

[SQUEAKING]

[RUSTLING]

[CLICKING]

HAZHIR I guess we can get started. Welcome back, folks. Good to see you all. I hope you have had a good week so far,
RAHMANDAD: and let's get us started today with our routine that will be repeated for the coming weeks, basically going through your models, critiquing and discussing them, and then preparing for the next session's modeling task.

Before we get started, anything administrative, any questions, any issues? Everybody has access to the Dropbox folder and has been doing-- I think most of you have uploaded your models. That's great. Looking forward to working through them today.

So one kind of minor point is when you're done with any file, just close it, because it creates conflicts, if, for example, I want to edit one of the-- kind of update one of the assignments because I had found some typos, if it is open in some other computer, I Can't, Or it creates multiple versions. So that's simple, but just keep that in mind because this is a shared folder with many people.

So the plan for today is basically spend most of the time going through the models that you have built and then discussing them. We'll see how much material there is inside this specific exercise, because it's probably a little bit on the simpler side, and hopefully we can cover several of the submissions we have had, and we'll take it from there.

So just to refresh our memories, we started with SIR model this week, and we wanted that to be expanded on a few fronts, relaxing some of the typical assumptions in SIR models. So adding a behavioral response to risk, adding the waning of immunity, thinking about equilibrium, trying to do even closed-form equilibrium to the extent possible, and then adding vaccination component. And so that was your charge.

And I didn't give additional information, so you would have quite a few kind of choices you needed to make. I think those choices are things we will be discussing. And how to think about those choices more broadly is the type of lesson we hope this class helps build.

So with that in mind, I have created a spreadsheet to help us just see who is going to present-- or which model we are going to work on. So I'm just showing you one so that this is a fair process. And then after that, I'll just throw the dice. So you have your names here. I look who has submitted models, add that here, so that we are only looking at the models that have been submitted.

And then I draw a random number. That random number goes and selects somebody. And the probability that you get selected is partly dependent on whether you are registered officially for the class or not. So those who are registered have twice as likelihood of being selected as those who are not.

And then once you're selected, I just highlight that that model is discussed. Then the next time it would not be selected. So all of that, I can now just type something and that redraws the random number, select somebody new. And I just do one random draw every time so they don't cheat anybody.

OK. So the first one is Jason's model. So I'm going to put Jason's model as discussed here and take this out of your view.

AUDIENCE: [INAUDIBLE] Jordan's name.

HAZHIR No, I have it here. I can just bring it up. It's probably easier than having-- OK. So this time, kind of we had three
RAHMANDAD: steps, so many of you have submitted three separate models, which is fine, even good, because we could go through them one by one. The models are not super complex, but it might be easier visually to start with a simpler one and just go to the next, more complex one.

So let's get Vensim open. So this is--

AUDIENCE: Might need to unhide to show all levels.

HAZHIR Yes. Oh, no, not here. And for those not familiar with this feature, you can hide variables in Vensim into different
RAHMANDAD: layers, which kind of allows you to have simpler visualization. Let me just turn off the light.

And maybe we-- is it-- no, this is too little. Is it good from the end of the room, you can see this? OK. So basic SIR model in the middle. We have death separately captured here. So that's pretty standard.

And then there is the question of effective risk perception on a bunch of things. That's the first component we have been assigned. So here, we have Jason's formulation of indicated risk, if I'm reading correctly, coming from indicated infection risk and indicated death risk. We will need to go into those formulations where they come from. And then that indicated risk is leading to perceived risk somewhere.

AUDIENCE: [INAUDIBLE]

HAZHIR Ah, it's further up, yes. So then there is a smooth function that leads to risk perception. This one has two time
RAHMANDAD: constants consistent with an asymmetric adjustment of risk-- or risk perception. And then that perceived risk level is going to impact behaviors through impact of perceived risk on contact rate, leading to changes in contact rate.

So conceptually, that's the idea. But there are a few choices here. First of all, where do you get the risk from? That is, what variables are driving the risk and risk perception? So here, we have it taken from total death. And seeing that, I'm a little bit suspicious, just because total death, for me, is the cumulative death, and therefore, itself directly shouldn't be driving it.

But Jason is doing something more complex here, is the expected fractional change in death rate versus expected death rate. So we need to actually look what exactly is happening to understand whether it does the function. And then we have a similar function from infectious population in this rate. So here, you have expected death rate, which is total death multiplied by expected fractional change in death rate.

So you get this expected fractional change in death rate, and you multiply that. And this is a trend function. So you are looking at the trend, and the trend is in units of unassigned unit here. So that's not good. And in fact, kind of good test at the very beginning of any-- of these is to do a unit test.

15 unit errors, that's a bad thing. You shouldn't have unit errors. That's a general rule. Never come to have a model that has unit errors. That means something might get messed up-- but OK.

So fractional change in death rate-- so if I remember the trend function correctly, it is 1 over year is the unit of that function, typically. That is what is the rate of change in whatever input variable you have per unit of time.

AUDIENCE: Fractional rate of change.

HAZHIR Yeah, that's fractional rate of change. So at 3%, or 0.03 means 3%, change rate in total death. You're multiplying
RAHMANDAD: that by expected death rate. So this, I'm not quite following the logic of that. So can you tell what you had in mind, what you wanted to do?

AUDIENCE: Yeah. The idea was to think about how people might estimate the changing death rate.

HAZHIR So why do you get it from total death if you're trying to get the changing death rate?

RAHMANDAD:

AUDIENCE: The thought was-- the goal, and I'm not sure this achieves it, was to get an estimate of the first derivative of that stock.

HAZHIR Well, the first derivative of that stock is what variable? What is the first derivative?

RAHMANDAD:

AUDIENCE: It's the flow.

HAZHIR Yeah.

RAHMANDAD:

AUDIENCE: But people can't observe that.

HAZHIR So first of all, there is kind of actual measurement of death. So it is not something that is completely
RAHMANDAD: unobservable in the system. In fact, if there is one thing that is reasonably observed, is measures of death. Now, it is not perfect measurement. Like any measurement, it has its own delays.

So the delays in measurement are important. We want to capture them. But also in a very simple model like this one, we are going to have another layer of delays from perception of risk.

So I don't need to separately capture the delay in measurement, because I can say that risk perception process delays will capture not only the measurement of the death rate, but also the changes in the overall sentiment towards risk. And that will take care of the inertia in both of these measurement processes. For a simple model like this, that would be perfectly fine.

Now, you may say-- you may have a model that you are trying to estimate, for example, and you would say, actually the death rate that is measured has a delay, and I want to capture that delay separately. And that's legitimate. For an estimation process, that might be an important consideration because those delays, usually in the order of days, but it might be varied. Some countries, some regions may have longer delays, shorter delays. So you may explicitly model that.

AUDIENCE: Yeah, that makes sense. I did do some testing on this, and as far as I can tell, my expected death rate and expected infection rate, which is formulated equivalently, mirrored the net flows for both of those stocks quite nicely.

HAZHIR It may well be, because you are basically saying something is fractionally changing with this rate-- so 3% per

RAHMANDAD: rate. And then you're multiplying that by the stock, so that becomes the flow. So in that sense, you are getting the same number, but kind of going this way a little bit in the formulation. So the trend function-- yeah, the trend function is introducing kind of two, three hidden stocks inside the trend function.

So we are all kind of understanding the challenges with that formulation, even though it is not wrong in the sense of conceptually it's getting to the same idea. Then there is the question of how things are happening in the aggregation of this indicated risk. So basically, you get this expected death rate. You compare with some reference death rate. And where does that reference death rate comes from?

AUDIENCE: Constant.

HAZHIR It's just a constant. And you get to indicated risk, which is--

RAHMANDAD:

AUDIENCE: It's the sum [INAUDIBLE].

HAZHIR --the sum of these two guys.

RAHMANDAD:

AUDIENCE: Can I ask a question before getting into this? because the differentiating [? mechanics ?] and differentiating both the total deaths and the infected people. So my question is, are we interested in the first derivative of the infected people, because the outflow of recovery is included there when we're differentiating?

AUDIENCE: Yeah, so--

AUDIENCE: Is that what we should be looking at or are we looking at infection directly?

HAZHIR What were you thinking, Jason, when you captured it?

RAHMANDAD:

AUDIENCE: I did think about this. I'll actually walk you through my thinking [INAUDIBLE]. It's probably not sound. The first most obvious thing to me would have been to just use the death rate and infection rate for a delayed version.

But in my mind, those are model variables that are not observable. They're actually driving the behavior of the system of the actual infection patterns. The behavioral cues are likely somewhat different from those true system cues. So I was thinking, OK, how would you construct the behavioral cue?

And then looking in the textbook, I chose this formulation. So that was the idea. Yeah, probably not necessary to have all that machinery for what we're trying to accomplish.

HAZHIR Yeah. So on the one hand, kind of infectious population is not any more visible than the infection rate, because

RAHMANDAD: infectious population is drained by a recovery that is not usually observable. So what we do have measure signals in the community-- yes, you get some signals about infection population. If your friends are sick, you know that they're sick. But also we are getting these numbers on the infection rates are not higher, we have this new number of cases, and so on.

So I'm more comfortable just going with the flow variables of infection rate. Now, going back to Hesam's question, if you capture infection rate, it has a different feature characteristic than infectious population, because infectious population can actually go negative, its rate, its net flow, which means the indicated risk of infection can go negative in this formulation. And if the indicated risk of infection goes negative, then you get weird behaviors down the line, or you have to avoid those weird behaviors. That is, you need to now make sure your risk perception response doesn't respond to negative values. Now, that opens up more challenges for containing your model formulations.

So I'm highlighting these because this is what is happening in my mind when I'm kind of seeing these formulations. I want you to basically build the quick simulation, mental simulation idea that you see the formulation and say, OK, but this creates additional robustness challenges down the line for me in terms of formulation.

AUDIENCE: But I definitely saw that problem, and I thought about it, but didn't come to a clean answer. So it's easy to turn around.

HAZHIR No, that's fine. So actually, this is-- the purpose of this class is to actually critique and learn from those
RAHMANDAD: challenges. And sometimes there is a good way of thinking about it that you have applied, but the result is not necessarily the best thing. So everybody feel comfortable being critiqued here. This is the only way we learn.

So that is actually probably the bigger challenge here in the indicated infection risk, just because when it goes negative, now we have a whole lot of other things like deaths are high, but the infection risk can go negative, cancel the death rate risk. So you may look at the data, and based on this formulation, sometime in January of 2022, when we had actually very high death rates and very high infection rates, you may get a risk, indicated risk, that is 0.

And why is that? Because the cases are falling because the cases peak at Christmas. So the cases are now falling in mid-January. The death rates are actually very high. So at that point in time, actually the risk level, or indicated risk level, is still pretty high. It shouldn't go 0 or even negative. But the formulation doesn't get that. So that's more fundamental conceptual issue.

Then there is this kind of weighing of the two risk factors. And it's OK, the way you have done it, basically, you have introduced two parameters and their values matter a lot. So this is 100. This is--

AUDIENCE: Smaller.

