Incomplete Contract Renewal:

A Repeated Prisoner’s Dilemma Model of Buyer-Supplier Relationships and Noncontractible Investments

Sinan Aral
IT Group - MIT Sloan

Note: This is a request for input – NOT a presentation of a final model… I hope to get lots of feedback!

Goal:

Extend Bakos & Brynjolfsson (1993) "From Vendors to Partners" *beyond a one shot game* setting…

to a setting that accommodates the influence of:

- *long term cooperative behavior* on buyer-supplier relationships by including considerations of:

- the *discounted value of future partnership* and

- the threat of partnership termination

in the traditional property rights framework.

A One Shot Game: Bakos & Brynjolfsson (1993) based on GHM

- $N + 1$ risk-neutral firms indexed by $i = 0,1,\ldots,N$, including a buyer firm ($i = 0$) and N identical supplier firms ($i = 1,\ldots,N$) connected via an interorganizational system.

0 ——— 1 ——— 2

Stage 0

- Buyer chooses n ≤ N suppliers to participate in a production network

Stage 1

- Each firm *i* makes a private, nonverifiable investment x_i at cost $c_i(x_i)$.

Stage 2

- Production takes place, goods are delivered and payments are made.
- MC = 0
- Bargaining over the appropriation of total value takes place. Each participant receives payoffs proportionate to its Shapley value.

- Stage 0: buyer chooses the number of suppliers $n \leq N$
- Stage 1: each firm *i* makes a private, non-verifiable investment x_i at a cost $c_i(x_i)$
- Stage 2: *Production*: A set of firms in a coalition $\underline{S}$ can generate value $v(\underline{S}, \boldsymbol{x})$, where $\boldsymbol{x}$ is the vector of investments.

Bargaining: According to Shapley

$$B_i(\boldsymbol{x}) = \sum p(S)\,[v(S, A|x) - v(S\backslash\{i\}, A(S\backslash\{i\})|x)]$$

First Order Conditions (2 Players):

Buyer: $\frac{1}{2} v^0(\{0,1\}, (x_0, x_1)) = c_0'(x_0)$

Supplier: $\frac{1}{2} v^1(\{0,1\}, (x_0, x_1)) = c_1'(x_1)$

General First Order Conditions:

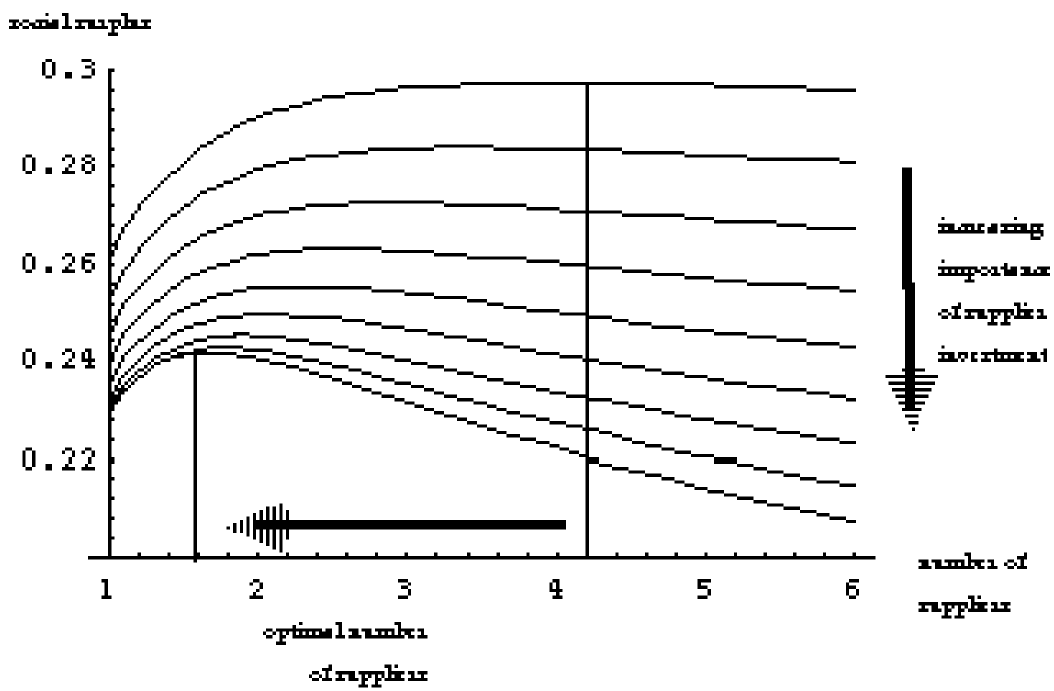
Buyer: $(n/(n+1))\, v^0(\underline{S}, \boldsymbol{x}) = c_0'(x_0)$

Suppliers: $(1/n(n+1))\, v^i(\underline{S}, \boldsymbol{x}) = c_i'(x_i)$

Adding Incomplete Contracts Theory: Bakos & Brynjolfsson (1993) based on GHM

- Suppliers have less incentive to invest when the buyer has more supplier options.

- As the non-contractible investments of the suppliers become more important, total social surplus and thus buyer surplus is maximized with fewer suppliers.



- They also model Coordination Costs and Fit Costs:

$$\text{Buyer Max}_n \quad B(x_i^*) - nK + \text{Max } \varepsilon_i$$

My Question: When you consider the benefits of long term relationships, (and the costs of losing long term partnerships) ***do the suppliers have an extra incentive to invest optimally?***

A Repeated Prisoner's Dilemma Game:

- An infinitely repeated prisoner's dilemma game $G^{\infty}(\delta)$, with $N + 1$ risk-neutral firms indexed by $i = 0,1,\ldots,N$, including a buyer firm ($i = 0$) and N identical supplier firms ($i = 1,\ldots,N$) connected via an interorganizational system, where δ $[1,0]$ is a discount factor.

