This summary presentation is based on: Roundy, Robin. "98%-effective Integer-ratio Lot Sizing for One-warehouse Multi-retailer Systems." *Management Science* 31 (11), 1985.

Presented by Riadh Alimi

Two words about the author

Robin
Roundy

- Professor at Cornell in IEOR
- PhD in OR from Stanford (1984)
- Fredrick Lanchester Prize (1988)

Problem to solve

How to minimize the average cost of a one-warehouse multi-retailer system ?

Details of the problem

- A single warehouse delivers product n to retailer n only n = 1…N

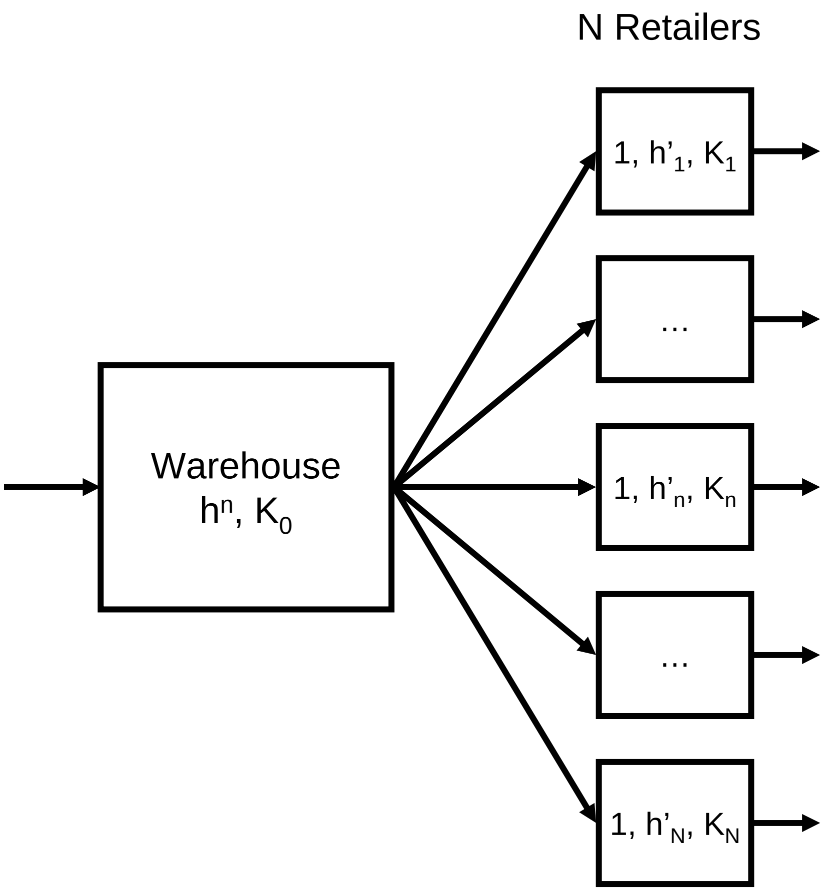
- No shortage allowed
- Delivery is instantaneous

- Set-up cost K_n
- h^n : holding cost rate of product n per time unit at the warehouse
- h'_n: holding cost rate of product n per time unit at retailer n

- Constant demand rate per unit of time at each retailer : 2

- Continuous time

- Infinite horizon



Previous solutions

and their limits

- There is no solution to the general case

- Optimal policies are either difficult to compute or have low effectiveness

➢ A new class of policies seemed to be needed

Integer ratio policies

a new class of polices

- Each retailer places orders every T_n
- The warehouse places orders every T
- A Policy $T' = (T, T_1, \ldots, T_N)$, is an integer ratio policy if either T/T_N or T_N/T is an integer
- They have at least a 94% effectiveness and are easy to compute

Average cost of a policy (1/4)

The average cost of an integer ratio policy $T' = (T, T_1, \ldots, T_N)$ is :

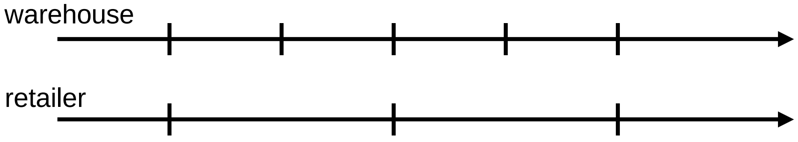
$$c(T) = K_0/T + \Sigma_{n>1}\, c_n(T, T_n)$$