HAZHIR Yeah, it should be smaller one. So basically, you're saying each death gives a signal of risk that is 100 times each
RAHMANDAD: case. It's OK. But also, it is a little bit ad hoc. And you have introduced two degrees of freedom in the formulation that-- just more parameters.

We don't want to have too many parameters. It just makes it harder for us to make sense of our model. So that would be another concern for me, that the formulations here, just we went through two parameters here for these-- for the trend function, and actually four the two-trend functions and two here. So that's now six parameters to get us to some measure of indicated risk.

We do need two parameters for the time constants. So that's eight parameters. So too many degrees of freedom that makes it harder for you to analyze because you change something, but you don't know whether it's the real thing that is changing the dynamics.

And then you have more parameters from the perceived-- oh, by the way, this formulation is important, and it's not as common as the standard delay. So let's take a look at it as well. So this is basically saying if the indicated risk that is the driver of that perceived value is higher than the perceived risk level, we get the indicated risk minus perceived risk divided by time to increase risk. Otherwise, it is indicated risk minus perceived risk divided by time to decrease risk.

So the idea is if the actual risk level that you are-- kind of in the real world is happening is higher than your current perception of risk, you get anxious. You see more signals that things are getting worse. And people are more sensitive to these distress signals. Therefore, the time to increase risk perception is typically shorter than the time to decrease risk perception.

And that asymmetry plays an important role in the dynamics of COVID, for example. And kind of people in a matter of days to weeks really start to respond. But once they are responding, it may take months to relax their responsiveness to those risks. And that plays a role. And you could empirically estimate these two, and you would see that there are actually quite different almost by an order of magnitude.

So useful formulation to keep in mind when you're dealing with asymmetries in how people respond to anything, whether it is good versus bad news or risk or customer service. So customer perception of quality is very asymmetric. If you have a bad experience, it has a much stronger negative effect on your assessment than when you have a good experience. You fly, the plane lands safely, you don't get impressed. You have a bad landing, it's not going to be an experience that you will ignore.

You may talk about it. You may record it and put it on TikTok. You may go and complain in a variety of networks. So that asymmetry plays a big role in organizational dynamics, in importance of keeping high levels of quality, because if you erode it, it's very hard to bring it back. It's erodes fast. Trust between people has this asymmetry. Anyway, this is generally an important conceptual thing to keep in mind.

So then you have this perceived risk that feeds back into effective perceived risk on contact rate. And you have one parameter here. So this is important to take a look at this formulation. This is an important formulation.

So you have 1 over e to the power of sensitivity of this-- some parameter that you have multiplied by perceived risk. So if perceived risk is a-- so for now, let's ignore the negative range for perceived risk because that negative range adds more complexity for our population. Yes?

AUDIENCE: For sensitivity of CR to PR mismatch?

HAZHIR So sensitivity parameter is just a parameter. So it is controlling how responsive to risk you are. So if you assume
RAHMANDAD: that parameter is zero, what this function does? So when you see a function like this, you need to actually think about alternatives. And so this is one alternative.

So what will happen? [? Say today, ?] if we set that sensitivity parameter to 0, what will happen to this response function? How would it behave? So this is 1 over e to the power of some parameter multiplied by some variable, perceived risk. Now we are making that parameter equal to 0. What would be the overall impact on this effect of perceived risk on contact rate? Jake, do you want to take a stab at it?

AUDIENCE: Goes to 1.

HAZHIR Basically, the denominator now has e to the power of 0. e to the power of 0 is 1. So it's 1 divided by 1. So it is
RAHMANDAD: always 1. So it means the effect goes away.

So as such, at least this extreme condition makes sense. That is, if this parameter is 0, it will put the effect at 1, which means the interaction contact rate will not change from its base value. So again, we are kind of running through a few steps, logical steps, and this kind of as you practice with these formulations, it becomes easier to digest those quickly.

At the beginning, it may feel, oh, what happened? You're doing too many things mentally. You need to break it down. If it is overwhelming, write down the equation. And Vensim's lingo is not necessarily making it easier to read these things.

So that extreme condition makes sense. But we also need to think about-- and the other extreme condition is if the perceived risk level 0, again, we get to a coefficient of 1, or a multiplier of 1, for effective perceived risk on contact rate, which is also meaningful, that we do want that to be the case. Now, if the perceived risk is high, multiplying that by this sensitivity parameter, and then 1 over e to the power of the sensitivity parameter.

So e to the power of this number is going to be a large number. So 1 over this will be a very small number. So the idea is that this basically brings down contact rate. Again, decent, meaningful way of formulating it.

Of course, you could actually just go e to the power of this sensitivity multiplied by perceived risk and make the sensitivity a negative number. Mathematically equivalent, just the functional form would look a little bit different. So this formulation works for numbers of risk, perceived risk, that are above 0. What about negative numbers for perceived risk?

So if, in Jason's formulation, actually it can go negative. So if we have negative numbers, what will happen to this formulation?

AUDIENCE: [INAUDIBLE]

HAZHIR Speak up.

RAHMANDAD:

AUDIENCE: [? It ?] [? increases. ?]

HAZHIR It goes up, the overall function because the denominator is now going to be below 1 because e to the power of a
RAHMANDAD: negative number is below 1. And if you have a below 1 value in the denominator, the total effect will become above 1. So it creates a large contact rate more than the normal contact rate.

And do we want that? Is that realistic? It is not. So this is going to be a failure mode with the formulation that you have for the infection rate going into-- not infection rate but the change in the stock going into the formulation. Everybody clear on this? Good.

So big picture, kind of in this formulation, this one I like because it has a single parameter. It does the job decently well. It's a good alternative. The only thing I would keep in mind is that exponential functions are very strong. That is, as soon as you have a perceived risk level that exceeds 1 over that sensitivity parameter, so that the multiplication of the two gets to the range of 1, this channel shuts off all of the interactions.

So again, this is a little bit you need to be familiar with the exponential functions and how strong they are. But when I see the use of exponential function in this, I would ask myself, do we see such a strong change in a kind of relatively short small range of the input to the function? Because the perceived risk-- keep it in mind. This is a scaling with the size of death and cases in COVID.

So in practice, you may remember in the COVID case, we had cases and deaths in the US that spanned an order of magnitude. So some days we had hundreds of deaths and some days we had thousands of deaths. So when you change an order of magnitude of input, the chances are this function will go from 1 to 0 between these two ranges. And in reality, the interactions of people in society was not changing between 0 and 1.

So that's one reason I would be a little bit hesitant about the use of exponential function. On the other hand, it has the beauty of being a single parameter giving me the right range. So I'm not going to say this is a bad formulation, but I want to highlight one of its weaknesses when we are looking at this. Any thoughts, questions? OK.

So this is good. Now you have the waning of immunity here that basically goes from recovered population after a delay with duration of immunity, go back to susceptible population. Very simple. This is a good approximation. People may consider whether the waning of immunity is a first-order delay or a higher-order delay. How would you think about that?

AUDIENCE: I thought about it.

HAZHIR So how did you think about it?

RAHMANDAD:

AUDIENCE: I thought it should be a higher-order delay.

HAZHIR Why?

RAHMANDAD:

AUDIENCE: Because in the first-order formulation, you are saying that there's equal probability that any recovered person leaves the stock at any point in time. But in reality, there's some memory of when you entered the stock. If you just recovered yesterday, you probably will start losing immunity in six months from now.

HAZHIR OK. So that's an argument in favor of higher-order delays. Basically, the argument is the chances that you lose

RAHMANDAD: immunity overnight is very low, at least for a single individual. So if I got COVID 100 times, and I look at the distribution of my immunity loss time, it probably will not happen within the first couple of months.

It starts to show off getting another COVID only after three or four months, and perhaps it peaks around five, six months or something like that. So that is the argument in favor of a higher-order delay. Any argument in favor of a lower-order delay or first-order delay?

AUDIENCE: Simplicity?

HAZHIR So simplicity, for sure. So that is absolutely the main reason most of these we use first-order delays. But any

RAHMANDAD: other more substantive conceptual argument for a lower-order delay?

AUDIENCE: I mean, [INAUDIBLE] is that we have, in this case, two first-order delays in sequence, the recovery rate and then the waning of immunity. And so like I think-- so the sum of the flows from infectious--

HAZHIR It becomes a second-order delay of sorts. So Alexander's point is that we have the recovery rate before this

RAHMANDAD: delay, and that is itself a first-order delay. So the two of them together add-- create a second-order delay. So we are capturing-- so first of all, this argument, although it kind of works because it looks like a second-order delay, but it is not.

This is two different delay times. So one of them is actually almost 10 times larger than the first one. So that means in practice, it behaves-- even if you had these two coupled together serially, it doesn't behave like a second-order delay. It behaves very close to a first-order delay.

So the more the two or the n different time constants are similar to each other, the more it behaves like a n -th order delay. And if they have very varied time constants, then it behaves closer to a lower-order delay. So that's good to keep in mind when you think about it. Cathy?

AUDIENCE: Is it a reason why, when [? we ?] first started, because there is maybe some small segment of the population that doesn't keep their immune response for very long.

HAZHIR Very good. So that is actually a much better argument in favor of a lower-order delay, that we have a lot of

RAHMANDAD: heterogeneity in the population. So yes, I might map my own recovery and getting more COVID, and see even over multiple times that it never happens two months later. But there are people for whom they get repeated COVID cases every other month if they are exposed to the virus. And basically, that individual-level variation acts to make a first-order delay a decent approximation here.

So that's kind of the trade-off we're talking about. Simplicity trumps for our case, which is kind of not trying to build a very detailed model. I think the first-order is a better choice just because of simplicity. But those arguments would play out if you were thinking about a more complex model-- is it worth adding that?

Now, if I wanted to have a higher-order delay, how would I capture that? So then you wouldn't have the stock going into the waning of immunity. What would be the right formulation?

AUDIENCE: The recovery rate would go right into the waning of immunity with the average duration. And then the difference accumulates in the stock.

HAZHIR Exactly. So that would be one way to do it. Of course, you could also spell out recovered population that have

RAHMANDAD: been recently recovered, recovered population that were the second cohort and third cohort. You can go to higher levels. But the simple delay function hides all of that complexity and is probably a better choice. So was that part clear for capturing the higher-order delay?

AUDIENCE: [INAUDIBLE] one more time.

HAZHIR So, sure. Do ask questions. I sometimes go too fast and I'm not seeing all your faces. And even if I'm seeing your
RAHMANDAD: faces, I'm not reading your faces well enough.

So the question was, how do we formulate a higher-order delay for waning of immunity? And the answer was there are two ways. One is to get an information link going from recovery rate all the way to waning of immunity, and then have a waning of immunity function that is a delay function that is-- or yeah, delay is the right one-- third-order delay, n-th-order delay.

AUDIENCE: So you wouldn't have [INAUDIBLE]. You would just have a flow out and then directly an arrow from that flow out to another.