- Stage 0: buyer chooses the number of suppliers $n \leq N$

- Stage 1: each firm *i* makes a private, non-verifiable investment *xi* at a cost $c_i(x_i)$

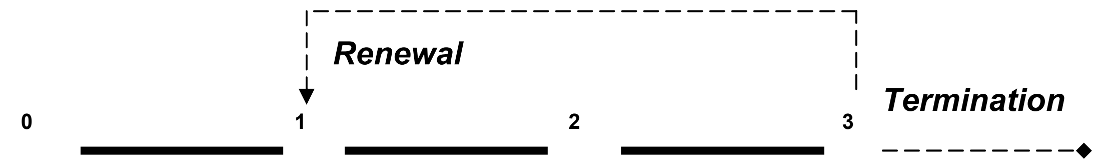
- Stage 2: *Production*: A set of firms in a coalition S̲ can generate value $v(\underline{S}, \boldsymbol{x})$, where $\boldsymbol{x}$ is the vector of investments.

Bargaining: According to Shapley

$$Bi(\boldsymbol{x}) = \sum p(S)\,[v(S, A|x) - v(S\backslash\{i\}, A(S\backslash\{i\})|x)]$$

- Stage 3: Contract Renewal or Termination

P(Renewal) = $f_i(x_i)$
P(Termination) = $1 - f_i(x_i)$



Stage 0
- Buyer chooses n ≤ N suppliers to participate in a production network

Stage 1
- Each firm *i* makes a private, nonverifiable investment x_i at cost $c_i(x_i)$.

Stage 2
- Production takes place, goods are delivered and payments are made.
- MC = 0
- Bargaining over the appropriation of total value takes place. Each participant receives payoffs proportionate to its Shapley value.

Stage 3
- The Buyer firm (i = 0), chooses whether to renew or terminate the network.
- The Probability of Renewal is a function of marginal contribution of suppliers' investments $f_i(x_i)$.
- Termination results in a fallback value of *z* for suppliers in the network, which represents the net present value of supplier firms' options outside of the relationship with firm 0.
- Renewal returns the game to P1, Termination ends the game for the participating suppliers who receive *z*.

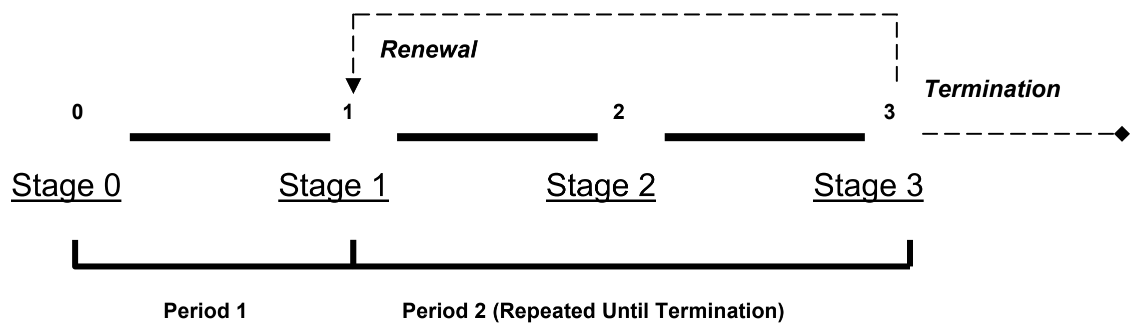
A Repeated Prisoner's Dilemma Game:

- Let $v^i(\underline{S}, \boldsymbol{x})$ denote the payoff per period to a supplier *i* while engaged in a production partnership with the supplier.

- The supplier chooses x_i to maximize the discounted present value v_i of being included in the partnership network, where the flow of payoffs per period is $v^i(\underline{S}, \boldsymbol{x})$.

- Given the discount rate δ and the present value of fallback options for the supplier z, the supplier's payoff from the repeated game is:

$$v_i = \frac{1}{(1+\delta)} \left[v^i(\underline{S}, \boldsymbol{x}) + f_i(x_i)\, v_i + 1 - f_i(x_i)\, z_i \right]$$



<u>A Repeated Prisoner's Dilemma Game</u>:

	C $v^0(\underline{S}, \boldsymbol{x}) = c_0'(x_0)$	D $(n/(n+1))\ v^0(\underline{S}, \boldsymbol{x}) = c_0'(x_0)$
C $v^i(\underline{S}, \boldsymbol{x}) = c_i'(x_i)$	?	? z
D $(1/n(n+1))\ v^i(\underline{S}, \boldsymbol{x}) = c_i'(x_i)$	?	? z

$$v_i = \frac{1}{(1+\delta)} \left[v^i(\underline{S}, \boldsymbol{x}) + f_i(x_i)\, v_i + 1 - f_i(x_i)\, z_i \right]$$

<u>Open Questions</u>:

1. If the Shapley value returns: (the sum of marginal contributions to coalitions*probabilities)… then why: $(1/n(n+1))\ v^i(\underline{S}, \boldsymbol{x}) = c_i'(x_i)$ … Shouldn't the number of other players be in $v^i(\underline{S}, \boldsymbol{x})$? This will affect the cooperate and defect plays…

2. Should the probability of contract renewal be based on $\boldsymbol{x}$, or rather on the total surplus generated by the coalitions and the investments – remember, the buyer can only see the total surplus as a signal of the investments according to GHM…. Maybe f(x) is just the total surplus - $v(\underline{S}, \boldsymbol{x})$?

3. How to model the game more appropriately…

BACK UP