

Answer:

(a) Solve the following equation:

$$\begin{aligned} S/(1.06)^1 + S/(1.06)^2 + S/(1.06)^3 + S/(1.06)^4 &= [.9 * (1.06/1.08)^1] / (1.06)^1 + \\ & [.9 * (1.06/1.08)^2] / (1.06)^2 + \\ & [.9 * (1.06/1.08)^3] / (1.06)^3 + \\ & [.9 * (1.06/1.08)^4] / (1.06)^4 \end{aligned} \quad (3)$$

$$S/(1.06)^1 + S/(1.06)^2 + S/(1.06)^3 + S/(1.06)^4 = 2.9809 \quad (4)$$

$$3.4651 * S = 2.9809 \quad (5)$$

$$S = \$.8602 \text{ per euro} \quad (6)$$

In each of the following 4 years you will pay a fixed amount of \$860,266 in exchange for 1,000,000 euros.