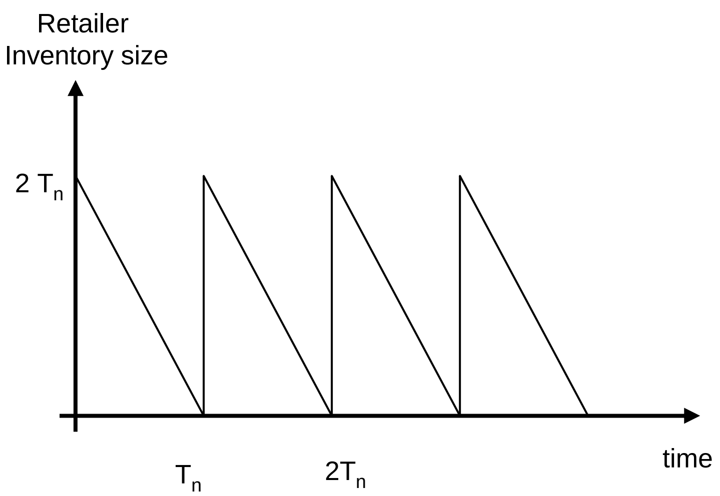
where $c_n(T, T_n)$ is the average cost per time unit of supplying the demand for product n

Average cost of a policy (2/4)

If $T_n > T$

No inventory is held at he warehouse





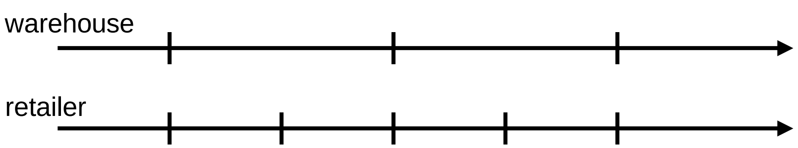
Average cost per unit of time for supplying product n :

$$c_n(T, T_n) = K_n/T + h'_n T_n$$

Average cost of a policy (3/4)

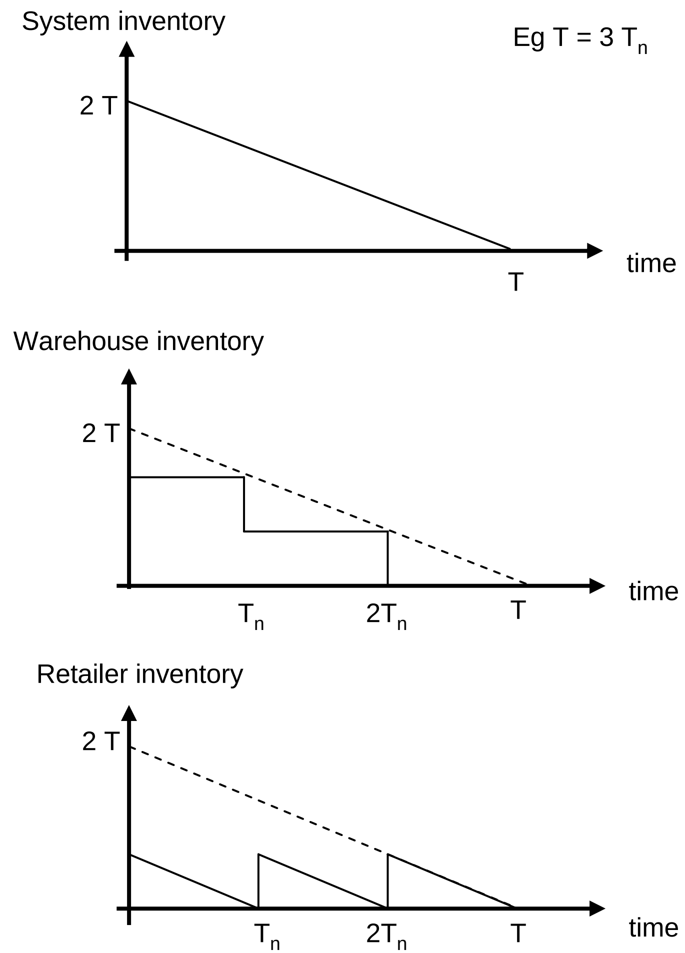
If $T_n < T$

Inventory is held both at he warehouse and the retailer



Average cost per unit of time for supplying product n :

$$c_n(T, T_n) = K_n/T + h'_n T_n + h(T - T_n)$$



Average cost of a policy (4/4)