HAZHIR Exactly. Yes. Now, there are stocks in that formulation. So every delay function creates a stock in the back end of
RAHMANDAD: Vensim, but we are not observing those in our screen. So computationally, it is doing that job. It's just hiding it from your view. For better or worse, it makes things cleaner, but also you don't see the stocks that are happening in your model. The other alternative is to add explicitly multiple recovered population groups, and then have first-orders going from one to the other to the other, and that would work as well.

So this model captures two of our key inclusion-- things that we wanted to consider. The third one was vaccination. Question, Alexander?

AUDIENCE: Before we move on, I wanted to ask about modeling of the death rate, because I like-- so I guess my concern is that if I assume the infection fatality rate is like a hazard rate of like 0.01 per year, but if there weren't--

HAZHIR Oh, so you are good-- you're bringing up an important problem here. So here, we have infectious population
RAHMANDAD: flowing to the right-hand side or flowing to the top side of the diagram. But the way they flow are through two separate parameters. One is infection fatality rate, one is the average duration of illness.

Now, this is going to work mathematically, but it is going to be a problem in terms of conceptual clarity. So what is the problem? The problem is that the average duration of illness is basically capturing both people recovering and the duration for people to die. So the duration of illness, at the end of it, people either recover or die.

In an extreme condition, when most of these people die, the average duration of illness is the same as in the extreme condition, but nobody dies. That is not affected by the infection fatality rate. So when you formulate it this way-- now let's go to the extreme condition in our mind that infection fatality rate, that is, if you get the disease, you will die. This is certain. This is a very sad, extreme conditions.

In that situation, the flow from infectious population to recovered population should be 0. But in Jason's formulation, that will not happen automatically. Now, of course, Jason can redefine average duration of illness to combine the probability that you survive and the duration of illness. But that is a convoluted way of defining your parameters, because now you can't fix that parameter based on biological knowledge you have. You have to play with these things in your head, and so you don't want that.

So what's the right way of formulating this? So basically, what you need is to split the total population that gets to the gate of recovery or death and split it between these two channels. So what's the right formulation? Do you have a solution yourself, Jason?

AUDIENCE: Could use some type of [INAUDIBLE], like a fractional allocation.

HAZHIR So what would it look like?

RAHMANDAD:

AUDIENCE: You could have fraction of-- OK. [? Hang ?] on. I have to think on my feet.

AUDIENCE: Well, you could have a separate-- you could introduce another stock.

AUDIENCE: I'm hesitant to do that.

HAZHIR Definitely don't do that. You don't need a stock.

RAHMANDAD:

AUDIENCE: So fraction--

HAZHIR So what's the total number of people getting out of the infectious stock? The stock of infectious population.

RAHMANDAD:

AUDIENCE: It's the sum of death rate and recovery rate.

HAZHIR Yes, in this formulation. But if you have a total recovery time, average duration of illness-- not recovery time,

RAHMANDAD: average duration of illness. So you wanted to calculate the total exit from this stock, what would it be from the stock of infectious?

AUDIENCE: It would be the duration times the infection rate in equilibrium.

HAZHIR I'm not talking about equilibrium. I'm talking about just you are trying to drain a stock with an average duration.

RAHMANDAD: What would the outflow be?

AUDIENCE: Oh, just a first-order--

HAZHIR Yeah, just the stock divided by the [INAUDIBLE]. So that's the total that should get out of the stock. Now you need

RAHMANDAD: to split that total between these two channels.

AUDIENCE: I see.

HAZHIR So you get that total, you multiply that number with infection fatality rate, which is what fraction of people die

RAHMANDAD: from disease if they got the disease. And that goes up to the death rate. And 1 minus that fraction goes down to the recovery rate.

AUDIENCE: Just to say it another way, it's a question of when the splitting happens.

HAZHIR Well, yes. In that sense, you could call it also about when a splitting happens, but it is really kind of separating

RAHMANDAD: these two parameters of infection fatality rate versus average duration of illness from each other so that they don't convolute. Right now, they are convoluted. You're calling them the-- something, but they're actually different things.

So the one that is going to the top right now is infection fatality rate multiplied by average duration of illness. The one that is going down is 1 minus infection fatality rate multiplied by average duration of illness. That's not a good way of doing the formulation.

AUDIENCE: Duration to determine the flow and then--

HAZHIR The total flow. And then fractional goes to the two.

RAHMANDAD:

AUDIENCE: Makes sense.

HAZHIR Yes, Alexander?

RAHMANDAD:

AUDIENCE: So that kind of implicitly assumes that it takes the same time to die from a disease as to recover from it?

HAZHIR Yes. Absolutely.

RAHMANDAD:

AUDIENCE: So if there's a disease where if you get it, you'll die within the first week, otherwise you're--

HAZHIR Yes. So this is kind of a first-order approximation for that idea. If you want to break it down, then you need to

RAHMANDAD: break down into different stages of the disease. So rather than have a single stock of infectious population, you would need kind of people with early disease and later stage and second week or third week or whatever you think. And then you would have different rates of death for them.

But it is a messy formulation. When you get to go down that route, you can't have a single infection fatality rate. Now you have to have stage-based infection fatality rates that kind of-- it's doable, but you will introduce more degrees of freedom and more places for confusion.

AUDIENCE: Another way-- I'm hammering this one because I had an issue with during [INAUDIBLE] sense of drug tests failing. [INAUDIBLE] moving on and [INAUDIBLE].

HAZHIR Yeah.

RAHMANDAD:

AUDIENCE: So what another way to do this be you have say like two socks, and then you split them in the [? influence. ?] Then you have infections going into both. And then some proportion of the infections are like destined to die, so to speak. And then they go into infecteds who will die, and then they flow out using their own [INAUDIBLE] rate like time constant. And then infecteds who will recover and they flow out--

HAZHIR So you're basically imagining two parallel stock and flow chains for people who are destined to die and people

RAHMANDAD: who are destined to survive? You can do it that way, but then you're introducing even more stocks. So the nice thing about that option is that from the very beginning, you use your infection fatality rate to divide people between these two chains. And then you are just doing the timing thing, that timing of recovery versus timing of death.

The downside of it is now you have extra stocks that you're tracking, rather than tracking all of the people in the same stock. You can do both of them. Really depends on what you have data for. So if you have data and first week, what is your risk of death versus risk of survival, then you can use the single stock chain. If you have only total infection fatality rate, then maybe it is worth separating them. Now, for this problem, I wouldn't even think about any of these, but for the general case.

AUDIENCE: [INAUDIBLE]

HAZHIR Could you speak up a little bit? I just want to make sure folks online can hear us?

RAHMANDAD:

AUDIENCE: So it's wrong to have the average duration of illness, for example, seven days, and then have average duration of illness that leads to death, let's say, 21 days. Even if you have the fraction allocated?

HAZHIR No. So if you want to do that, then you're dealing with the problem that Alexander is talking about. That is, then
RAHMANDAD: you probably benefit from splitting them earlier into two groups. The people who are destined to die. You put them into that stock, and after 21 days, you kill them. The people who are going to survive after seven, then you take them out and put them into the recovered population.

AUDIENCE: Well, can you explain why it's wrong to do that?

HAZHIR So it's not wrong. It is just now you have two different types of people, people who spend in the kind of sickness
RAHMANDAD: stage for 21 days, and people who stay in the sickness stage for seven days. And if you really want to track this distinction, you would need at least two different stocks.

Now, you could have those two different stocks either serially or in parallel. So if you did it serially, you would say, OK, I have a group-- I have the first stock. After seven days, people leave this stock, and most people leave that stock after seven days. But then a small chunk of people go to the second stage, which is the stage of-- I don't know-- being condemned to death. And they will stay there for 14 days, and at the end of that period, they would die.

So that's the alternative way of capturing it. But it is a little bit of a more convoluted way of doing it, because then that is splitting going down versus going up. You need to do some calculation, what is the exact fraction that goes versus stays? That's not obvious at the beginning, although it is-- basically ends up to be your infection fatality rate for that split. So you can do it that way as well.

AUDIENCE: So, basically, here you would have the average duration of illness being the same as for the people--

HAZHIR So this is not happening, not in Jason's formulation. Jason's formulation is just problematic. So the meaning of
RAHMANDAD: average duration of illness and infection fatality rate are just not correctly captured here. And therefore, in this formulation, it is doing some weird thing. We are not going to belabor that point.

But in the right way of doing it, you would have an average duration of illness that affects both of these flows. And then that would be the average time people stay in the infectious population stock.

AUDIENCE: Makes sense.

HAZHIR OK. So I think we kind of critiqued this model well enough. The biggest lesson, always be unit consistent before putting your model up. So let's do the next part, the vaccination on some other model, just to have a little bit of variance.

So this time we have Russ. OK. Let me open Russ's model. So let's actually just get our bearing around this diagram. Let's see.

So we have unvaccinated population and their movement between susceptible, recovered, and infected. So this is the three stocks we have in the center are the stocks we already saw in Jason's model. And then we have the vaccination process. So people get vaccinated, they go into vaccinated populations susceptible to COVID. They may get COVID. So you have basically the three equivalent parts-- three equivalent stocks on the left-hand side.

So, conceptually same idea. I would probably just make them parallel on the top or bottom so that it is a little bit visually easier to see the parallelism between the two, but there is nothing conceptual there. Let's now-- just so that I quickly understand what Russ's model is doing, we have the risk perception, and then we have the impact of risk perception on contact rate or on infection rate. And that process is contact rate multiplied by infectivity multiplied by unvaccinated population susceptible to COVID multiplied by population infected with COVID divided by total population multiplied by 1 minus perceived risk.

So what are the challenges you see with this formulation? So any things that come to your mind looking at this formulation quickly?

AUDIENCE: The range [INAUDIBLE]?

HAZHIR Can you speak up, Hesam?

RAHMANDAD:

AUDIENCE: What is the range for perceived risk in this model?

HAZHIR Very good question. So the range for perceived risk matters a lot here, because if it is 1 minus some number, if the perceived risk is large enough, what will happen to this 1 minus thing? It goes negative. Now you will have negative infection rate, which is terrible. It will just mess up all of your model.

So I haven't looked at your perceived risk formulation, Russ. It might be already captured in the perceived risk part. So we shouldn't judge it yet too fast. So we'll look at that. The other part that I'm concerned about this is population infected with COVID here. Why am I worried about that? Anybody can tell?

So this is very standard SIR model. So you have the total infection rate is infected fraction of population multiplied by the susceptible multiplied by some contact and infectivity. So what could go wrong here?

AUDIENCE: [INAUDIBLE]

HAZHIR Jake.

RAHMANDAD:

AUDIENCE: I mean, one thing is-- I don't know if this is the thing you're talking about, but one thing is I think this is a separate-- like he also has a separate stock for vaccinated population.

HAZHIR Very good. So the vaccinated population who are infected are missing from this equation. And those people can

RAHMANDAD: also infect people. So you need to actually include the vaccinated population that is infected, otherwise the accounting will be all messed up.

