

A Simple and Effective Component Procurement Policy for Stochastic Assembly Systems

A paper by Jeremie Gallien and
Laurence M. Wein

Presentation by Theophane Weber

This summary presentation is based on: Gallien, Jérémie, and Lawrence Wein. “A Simple and Effective Component Procurement Policy for Stochastic Assembly Systems.” *Queueing Systems Theory and Applications* 38 (2), 2001.



Introduction

- Framework: Procurement policies for assembly systems
- Make to stock / assemble to order
- Uncapacited / capacited suppliers
- Single / Multi items
- Stochastic / Deterministic



Literature review

- Usually capacitated
- When uncapacitated, usually deterministic
- When stochastic, usually algorithmic / computationally intensive
- Glasserman and Wang develop a simple and effective stochastic, multi items system with capacitated suppliers



The problem

- Make to stock
- Uncapacitated suppliers
- Stochastic supply
- Single item, instantaneous assembly

=> Simple and efficient policy



Presentation

- **Model**
- Analysis
- Simulation
- Conclusion



Model

- Suppliers
- Cost and objective
- Policies
- Synchronization assumption



Suppliers

(See Figure 1, page 7, in the Gallien and Wein paper.)



Cost and objective

- Z = steady state RV of the net inventory
- Z^i = component i , cost h_i
- $Z^+ = \max(Z, 0)$: finished goods, cost h
- $Z^- = \max(-Z, 0)$: backorder, cost b

$$C = h \cdot E[Z^+] + \sum (h_i E[Z^i] + b \cdot E[Z^-])$$



Policies

Pre specified class:

- Component base stock policies

$$[s_1, s_2, \dots, s_n]$$

- Finished goods base stock policies

$$[S, l_1, l_2, \dots, l_n]$$



Synchronization assumption

- Assembly performed with components belonging to the same set of replenishment orders
- Typically suboptimal as opposed to asynchronous FCFS behavior



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Analysis

- Problem formulation
- Deterministic lead times
- Approximation for solving
- Gumbel distributions and generalization



Problem formulation

Assembly line Queue:

Q=number of replenishments

- Arrival of orders : Poisson(λ)
- Number of servers : infinite
- Service time: $\text{Max}(X_i+L_i)$

$\Rightarrow Q=\text{Poisson}(\rho), \rho= \lambda E[\text{max}(X_i+L_i)]$



Problem formulation

Steady state inventory and back-order

- $Q = \text{Poisson}(\rho)$, $\rho = \lambda E[\max(X_i + L_i)]$
- $Z + Q = S$
- $Z = Z^+ - Z^-$

$$\rightarrow E[Z^+] = e^{-\rho} \sum_{j=0}^S (S - j) \cdot \frac{\rho^j}{j!}$$

$$\rightarrow E[Z^-] = E[Z^+] - S + \rho$$



Problem formulation

Component queue
(queueing network)

(See Figure 3, Equation 5, and Equation 6, all on page 10, of the Gallien and Wein paper.)



Deterministic Lead Times

- Separation of S and (l_1, l_2, \dots, l_n)
- For (l_1, l_2, \dots, l_n) , S^* is the smallest integer such that
$$P(Q \leq S^*) \geq \frac{b}{b+h}$$
- Reduction to single variable
$$l_i = \frac{\rho}{\lambda} - X_i$$
- Resolution:
$$l_i^* = \max_j (X_j) - X_i$$
- Interpretation



Approximation

(See page 13, section 3.4, in the Gallien and Wein paper.)



Gumbel CMT1 Lead times

- Problem: How to find an analytical solution?
- $E[\max(X_i+L_i)]$?
- Distribution with CMT property:
The only uniparameter family is the Gumbel univariance family

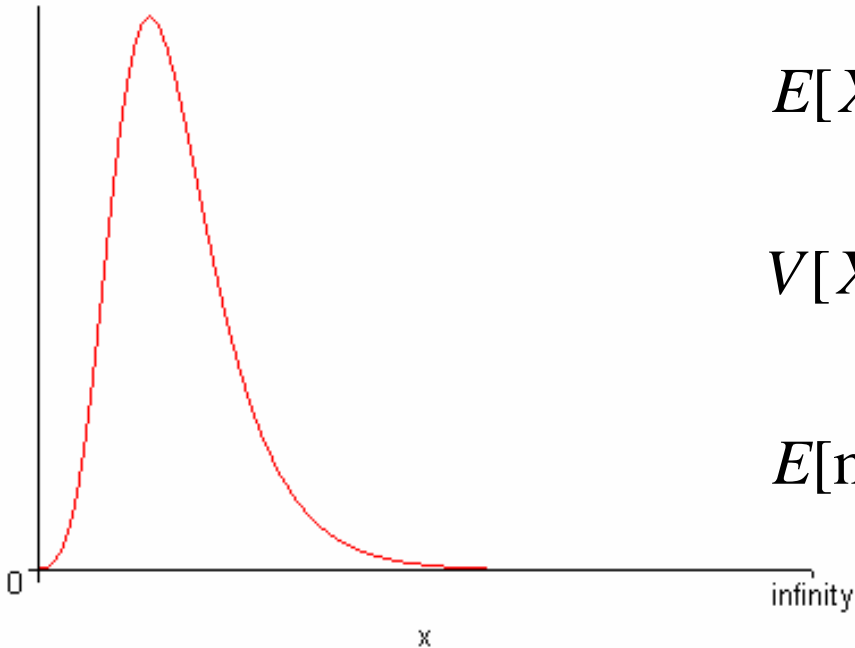
Gumbel CMT1

$$F_m(x, \alpha) = \exp(-\alpha \cdot e^{-m \cdot x})$$

$$E[X] = \frac{\gamma + \ln \alpha}{m}$$

$$V[X] = \frac{\pi^2}{6m^2}$$

$$E[\max(X_i + L_i)] = \frac{\gamma + \ln(\sum \alpha_i e^{m \cdot l_i})}{m}$$





Gumbel CMT1

Gumbel Distributions allow analytical results

$$l_i^* = \max(E[X_j] - \frac{\sqrt{6}}{\pi} \cdot \sigma[X] \ln(h_j)) - E[X_i] - \frac{\sqrt{6}}{\pi} \cdot \sigma[X] \ln(h_i)$$



Gumbel CMT2

- Allow different variances
- Numerical results



Presentation

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Simulation

- Data from HP (Appolo 260 in Exeter)
- Five Policies:
 - Proposed
 - Numerical I
 - Numerical II
 - Deterministic
 - Independent



Simulation

(See Figures 4 – 7 in the Gallien and Wein paper.)



Conclusion

- Robust, efficient and simple policy
- Major drawback: Synchronization assumption
- Generalization: multi systems