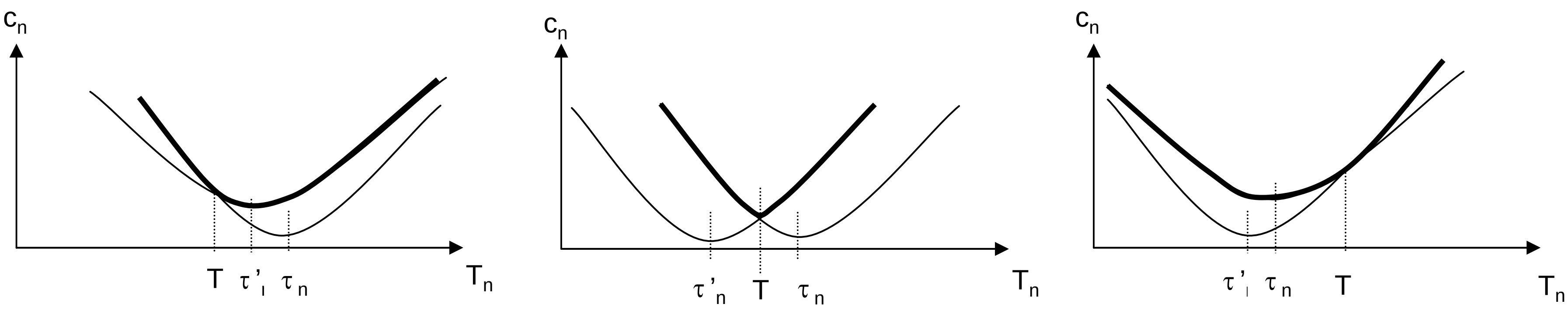
The average cost becomes : $c(T') = K_0/T + \Sigma_{n>1} c_n(T, T_n)$

With $c_n(T, T_n) = \begin{cases} K_n/T_n + (h^n + h_n)T_n & \text{if } T_n > T \\ K_n/T_n + h_n T_n\ h^n\ T & \text{if } T_n < T \end{cases}$

Where h_n is the echelon holding cost : $h_n = h'_n - h^n$

Lower bound of the average cost of all policies by relaxation (1/2)

1. We minimize c_n over all T_n for a given T :



with $\tau'_n = [K_n/(h^n + h_n)]^{1/2}$ and $\tau_n = (K_n/h_n)^{1/2}$

which introduce 3 sets we will need further :

$G(T) = \{ n \,/\, T < \tau'_n \}$ $\quad$ $E(T) = \{ n \,/\, \tau_n < T < \tau'_n \}$ $\quad$ $L(T) = \{ n \,/\, \tau'_n < T \}$

$$\text{let } b_n(t) = \inf_{T_n > 0} c_n(T, T_n) = \begin{cases} 2[K_n(h^n + h_n)]^{1/2} & \text{if } n \in G \\ K_n/T + (h^n + h_n)T & \text{if } n \in E \\ 2(K_n h_n)^{1/2} + h^n T & \text{if } n \in L \end{cases}$$

Lower bound of the average cost of all policies by relaxation (2/2)

2. We minimize now the cost for T :

$B(T) = K_0/T + \Sigma_{n>1}\ b_n(T, T_n)$

Using the fact that B(T) = K(T)/T + M(T) + H(T)T
where B,M and H are constant piecewise functions

We can write an algorithm to minimize B in O(N logN) time

B* = B(T*) is a lower bound on the average cost of all integer-ratio policies. We can show that it is also the lower bound for all policies

Order preserving policies

The cost of an order preserving policy can be seen as the sum of costs of single-facility lot-sizing type

Let T'* be the optimal relaxed order policy, notice that :

$T^*_n > T^*$ when $n \in G$
$T^*_n = T^*$ when $n \in E$
$T^*_n < T^*$ when $n \in L$

$T' = (T, T_1, \ldots, T_N)$, is said order preserving if

$T_n > T$ when $n \in G$
$T_n = T$ when $n \in E$
$T_n < T$ when $n \in L$

The average cost can then be rewritten as $c(T') = (K/T + HT) + \Sigma_{n \in G \cup L} (K_n/T_n + H_n T_n)$

With $K = K_0 + \Sigma_{n \in E} K_n$ aggregate set-up cost for $W = \{0\} \cup E$
$H = \Sigma_{n \in E} (h^n + h_n) + \Sigma_{n \in L} h^n$ aggregate holding cost rate for W
$H_n = (h^n + h_n)$ if $n \in G$, h_n if $n \in L$ average holding cost for GUL

It follows that $B^* = c(T'^*) = M + \Sigma_{n \in G \cup L} M_n$

With $M = 2(KH)^{1/2}$ and $M_n = 2(K_n H_n)^{1/2}$

Let's define H_n by $K_n/H_n = T^{*2}$ for $n \in W = \{0\} \cup E$

The lower bound theorem

The minimum relaxed average cost B is a lower bound on the average cost of all feasible policies over every finite horizon*

Let C(t’) be the cost of an arbitrary policy over the interval [0,t’)

Using the idea that it is possible to allocate the costs incurred by an arbitrary policy to individual facilities

We can show that C(t’) > B*

The q-Optimal Integer-Ratio Lot-Sizing

It is possible find an integer-ratio policy whose effectiveness is at least 94%

The Optimal Power-of-two

It is possible find a integer-ratio policy whose effectiveness is 98%