Now, if you want to include the vaccinated population that are infected into your calculation of infection rate, so the first-order way of doing it is just add that population infected with COVID plus vaccinated population infected with COVID, divide that by total population. What's problematic with that? Alban, do you have?

AUDIENCE: I think they add this population infected with COVID is probably the--

HAZHIR Oh it might be the combination of the two. Sorry, I might have judged that too fast. So let me see--

RAHMANDAD:

AUDIENCE: It's on the right side of the [INAUDIBLE]. I put all the--

HAZHIR Yes, let me-- you're right. So this is not unvaccinated. This is the total. Thank you. And so note that Russ did this

RAHMANDAD: right. We just didn't read his model before commenting on it.

So the unvaccinated population with COVID plus vaccinated population with COVID are added up in this formulation. So that's good. That takes care of the first-order problem I highlighted. It does have one minor issue, and that is the question of whether the vaccinated people have the same infectivity, have the same risk of infecting others, as unvaccinated people. Russ, what was your thinking on that?

AUDIENCE: I also calculated the infectivity as a weighted sum-- a weighted average between the--

HAZHIR That's an interesting idea. So basically, for infectivity, you used some kind of a weighted average. So let's look at

RAHMANDAD: this formulation. Now, this is messy because Vensim diagram is not easy, and this long variable names don't help. But it's baseline infectivity multiplied by unvaccinated population divided by--

AUDIENCE: Proportion of unvaccinated--

HAZHIR Uh-huh.

RAHMANDAD:

AUDIENCE: And then for the proportion who is vaccinated, there is a vaccine effectiveness in reducing infectivity.

HAZHIR Got it. Got it.

RAHMANDAD:

AUDIENCE: So we have the baseline [INAUDIBLE]?

HAZHIR Yes.

RAHMANDAD:

AUDIENCE: --by that.

HAZHIR Yes. That makes sense. So this takes care of the problem we were just talking about. So this is a good

RAHMANDAD: formulation addressing the challenge we were talking about. That is, the total population that is infected works fine. The infectivity also works fine. And this is population weighted average. So this should be fine. Let me just-- yeah. This is good.

So first, any questions on what is the problem we are trying to solve here, because we jumped through it a little bit too quickly? So basically, the basic problem was we have two types of infectious populations who could infect anybody. Those two types are those who are vaccinated and unvaccinated. Those who are vaccinated probably have lower risk of infecting others.

So when we are calculating the total fraction of population that is infected, we want to calculate both of these as infected. But then the infectivity will change depending on there are more people who are vaccinated or less people who are vaccinated. This formulation creates a weighted infectivity that says, depending on which of these is a larger fraction, infectivity goes up or comes down. And this is doing it in an internally consistent way. So this would be a nice way to formulate it.

A mathematically equivalent but a little bit simpler, even though potentially, conceptually, a little bit more murky way of doing it, would be to keep your baseline infectivity that you have as your infectivity. But then into the population infected with COVID, rather than calling it population infected with COVID, call it kind of population infected with COVID normalized for infectivity, or something like that. And then within that formulation, rather than using unvaccinated population plus vaccinated population, use unvaccinated population plus vaccinated population multiplied by that parameter that is the relative infectivity of vaccinated population to unvaccinated.

So that parameter is always below 1. That kind of down weights this population. I'm 80% sure it's mathematically equivalent to what we are seeing here, but I haven't written down the equations. So both of those work, would be fine. Thanks, Russ, for doing it in a solid way.

So now let's go to the perceived risk, because we said perceived risk, if it is-- oh, you have a question, Alexander. Go ahead.

AUDIENCE: So is it an issue that there is no parameter controlling the effect of perceived risk on infection rate? [INAUDIBLE], yeah.

HAZHIR
RAHMANDAD: So it really depends on the meaning of perceived risk. So if the perceived risk is defined in a way that is well contained between 0 and 1, and then it would need parameters that measure its kind of strength, how fast it responds to cases, deaths, and so on. So let's take a look at this formulation here.

We have indicated perceived risk. We have the COVID trend. And the COVID trend is coming from this COVID statistics, which depends on death versus infection rate, and the parameter that controls which of these has a bigger impact of sorts. So this sensitivity to death is-- so this formulation is OK. It will take care of what you are trying to do. But then you have to have a large number for sensitivity to death, probably, to make it comparable with the infection rate.

So you're going with 10 times here. That's fine. And to be honest, I'm also fine with doing no impact of cases, just doing an impact of deaths. And by the way, we don't want unit errors. Then this COVID statistics things goes into basically a delay, a trend function of sorts. What's happening? Ah, yes.

So this is a fixed delay of COVID statistics with a time-- with a delay length of 1. So first of all, what is this formulation trying to do? So Russ, do you want to tell what it is trying to do?

AUDIENCE: So I remember every day we had the COVID statistics during the COVID time, and I was trying to capture the slope.

HAZHIR Very good. So basically, to capture the slope, Russ is creating a one-day delayed version of that thing, and then
RAHMANDAD: compares it with whatever the thing is, and then creates the slope using that. So it's basically one way of doing the trend function. Now, I don't like fixed delays. So by and large, in this course, don't use fixed delays.

There are two reasons for it. One is that under each fixed delay, Vensim creates a large number of stock variables-- state variables-- to capture the fixed delay. The number it creates is the time of the delay divided by the time step for simulation. So if your time step for simulation is 1/100ths of a day, then you have 100 stocks hidden behind this fixed delay, which you have no idea of how things are happening inside them. So that's number one.

Number two is that partly because of this reason, fixed delays do behaviors that are weird, that are hard to explain because you can't trace their-- they're very sharp behaviors. So even though mathematically, sometimes it is a useful thing, you may want to use it, it's not usually the best thing. And that noisiness is also-- in a noisiness a stochastic model, creates lots of problems. So that's kind of you don't want it for that purpose.

But conceptually, what Russ is doing is fine. He's just trying to take a derivative of some variable, which is basically these COVID statistics. And this creates the COVID trend. In fact, you could use the built-in trend function in Vensim to get perhaps to the same thing. It's just the difference between the two divided by delayed COVID statistics plus this number to avoid division by 0.

Now, all of that is fine, but adding a number to the equation is not really the best way to avoid division by 0. Even if you want to add it, you want to have a separate parameter named for it so that you don't lose unit consistency. This will be a source of unit problems for you.

The better way might be using a ZIDZ or XIDZ function, which is just a division. But if the denominator is 0, you can specify what is the output of that division in XIDZ or ZIDZ, which is the output is 0. And I think that's fine.

Now, conceptually, this formulation is taking the derivative of deaths or cases, infection rate or death rate. So why do we care about the derivative, Russ, rather than the current value of death rate or infection rate? What is that idea? What's the driver of that idea for you?

AUDIENCE: I just thought the risk response to risk is usually comes from the philosophy that people feel about whether it is getting better or worse compared to yesterday, I guess.

HAZHIR OK. So this is actually an important kind of question. What is driving people's risk perception? Is it the change in
RAHMANDAD: risk, the way they see it? Or is it the current level of risk?

So if the number of cases are coming down, does it mean I would feel completely safe now because it is a negative number, and therefore, I should feel even better than when there was no COVID, because when there is no COVID, the trend is 0. So now I feel I should go out and party every day because the number of cases per day are coming down from 3,000 to 2,500 deaths per day. Or I should feel less worried than yesterday, but not completely unworried. So that's the question.

And I think the answer is largely clear in this case. That is, it is not the trend of death and case that determines the absolute value of risk perception. Rather, it is the absolute value of risk that determines the absolute value of the perceived risk. So you actually don't even need all of these trend stuff here. And then the indicated perceived risk is doing--

AUDIENCE: I'm using a sigmoid function, like a logic function, that is centered on 0.

HAZHIR Yes.

RAHMANDAD:

AUDIENCE: And then-- because the trend is something that is just any real number, such that it might fit into some interval.

HAZHIR Yes.

RAHMANDAD:

AUDIENCE: And then center in into 0.

HAZHIR Yes.

RAHMANDAD:

AUDIENCE: So when there is 0 COVID trend, then--

HAZHIR The function becomes minus 0.5.

RAHMANDAD:

AUDIENCE: The whole thing becomes 0.

HAZHIR Oh. Does it? OK. Sorry.

RAHMANDAD:

AUDIENCE: Because it is centered at 0 by subtracting 0.5.

HAZHIR Oh, Yeah, yeah, yeah. You're right. You're right. So e to the power of thing. Yeah.

RAHMANDAD:

AUDIENCE: Yeah. So when trend is increasing, then it's going to be some positive number. When it's decreasing, it's negative number.

HAZHIR So let's say trend is negative infinity. Then this function becomes minus 5-- 0.5. Correct?

RAHMANDAD:

AUDIENCE: Yeah.

HAZHIR Which means people will be interacting more than they normally do.

RAHMANDAD:

AUDIENCE: Yeah.

HAZHIR So there is kind of the possibility in this function that people actually interact. Remember, in the next step,

RAHMANDAD: infection rate is 1 minus this thing. So if you make that 1.5, you're increasing the infection rate--

AUDIENCE: Yeah.

HAZHIR --when the trend is a very negative number. So this is not going to work. First of all, your next function, which is
RAHMANDAD: when this trend thing goes into the infection rate, this has 1 minus perceived risk. So this 1 minus perceived risk, I don't want it to go ever below 0 or above 1, because otherwise I lose control of how I'm thinking about this function. It becomes a very messy function. So this is my constraint for this function. And right now, I can see that it can actually go to above 1.

Now, could it go below 0? For it to go below 0, you need the other part to go above 1. Could this ratio go above 1? No, it can't go above 1. So that ratio will always be below 1. So at least it wouldn't have the very extreme lack of robustness that pushes your infection rate to become negative, because-- yes, Russ?

AUDIENCE: I was trying to capture the perceived risk, and then make it adjust the infectivity infection rate from 50% to 150% compared to the regular level. I guess it is restricted.

HAZHIR Well, one thing is being restrictive. The other is that by making it 150% compared to the baseline, you are
RAHMANDAD: basically incorporating a situation that, as a result of the dynamics in the model, people's infection rate and people's interaction levels that create those infection rates go above and beyond what they would be absent any of the disease. That is, you are saying people will start to interact even more than when there is no disease. That's just not physically right. That's the problem.

AUDIENCE: Can I add another concern? The fact that this cannot actually shut down infection, even in a very extreme case, I think that is concerning, too.

HAZHIR Yes. So you are saying that if the trend-- and the trend is a problem. So let's put that aside. But let's say this
RAHMANDAD: goes to infinity, the trend is very large number. So this ratio becomes minus 0.5. And therefore-- sorry, not minus 0.5. Sorry. It becomes a 0.5 because it is 1 minus 0.5.

So then it will never go below 50% of the baseline. So you are imposing another kind of a lower bound on the interaction in the population that is baked into your formulation. So it is OK to have a lower bound. You may say the population interaction will never go under 10% of its baseline, or whatever you think is feasible. But if it is baked into the structure of your formulation, you will never recover understand that failure mode. So that's another reason why this formulation is problematic.

So I think that was good discussion on the formulation. Let's take a look at the vaccination process. So we are getting some fraction of population getting vaccinated here, or some number of population getting vaccinated here. And the formulation is the maximum vaccination capacity per day, minimum of maximum vaccination per day, multiplied by fraction of population willing multiplied by unvaccinated population susceptible to COVID.

So I think the idea that Russ is trying to capture here is there is a maximum capacity-- so we put the maximum capacity-- and then a fraction of population is willing to vaccinate. Out of that fraction-- so we multiply that fraction by unvaccinated population that remains susceptible to COVID. So what's problematic here?

AUDIENCE: Unit?

HAZHIR Actually, units should be fine if you make the fraction willing to get vaccinated 1 over year. Otherwise, you're

RAHMANDAD: right, the units will be a problem. So I think the unit part is actually informative, because if you think about it, then you see that the variable naming is problematic.

If you're calling it fraction willing to get vaccinated, that should be a dimensionless concept. But here, for the unit consistency, it has to be 1 over unit of time. My guess is that it is dimensionless. We'll see. Russ, which one do you want to do? Yes, it is. It is a dimensionless one.

So basically, it is missing the whole time for vaccination. And therefore, this is a good unit problem. That is good in the sense that if you had done the unit analysis, you would have said, oh, I'm missing actually a conceptual variable here about time to get vaccinated. What else is problematic? Alexander?

AUDIENCE: If there were no outflow from the unvaccinated population susceptible to COVID, like let's say that was the only outflow and the only flow from it, then everyone would eventually get vaccinated, when we were saying that it's only a fraction that would.

HAZHIR Good. So are you folks following this critique? So Alexander's point was that-- take an extreme condition case

RAHMANDAD: where the unvaccinated population susceptible to COVID was our only stock-- we didn't care about their infection, and so on-- and we just wanted to track vaccination. In that case, mentally simulate-- and let's say you don't care also about the infections on the top just to keep things simple. You're just looking at people moving to the left-hand side-- two stocks.

In that extreme case, what you want-- you're trying to set it up so that 80% of people get vaccinated and no more. But would 80% get vaccinated here?

AUDIENCE: All of them would.

HAZHIR All of them would because the flow variable will never shut off. The flow variable of vaccination rate will always

RAHMANDAD: remain positive as long as you have unvaccinated population. And therefore, it will vaccinate everybody. So how would you tackle that problem? Jason?

AUDIENCE: This might not be correct. You could have some sort of gap-closing logic where you calculate indicated vaccinated population, and then as the actual vaccinated population approaches that level, the flow goes to 0, because now we're at the indicated--

HAZHIR So how do you calculate that indicated? Yes.

RAHMANDAD:

AUDIENCE: The way I did it was similar to Russ. I had a fraction willing to get the vaccine, and an indicated level, the indicated vaccinated population here is the sum of the unvaccinated susceptibles and the vaccinated population times that fraction. So yeah.

HAZHIR OK. I think to follow what you did, we need to open your model. I couldn't mentally follow all of those steps.

RAHMANDAD:

AUDIENCE: So mine-- I did do this robustness test and it seemed to work.

HAZHIR OK. Well let's take a look at it. So basically, here is Jason's third model--

RAHMANDAD:

AUDIENCE: Vaccination pop. You see it right there in the center of the screen.

HAZHIR So this is the--

RAHMANDAD:

AUDIENCE: To the left.

HAZHIR --susceptible vaccinated is left, yes. And what we are looking at is vaccination rate. And vaccination rate is here.

RAHMANDAD:

AUDIENCE: Yep.

HAZHIR So you have capacity. That's fine. And then you have the desired vaccination. Desired vaccination is fraction of

RAHMANDAD: population interested in vaccine into the indicated vaccine population. What is indicated vax population?

AUDIENCE: That is the Size Of the population who desires vaccine? It includes the people who have vaccine.

HAZHIR OK, let me see how it is formulated. So it's susceptible vaccinated plus susceptible unvaccinated multiplied by

RAHMANDAD: fraction of population interested in vaccine.

AUDIENCE: Yep.

HAZHIR So this is not going to work. So the reason this is not going to work is that this is not accounting for the total

RAHMANDAD: population. It is kind of you have all these other stocks that are missing from the calculation of what fraction-- how many people remain susceptible who are not vaccinated but willing to vaccinate. That's the number you need.

AUDIENCE: And we actually have some unvaccinated infected, too, which should be--

HAZHIR You have recovered people who are vaccinated and unvaccinated.

RAHMANDAD:

AUDIENCE: That's true.

HAZHIR It is missing from-- yeah, you we could-- so OK. Let's look at one of the other models just so that we have one that

RAHMANDAD: is working on this front, so that-- because this is a little bit more complex, just talking on the air wouldn't work. So let's look at Hesam's model. Do you have that?

AUDIENCE: Yeah, I have something. So I don't have the pressure there, but I have the indicated vax population is the fraction times the total population.

HAZHIR Yeah, so the other thing was that pressure thing was a little bit harder to follow for me, so we needed to unpack

RAHMANDAD: that one.

AUDIENCE: I have another one, I have a subscribed model.

HAZHIR I see

RAHMANDAD:

AUDIENCE: It has an assumption, which is true.

HAZHIR So this is not vaccinated.

RAHMANDAD:

AUDIENCE: It does have vaccination, but this one's assuming perfect immunity for vaccinated. So I have another [INAUDIBLE] which is a subsequent model.

HAZHIR Got it.

RAHMANDAD:

AUDIENCE: It has a [? C ?] [? sub. ?]

HAZHIR OK. OK. So this one has a subscript. For those of you who are not familiar with subscripts, basically they allow you

RAHMANDAD: to look at-- you look at any variable and have multiple copies of it. So I don't know whether it's subscripted on these. Yes. So it's--

AUDIENCE: You have no vax and a vax [? rate. ?]

HAZHIR OK. So you have no vax and vax as your-- so this is where the subscript shows up in Vensim's user interface. And

RAHMANDAD: you can have separate equations or have the same equation that applies to all of the members. OK. So now again, here we have the vaccination process, so let's look at how the vaccination is formulated. So total vaxed is a sum--

AUDIENCE: It's the sum of all-- both susceptible vax and--

HAZHIR But why not recovered vax?

RAHMANDAD:

AUDIENCE: I think I don't have a recovered. I add them up all into the recovery. I didn't separate the recovered.

HAZHIR OK, but conceptually-- so what Hesam is trying to do is to calculate the total number of people who are

RAHMANDAD: vaccinated in the population, then compare that against the total population and see whether we have people left out who are not yet vaccinated, but are willing to get vaccinated. So that's the general idea you need. You calculate how many total vaccinated people versus how many are willing, which is the total population multiplied by fraction vaccinated.

And then comparing those two, you get a gap, a fraction of the total-- then you turn that into a fraction of total population. That becomes out of all the rest of the population, that fraction is willing to get vaccinated. So then from your susceptible unvaccinated, you use that fraction of them only to drive the vaccination rate.

AUDIENCE: Why are infections people included in that?

HAZHIR Because there are people who were vaccinated and are now infectious. You don't care about their infection. You

RAHMANDAD: care about the fact that they were willing to be vaccinated, and therefore, they are in the vaccinated group.

Anybody has done it in a way that they think is consistent with the story?

AUDIENCE: I just want to check mine. I don't know--

HAZHIR Sure. So let's take a look at Ali's. So we have susceptible that go to vaccination rate, and then we have the
RAHMANDAD: susceptible fraction, fraction interested. So I'm not sure if this--

AUDIENCE: I actually did the fraction of susceptible over the population.

HAZHIR But this is over the total population.

RAHMANDAD:

AUDIENCE: Yes.

HAZHIR So you have the susceptible fraction, but just this is very similar to the-- Russ's equation. That is, if you have the--
RAHMANDAD: if you ignore all of the other things that are happening, just look at the s moving to vaccinated, that everybody from the s will move to the vaccinated.

AUDIENCE: No, I hear you. So if the fraction, susceptible fraction, is-- if you can't control--

HAZHIR Yes. So let's open the vaccination rate.

RAHMANDAD:

AUDIENCE: [INAUDIBLE]

HAZHIR Yes. So you're looking at the vaccine capacity multiplied by what fraction of the population-- whether the fraction
RAHMANDAD: of people who are susceptible are above fraction interested. So there are a couple of issues here. First of all, I'm worried about this equation because it can become negative if those fractions are not correctly matched.

Now, maybe they would never happen if they are very well formulated, but it is a risk that worries me by looking at this formulation, because vaccination rates should never be negative. So thinking about this equation and not letting the robustness to be dependent on other formulations will help you. So that's a general lesson.

The other then part is susceptible fraction is what fraction of population is susceptible. So that is conceptually different from fraction that is interested in getting vaccinated. So if you have a big wave of COVID, and you can't do everybody-- you can't vaccinate everybody, so everybody will get out of the susceptible population because they get sick. Susceptible fraction goes to 0, whereas fraction interested is not. And therefore, this will lead to negative numbers that shouldn't happen.

So this is also not working. But let's see if-- at least the units are good. OK.

[CHUCKLING]

Ah. Let's see. Actually, Ann is not here-- or let's bring up-- yeah, she submitted. She has her model here. So let's open Ann's model.

So this is the same problem. So it's not going to work, because basically, it is using the fraction that can be vaccinated, willing to vaccinate going to the multiplied by susceptible. Same issue.

AUDIENCE: Can I ask a question? In my formulation, if I had defined the fraction interested as a function of the total population, wouldn't that solve the negativity issue?

HAZHIR So if it was as a function of the total population? So let's bring it back up. What do you mean? Oh, not the

RAHMANDAD: formulation you currently have. You're thinking about a fraction interest, that that would be--

AUDIENCE: [INAUDIBLE]

HAZHIR Yeah. Yeah. Mmm, It's going to be tough. I don't like to change the fraction interested from a constant. I want it

RAHMANDAD: to be a constant because it is a well-defined thing. You can say, OK, this population we have surveyed suggest there are 80% interested, or 90% interested, or so on. So keep that, whatever it is.

The way you need to actually tackle this is to calculate the total number of people who are vaccinated, compare that with the total population. You get to a fraction that is currently vaccinated versus the fraction that is maximum fraction that is willing to be vaccinated. So it might be 5%. That gap is the fraction of the rest of the population who are still willing to get vaccinated.

So use that gap, that fraction, to multiply it by the susceptible population. So the susceptible unvaccinated, the fraction of those who are willing to get vaccinated is that gap between the two. So is the logic clear? Question, Cathy?

AUDIENCE: Can you do it without-- so the way I did it-- can you do it without the fractions? The way that I did it was just having the gap of-- the gap is in the units of people.

HAZHIR OK, let's look at your model.

RAHMANDAD:

AUDIENCE: Frequency model.

AUDIENCE: Yeah. Yeah, [? frequency ?] model. And then it's capacity constraint that I don't think--

HAZHIR Yeah, so capacity constraint is more straightforward. This is kind of the more challenging part of the formulation.

RAHMANDAD:

AUDIENCE: So this is all up in the upper right.

HAZHIR So you have the vaccinated population. And then the vaccination rate is-- let's--

RAHMANDAD:

AUDIENCE: On the upper right of the screen. Scroll up.

HAZHIR Oh, do you have formulated it up there.

RAHMANDAD:

AUDIENCE: Yeah.

HAZHIR OK. So effective capacity utilization.

RAHMANDAD:

AUDIENCE: So that's just a fuzzy min.

HAZHIR Yeah, but this is all about capacity. So then we have to really figure out what is the indicated capacity utilization,

RAHMANDAD: correct?

AUDIENCE: Yeah, [INAUDIBLE].

HAZHIR OK. And this is about indicated vaccination rate, so we need to figure out the indicated vaccination rate, which is coming from the vaccinated population versus indicated vaccinated. So this is OK. This is a good calculation that is-- so what Cathy is doing here is looking at the indicated vaccinated population that is the total number willing to get vaccinated versus the current vaccinated population.

Now, how is this one calculated? This is actually the stock of total vaccinated population. I see. I see. So in this model, the way Cathy is dealing with vaccinated people is that they don't get sick-- well, they do get sick.

AUDIENCE: They get sick.

HAZHIR Ah, so then what wouldn't work. So if these people get vaccinated, and then they get sick, in your calculus, they are back into the main stock and flow chain. They are now into the infectious population. And you have lost track of whether they were vaccinated or not.

AUDIENCE: OK. Yeah. So I was thinking about-- in terms of-- right. So I think partly, I'm tracking whether they're vaccinated and subscripts for the recovery.

HAZHIR You have subscripts here?

RAHMANDAD:

AUDIENCE: Yeah.

HAZHIR Oh, OK.

RAHMANDAD:

AUDIENCE: So I think if what you mean is that they become as if they were never vaccinated?

HAZHIR So that's a good question. I mean, if you have subscripts, you can track that. So the vaccinated infectious population is vaccinated infection, rate recovery rate for vaccinated, this rate vaccinated. So then you can track that. That's good.

But then in your equation up here for vaccinated population V , it is actually not just the people who are vaccinated but are not sick or recovered. It is also all of the sick and recovered vaccinated people who should be counted towards creating that indicated vaccination rate, or to some gap variable.

AUDIENCE: I was thinking that once you're sick, it doesn't matter if you were previously vaccinated, and then, like the vax--

HAZHIR It doesn't matter for what?

RAHMANDAD:

AUDIENCE: It doesn't matter for-- you would still decide in six months, I want to get vaccinated again.

HAZHIR Sure. You want to get vaccinated again. But here, what you're trying to calculate is how many people are left willing to get vaccinated, but not yet vaccinated. The second part is important. The ones who are already vaccinated, and their vaccine has not waned, and are willing to be vaccinated should be deducted from the total number of people who are willing to be vaccinated in the total population.

So let's say I have 100 people in the population, 80 of them are willing to get vaccinated. Out of this 80 people, 50 of them are in your vaccinated-- in the vaccinated population. Now I have 30 people in other are stocks, in the stock of infected vaccinated and recovered vaccinated, who are not willing to get vaccinated. So the indicated vaccination rate I would get needs to only look at anybody out of the 80 who is not yet vaccinated. But you're taking out 50 of them, but not the other 30, and that makes it harder.

Now, this indicated vaccination rate-- and it's kind of a-- good. Your units are-- so this is more complex formulation. So you have the V gap divided by time to close the gap plus vaccinated infection rate plus vaccination waning rate. So you are saying as soon as somebody is infected, somebody who is vaccinated is infected, you want them to get vaccinated.

AUDIENCE: [INAUDIBLE]

HAZHIR Because this is the indicated vaccination rate. Now, assuming there is no capacity limits.
RAHMANDAD:

AUDIENCE: Yeah, I want somebody to get vaccinated.

HAZHIR You want some--
RAHMANDAD:

AUDIENCE: Infected people?

HAZHIR As soon as somebody infected becomes-- gets infected, somebody vaccinated gets infected, you want
RAHMANDAD: somebody-- you're right. You want somebody to get vaccinated. What's the logic for that?

AUDIENCE: Well, I think I was just doing the stock control [? towards ?] that problem.

HAZHIR Yes, but this is not exactly the stock control that you are trying to do. This is kind of a little bit of a-- I don't know
RAHMANDAD: what is happening exactly here, but you're saying as soon as somebody gets the disease, somebody else in the unvaccinated population, you are going to vaccinate them.

AUDIENCE: Yeah. I think my-- the thread of thought here is that once somebody is infected, it doesn't matter. I mean, it matters for the death versus recovery, but otherwise, it doesn't matter if they were vaccinated before because they got infected anyways. So they're kind of back in the pool of everybody.

HAZHIR So they are back in the pool of people to be vaccinated, but they are not back in the pool of everybody. So that's
RAHMANDAD: the problem. That's one of the problems. So this flow ultimately is draining people who are susceptible and taking them to the vaccinated population.

But once somebody gets infected, they are not in the susceptible population. They are in the infectious population. So this could lead to rates of vaccination that exceeds the available susceptible population. So it is very important not to even go down that road. That creates all kinds of messy problems.

But all of the problems start with lack of clarity on the underlying mechanism. So the mechanism is not that when I get sick, or if I recover, somebody else should get vaccinated from the susceptible group. You should let these people-- to go back into the susceptible group with waning of immunity and waning of vaccination, they could go into the-- being a susceptible who is in need of vaccination, and then get them vaccinated with whatever rate you want. So from that group, you take them out.

AUDIENCE: OK. So resolving this would be taking out those correction-- quote unquote "corrections," that I tried to--

HAZHIR Yeah, those definitely need to go. Then you need to have the vaccination waning rate to not be vaccinated-- sorry
RAHMANDAD: not vaccination waning rate. This V gap to not only be based on vaccinated population, but also other vaccinated groups that are infected vaccinated and recovered vaccinated. So that way, you can calculate a V gap, which is good.

Then you divide that V gap by total population. You get the fraction of the population that is not vaccinated, but willing to be vaccinated. That's the thing that you really need, fraction that is willing to get vaccinated, but is not currently vaccinated. Then you take that fraction and use it on the susceptible population that is not vaccinated. And you say that fraction of the susceptible population is going to be open to vaccination. And that becomes the population that you drain through vaccination. Hesam?

AUDIENCE: Can you take a look at my population here. I think I missed out on the vaccinated recovery population [INAUDIBLE].

HAZHIR Yes, but other than that, we can--

RAHMANDAD:

AUDIENCE: That's the question-- because I wasn't sure of what I was doing myself.

HAZHIR Yes.

RAHMANDAD:

AUDIENCE: [INAUDIBLE]

HAZHIR Yes, so the total vax--

RAHMANDAD:

AUDIENCE: Needed a recovered--

HAZHIR Yes, it needs to have the recovered. But other than that, it is OK. Good. So then we have the total vaxed, and
RAHMANDAD: then we have-- so this formulation has five inputs into it, so it makes me a little bit anxious. Just kind of it may become messy to understand.

And then it has two mean functions that will kill us. For now, let's ignore capacity. So that's easy to do. Then you have the minimum of susceptible population and interested in vax minus total vaxed divided by time to vax. So this is--

AUDIENCE: [INAUDIBLE] just a unit.

HAZHIR Yeah, I know. So this is closer, but it is still problematic. So can anybody tell what's the problem here? So what
RAHMANDAD: Hesam is trying to do is saying we have a number of people interested in vax minus total vax that are interested in getting vaccinated, because the interested in vax is the total number of population that is interested in vaccination. Those that are vaxed already have vaccine, this is the remaining population. This is the group we want.

And it says, OK, we want to drain that group with some fractional rates-- time to vax. So we get those out of the susceptible population, except that we don't want it to overflow-- we don't want it to be bigger than susceptible population. So if the susceptible population is smaller than that gap, it would shut down to whatever susceptible population is. And if susceptible population is 0, it would not drain anything. So what's the problem.

AUDIENCE: Are you saying everyone in that case, the susceptible population is smaller than that gap, you're assuming all of them want to be vaccinated.

HAZHIR To get vaccinated. Yes. So the problem is what Jason said. That is, if the minimum is the susceptible population,
RAHMANDAD: that is, you have a lot of unvaccinated people who are willing to get vaccinated, they may be still a small fraction of the total population. But you are draining all of the susceptible population with your rate of vaccine. For those people, they might be only 10% likely to accept vaccination, but you're assuming everybody is going to accept vaccination.

Now, you wouldn't have problems with running too many people totally vaccinated, but the speed of vaccination would be unfairly high. That's the problem. Is that clear what happened here?

OK, so I think it's a good time to take a break. We have had enough pressure. So 10 minutes, and then we come back. We do a few more of these and then--

So I think actually we went through quite a few of the models quickly for this vaccination question. So let me actually record those so that we know we have discussed them a little bit.

OK, so we have a few of them now noted. Who else wants to look at their models? We have a little bit more time for going through this. Gerardo, do you want us to take a look at your model?

AUDIENCE: [INAUDIBLE], but I just worked on the first part of the model, part A. And I think that I have to do some improvement [INAUDIBLE]. But if you want [INAUDIBLE], you can just open and try to see something about it.

HAZHIR Sure. Let's open that again. Today we are not doing that much on the analysis of the model, but that does
RAHMANDAD: become a very important-- so you have a big risk perception structure, and then you have the basic SIR model another view, and then the risk perception goes into current contact rate and current infectivity.

So let's take a look at your risk perception sector. You're saying that we get started with the infection rate. Then this creates a reference infection rate. And you also get a reference death rate, which is basically delays or smooth functions. And you turn that into some risk perception. How does that work?

So ah, interesting. So you're saying that people adjust their tolerance for risk based on recent infection rate and death. So folks, notice in the other models we have been using, we have had these reference infection rate and reference death rates, or equivalents of those, that says, how do you normalize? How do you decide if this is a high level or low level? And we have had them as a constant.

What Gerardo is doing is saying, oh, actually, let's make these endogenous with some-- that is basically over a longer time horizon, you change your norm and you get used to basically the current rate of death. But the adjustment time for that reference here is one day, and I guess not very long here either. And then one day here.

AUDIENCE: So it does not focus so much on the parameter. But the idea is that basically, people can adjust their thoughts based on what is happening in the--

HAZHIR Yes, but adjust their thoughts about what is very important. So the way I saw this, at least at the beginning, was

RAHMANDAD: OK, you have your infection rate, but you need to decide whether this is a high number or a low number. You compare that with a baseline, which is your reference infection rate. And using that, you may say now we have high levels of infection and that means more responsiveness or less.

And that is logically correct. It's kind of in the right space. I'm OK with that. And I'm OK with the idea that people adjust that reference point because, yeah, you get used to things and your standards change. If today, we see 1,000 deaths from COVID in one day, it would be a very different emotional reaction compared to when at the height of the pandemic, we might have had 3,000 deaths per day.

So all of that makes sense, except that the time constants for these reference adjustments are not in the order of one day or two day. These are in the order of hundreds of days. That is, people don't get used to death or infection from one day to the next day. It's kind of a very slow adjustment to reset their norms.

So as such, if you wanted to go down that route, I wouldn't use the short time constants. But otherwise, the basic idea is fine. It works. I don't necessarily vouch for it as-- for this simple model. I think it's probably too much to capture, but it introduces some interesting dynamics. So you may want to try something like this for a theoretical model as well.

Then the next step, he brings these together through an attended risk perception, which is basically a weighted average of these two, except that Gerardo is using two different weight parameters to create the weighing, and he basically is multiplying one with one weight and with the other weight, and dividing by the total weight. So good or bad? What's good, what's bad with this formulation?

AUDIENCE: I think you can simplify it to just one parameter.

HAZHIR Yes. So basically, you can easily simplify and get the exact same behavior with a single parameter. So how would

RAHMANDAD: you do that, Alban?

AUDIENCE: So it would be weight of death, and then you use 1 minus death.

HAZHIR Yeah. So basically, you use weight of death and 1 minus weight of death. And with a single parameter, you get

RAHMANDAD: the exact same behavior, except that now you don't need two degrees of freedom and figuring out between two degrees of freedom how to parameterize that, and so on. Is that clear why they are equivalent? Basically, you divide this whole equation on the numerator by one of these things, weight of death, let's say. You will get to a single parameterized version of it without any loss of generality.

OK, but conceptually, this is fine. There is nothing problematic. It's just that you can do it more efficiently. Then you get risk perception, simple formulation we are familiar with, except asymmetry is not captured here. But keep that in mind.

And then you have the impact on use of safety measures versus impact on contact rate. So this risk perception is a simple-- a smooth function. So means the risk perceived, the risk perception in this function, what's the range of it? That is, how far up can it go?

AUDIENCE: From 0 to something very high.

HAZHIR Yes. So it can go from 0 to something very high. But basically, it will not go below 0 because the risk perception

RAHMANDAD: things here are fractions. And therefore, they would never go below 0. But it has no limits on the top side. Alexander, you have a question?

AUDIENCE: Yeah Just going back to just the risk perception, Gerardo divides by the reference.

HAZHIR Yes.

RAHMANDAD:

AUDIENCE: So if the reference is 0, which it could be like, there's no pandemic going on, how would you suggest--

HAZHIR Yeah. So that would be another problem. So basically, you would need some kind of a minimum reference of

RAHMANDAD: sorts to set that actual reference. That the indicated reference should also not go below the minimum reference or something.

AUDIENCE: So is that realistic, though? Because let's say like before COVID, people's reference for COVID deaths was 0 because it didn't exist.

HAZHIR Yeah. So it is not-- you shouldn't think of it as reference-- so you should define what is reference for yourself very
RAHMANDAD: clearly. So we should define reference in this case as the level of COVID case, or the level of disease that basically triggers people's attention. So if you define it that way, that triggering level is not 0. It's something positive, but not a large number.

And it would never go completely to 0 because there are all sources of death and misery around us. So if there is one deaths per million per year for some topic, we probably don't care about it. We don't hear about it. It doesn't come to our attention space.

So therefore, the risk perception-- now going back to the conversation-- is between 0 and a large number. So how does that impact current contact rate? So this part is not clear to me.

AUDIENCE: Here, I start to struggle to try to make sense with this formulation. But I think that on contact rate, I have some trouble-- or some problem, probably on the other part, like the detection is a little bit--

HAZHIR Yeah. So let's look at this formulation, not to try to correct it, but just to highlight a few general observations. So
RAHMANDAD: this formulation is messy. Kind of, I look at it, and I say, OK I can't follow. And because it has a few operations happening into it, you have some subtractions, some multiplication of three different things. You have this extra parentheses that is just syntax that confuses, but it still makes it even harder.

And so I can't really make sense of it. It just would suggest that the logic behind it is not very well thought through. You don't have a very clean logic. It makes it very hard for yourself and for others to understand it. So this is kind of an example of try to make the logic that you are trying to follow very clear, operational, then write the equation.

Now, the other thing is that basically this formulation is not robust because the current contact rate that will come out of it is likely to go outside of the range that you want it to be. So the current contact rate should always be between 0 and the maximum contact your, normal contact rate absent cases and deaths. So enforcing that constraint within this formulation is, I think, what Gerardo was trying to do. And that's how he got into this more complex formulation.

He was testing it and not getting the result that he wanted, and coming up with other formulations to fix the problem. But it started with a wrong foundation. It just-- you can't get to the right thing. You actually don't need a stock for sure, because you already have the delays in the risk perception process captured. So adding a stock is already kind of creating lots of confusion, because now you have to worry about the stock and flow relationships. It's a mess to figure that problem.

So I think that was a good example to critique a few other-- or identify a few other challenges. Any other volunteers? I volunteered you.

AUDIENCE: Can we see Alban's?

HAZHIR Alban, do you want us to see yours?

RAHMANDAD:

AUDIENCE: Sure.

HAZHIR OK. Good layout. One unit error.

RAHMANDAD:

AUDIENCE: I didn't normalize the perseverance to the impact of the--

HAZHIR I see. OK. So you are getting the indicated risk from death rate. That's OK. As we discussed, we could get it from

RAHMANDAD: the-- we could get it from the infection as well, but I'm fine with just using death. Death is probably a stronger driver of risk perception in most communities.

And then indicated risk becomes perceived risk. You can see that the asymmetric risk adjustment is approached, is applied here, which is good. And then the impact uses a sensitivity parameter which is similar to what Jason had. So this is very similar to the formulation Jason had.

It works fine. It's a good first-order approximation. We could come up with alternatives. We can actually think about-- so a simple alternative would be $1 / (1 + \text{perceived risk} \times \text{factor})$, a kind of a scaling factor. So that also will be remaining between 0 and 1.

That is, if the perceived risk is high, or the multiplying factor, that that factor is high. It will go towards 0. But it would go towards zero much more slowly. So when we are dealing with these things that may change orders of magnitude, that is an easier, simpler formulation to work with.

You could also add another parameter that controls the strength of this slope by, for example, $1 / (1 + \text{perceived risk}^\alpha \times \text{factor})$ to the power of some alpha multiplied by that factor, whatever it was. And that alpha can control how responsive you are to risk. But this is fine. Those are a couple of simple alternatives you could use.

Let's now go to the vaccination part. So we have the vaccination process with vaccination-- fractional vaccination. So it is kind of missing the formulation for people not being interested in vaccination. So it is assuming that everybody is interested in vaccination. And then the vaccinated population, I'm assuming is going through the same process. You have the unvaccinated also impacting the infection rate. So you have the unvaccinated plus vaccinated divided by total population.

So remember the assumption that comes with this formulation. The assumption is here that unvaccinated and vaccinated have the same level of infectivity. So that assumption is built into this formulation, but it's easy to relax that by adding that factor that we saw in Russ's formulation.

And then-- let's see, what else here matters? So an interesting part of it is loss of vaccination, waning of vaccination. Yeah. So the immunity period takes people out into unvaccinated. And then the duration of-- oh, OK. So here, we don't have vaccinated recovered, correct?

AUDIENCE: Right.

HAZHIR And that is a little bit of a judgment call. So this is actually-- I think Hesam had the same assumption, that when
RAHMANDAD: people get sick and they recover, should we keep them separate into a stock and flow chain of vaccinated, but recovered, or just dump them into the group of unvaccinated recovered or total recovered? So that distinction may matter in two settings.

One is for our vaccination rate, because for being able to decide how many people we need to vaccinate, we need to know how many people are vaccinated and recovered, but not interested in vaccination right now because they have recently been vaccinated. So that part would not be able-- we would not be able to separate if we don't have the stock of recovered vaccinated versus unvaccinated recovered.

The second part that it may matter is about differences in delays. So if my vaccine immunity lasts for a longer time than my natural immunity-- so let's, for extreme conditions, say natural immunity lasts for a month. Vaccine immunity lasts for a year. Now, if I don't separate these two groups, once somebody vaccinated gets sick, we are dumping them into the group that will quickly lose their immunity because they got the disease, and through the natural basically loss of immunity, they will become susceptible very quickly.

So I have basically nullified the value of vaccine that would otherwise last for a year for these folks. So that's the second reason why this separation does make sense. Now, there is another interesting point here. That is, what about the reverse? That is, if we had the case that people who get vaccinated have much shorter waning time than people who get natural immunity.

So it takes a year after getting the disease to recover. But after a month of vaccine, you are going to lose your immunity. What should we do. So in that case, having this stock of vaccinated recovered, you would let them wane-- by waning of their immunity from vaccine, they would go into the recovered population without vaccine immunity. And then they would lose longer-term immunity from natural vaccination, some natural disease, and go back into the susceptible population. So that way, you get their double protection. You don't lose that double protection. Is this conversation clear? Jake, you had a question or that--

AUDIENCE: You've kind of answered?

HAZHIR OK. Alexander?

RAHMANDAD:

AUDIENCE: So in that structure, people go from vaccinated infected to vaccinated recovered. And then because vaccination immunity is so little, you go into recovery population. And then so that's what happens if waning for vaccination happens much shorter than natural. So then what if there-- maybe you already said this, and I-- like if the reverse is true, if the natural immunity is much shorter than vaccination period, does that mean that you go from vaccinated--

HAZHIR It is OK to then still drain them into recovered but unvaccinated because then they will quickly go into the

RAHMANDAD: susceptible part. It is OK to do that, but it is not necessarily the best way to do it. You could also go straight to susceptible unvaccinated because you--

AUDIENCE: Vaccinated recovered.

HAZHIR Yes, because you got out of the vaccine-- yes. You lost your immunity after a long time. So you have by default
RAHMANDAD: already lost your immunity because of the natural immunity. So in a sense, this is an interesting situation where the value of the parameter could potentially change the structure you use for the flows.

That is, if the natural immunity is much longer than the vaccine immunity, you definitely want to drain the vaccinated recovered into the naturally recovered and let them stay there for as long as the natural immunity helps them with. But in the reverse case, if natural immunity is shorter than the vaccine immunity, it may be actually more objectively correct to go from vaccine recovered all the way to susceptible unvaccinated, because by the fact that you have finished your vaccine immunity, by definition, you will lose your natural immunity as well.

AUDIENCE: So couldn't you do that in the other case as well, that you just have a separate immunity waning time constant for people who are vaccinated and recovered? And you just make that longer than that of people who are unvaccinated but recovered? And it's longer because it's not-- I guess I don't [INAUDIBLE].

HAZHIR So it's not exactly the sum. Yeah, you could do that as well.

RAHMANDAD:

AUDIENCE: But that's like-- it's the same as going from vaccinated recovered to unvaccinated recovered. So it would just be first-order instead of two. But it's still just sum of the time constants.

HAZHIR Yeah. Not exactly the sum, but yes. So yeah, I think you could do that as well. Cathy?

RAHMANDAD:

AUDIENCE: I think I'm still a little confused in my mind. I think if a vaccinated person is infected, then their immunity wanes. They didn't have immunity, too, and so they might have a better outcome. But do we need to consider-- like so the vaccinated person gets sick, but then once they recover, they still have some vaccination immunity. To me, that doesn't equate.

HAZHIR So it is actually kind of a very objective biological question that I don't have a good answer for it. And I don't
RAHMANDAD: think even the literature has a good answer for it. This is kind of one of those areas that I've seen enough models to know that people don't know what exactly is happening. So depending on the story you tell, you could actually build the model a little bit differently.

So one story is that the fact that you got the disease is a signal that you didn't have the immunity-- much immunity from the vaccine. And therefore, by the time you recover, you should just go like the people who don't have any immunity from vaccine. There is no residual vaccine immunity left for you.

The other story is that no, it's a stochastic process. And if you get unlucky or enough exposure, and so on, it may penetrate your defenses through vaccine. But it doesn't mean those defenses don't exist. And therefore, those defenses may still last actually for a while after your natural immunity, and it's worth keeping track of those.

Those two different stories, probably both of them are operating. The thing is that we do have heterogeneity, and some of it is that people who are getting sick are the ones who don't have as much-- don't benefit as much from the vaccines. But also there is the stochastic nature of it. So yeah, we're going more with the stochastic story than with the complete heterogeneity.

In fact, if you wanted to capture that heterogeneity really robustly, the better way to do it would be to separate the population, those who are benefiting from vaccine versus those who are not benefiting that much from vaccine because their immune system is not compatible or whatever. They don't get enough a response and so on.

AUDIENCE: I'm curious to know how formulated would be at risk.

HAZHIR So I think it should be pretty much the same as--
RAHMANDAD:

AUDIENCE: It looks like it's just the sum

HAZHIR It's just indicated risk. It's just-- yes, the sum. Yes.
RAHMANDAD:

AUDIENCE: So this is the simplest possible way to--

HAZHIR Yeah, this would be the simplest possible way. It's a pretty decent formulation. So actually, for most of our simple
RAHMANDAD: models of COVID, this is actually what we use. We didn't even work through the infection rate as a source of risk. We just use death rate, because then it kind of captures very well the fact that over time, as the risk of death goes down, people become much more comfortable and are not as concerned. These days, people don't talk about COVID.

COVID is happening around us quite a bit. In fact, the infection rates are similar to 2022. It's not that different. We don't have the Omicron peak, but other than that, you look at the wastewater data from Boston, you see that we are dealing with COVID rates that are very similar to the normal pandemic year-- not normal, very unnormal pandemic year. But people are not dying as much, even though people are dying.

So I don't know what's the number today, but it's in the order of I think 60,000 people per year in the US. So it's not trivial at all. But don't quote me on that number. It's just kind of a guesstimate for where we are. So I think we--

AUDIENCE: I have a general question

HAZHIR Yes.

RAHMANDAD:

AUDIENCE: So from here, we obviously have a lot of parameters and different weights. How are we thinking about moving forward in terms of actually tuning all of those?

HAZHIR Very good question.

RAHMANDAD:

AUDIENCE: [INAUDIBLE]

HAZHIR Yeah. So first of all, how many parameters do we really need in this model? OK, let's go through this-- actually,

RAHMANDAD: Alban's model is a good exercise for that. And I think this is a good exercise in general.

So we need two parameters here for the risk perception adjustment. We need one parameter for infection fatality rate and one parameter for impact of vaccination on fatality rate. Those are mandatory. We need something for the contact rate and infectivity multiplied with each other. So we don't need both of them, really. But in the SIR model, in business dynamics, we use those for clarity of the logic, because they are completely overlapping with each other. So there is one parameter there.

There is one sensitivity parameter here. There is the recovery time and waning period. So you would have two waning periods for vaccination and for the normal. And then you have something about fraction of population that is open to vaccination. So that's about a dozen parameters total.

Would this model be identifiable-- so first of all, if you make it very kind of generic simple, you can get away with about a dozen parameters, maybe even a little bit less, but probably you need a dozen. So I think most of the models you folks have built have more parameters, probably some of them double that size. So that's one of the ideas, to try to keep things simple to the extent that you can. Don't introduce parameters if you don't have to.

Then there is this issue of estimation. That is, the model identifiable with those 12 parameters and the time series we typically get, which are on cases and deaths. Usually those are the two things you can get. For this model, yes, usually it is identifiable. That is, most of those parameters you can get.

Although if only cases and deaths, maybe not. I haven't done this version of the model with vaccination. The vaccination part makes things more complex to get to see. If you have data on vaccination so that you drive the vaccinated population through the data, it helps with your ability to estimate the model.

But that's always in the back of my mind when I do have an empirical leg to the project, that-- can I estimate this or do I have too many parameters? Can I simplify it? Or can we nail down some of the parameters using other type of data? Like the fraction of people willing to get vaccinated, we don't really need to estimate it from this model. We can look at trends in vaccination, and what fraction of the population got vaccinated and second vaccinated. You can look at surveys, you can look at other sources.

So to the extent that you can pin down some parameters without using-- without letting them determine by calibration, do that. It's another choice. Sepideh?

AUDIENCE: I was thinking that vaccine efficacy also is important in this model--

HAZHIR Yes.

RAHMANDAD:

AUDIENCE: --because efficacy--

HAZHIR So vaccine efficacy, where does it play a role? What's your intuition for where it is impacting things, Sepideh?

RAHMANDAD:

AUDIENCE: So I'm thinking that if we consider vaccine efficacy, its impact on waning rate.

HAZHIR Yes. So vaccine efficacy is not about waning rate, though there is something about it. That is, waning rate is

RAHMANDAD: about how long the vaccine remains effective. So that's part of it. That's important.

So the parameter determining the immunity period from vaccine is an important parameter in assessing the overall effectiveness of vaccine. What else is happening here that addresses-- or what other parameter informs the question of vaccine efficacy here? Anybody else wants to chip in? Hesam?

AUDIENCE: The color [INAUDIBLE], the two red ones?

HAZHIR Yeah, the two red ones are big ones. So there is impact of vaccination on infectivity and impact of vaccination on

RAHMANDAD: death rate. So impact of vaccination on infectivity, this is determining whether the vaccinated people are more or less-- how less likely they are to get sick. So if this parameter is 0, then it means vaccinated people will not get sick at all. And if it is 1, it means vaccine doesn't bring down your risk of getting the disease at all.

And then vaccine impact on death is the other key one. Again, if it is 1, it means it has no benefits for death.

Once you have got it, and if it is 0, it means it completely protects against death. So those are the three parameters really that matter here in terms of vaccine efficacy.

There is a fourth parameter that is not in Alban's model. It was in Russ's model, which is the impact of vaccine on infecting other. And that is also important in these models. That's kind of the path through which vaccines protect the rest of the population, and it's an important factor in these models.

OK. Great job. So keep up the good work. I think, as you can see, even relatively simple models are not very simple. When you're going through the formulations, there are so many things that are happening. But kind of process lessons from this exercise, I hope, keeping things simple to the extent possible, kind of not trying to add a lot of components. Especially at your current level of skills, it is hard.

On the one hand, adding details sometimes make it more operational, makes it easier to deal with it. On the other hand, it makes it harder to integrate things. So trying to think hard, What do I really need to add? becomes important.

Where we are getting the signals for different decisions. So the decision about perception of risk, where we are getting the signal is a key decision in this model, how you are getting it is important. The other is the signal for vaccination. How do we decide the number of people we need to get vaccinated?

You saw that we went through thinking about robustness of every equation. Every equation we clicked on, we were thinking in our mind, OK, how do I enforce the input and output robustness for this model? Do I get the right range for inputs, and does it give the right range for outputs? And are they logical?

This is very important, and this doesn't come overnight. You need to be familiar with how these formulations behave. And that is why we want to work with simple formulations. The most complex formulations we had here were those exponential functions that we looked at in Jason's model, for example.

So let's switch for the last 25 minutes for our next week's modeling challenge, which is energy coflows. So hopefully you have had a chance to look at it. Basically, this is something that Alexander is very familiar with. He has done quite a bit of modeling with John around coflows and energy and retrofits and so on.

The idea is you want to model buildings in a city, let's say, and their energy consumption. So it's a fairly physical problem, which makes it easier to conceptualize and put numbers on. And in the process, we want to learn or improve your understanding of a key formulation, coflow structures.

So the way the coflow structure works is that you have the stock that you're tracking-- so buildings. And then you have some characteristics of that stock, some characteristics of buildings, that you need to track. For example, their energy efficiency. And that becomes another stock that tracks how energy efficient the buildings are in that-- the buildings you are modeling.

So you have the key stock and flow, the main stock and flow for buildings. And then you have parallel stock and flow that is very much parallel to it for energy efficiency of these buildings, or total cumulative energy efficiency-- total energy efficiency for those buildings. And then we want to capture three different effects in this situation.

One is the vintage effect, which basically says older buildings have lower energy efficiency. So as new buildings are introduced, you will have better technology, better insulation, less degradation of energy efficiency because of depreciation and so on. So all of those make the newer buildings more energy efficient. You want to capture that. So that's fairly straightforward.

Then you have retrofits. So what is a retrofit? You may go and add insulation to your walls. You may go and change your windows. You may go and change your heating system. You may add these things at the windows and doors that block the air coming in from cracks.

So there are various ways you can invest in reducing energy consumption or increasing energy efficiency of your building. So capturing that explicitly is important. That is, you should be able in your model to change the energy efficiency of these buildings over time endogenously by investing into retrofits.

So for example, for simplicity, you can have a rate of investment in retrofits, and that should turn into some reduction in energy efficiency-- an increase in energy efficiency, reduction in energy consumption. Now, retrofits cannot solve the problem altogether. They can't drive your energy consumption to 0. They can improve, but they have limits. And those limits are partly a function of the type of a building you have.

So if I have a building that was built in the 1950s with that technology, how much the retrofit in an economical sense can go in terms of addressing the energy consumption has a limit. So there is a lower bound of energy consumption per square foot of building, for example, that each vintage has. So that needs to be accounted for in your formulation for retrofits.

And finally, we want to introduce new technologies that may change that minimum energy consumption. That is, if now you have a better technology for retrofitting the insulation in the walls that makes it cost effective, reasonably economical, and brings down that threshold significantly, you want to be able to capture that. So being able to explicitly have some investment in technology, or some exogenous technology impacting how far retrofits can go, becomes key.

So this is the model you need to build. Basically, a model that allows you to capture energy consumption of a series of buildings over time, exposed to different investments in retrofits and in the technologies that enhance the effectiveness of retrofits. So with that in mind, talk about architectural choices.

So who is a good audience? Alexander, who was your audience when you worked on a related topic? It's not exactly retrofits. You worked on heat pumps. But also--

AUDIENCE: I mean, we include retrofits in our model. I mean, it's still not done yet, but the hope was to show this to policymakers.

HAZHIR
RAHMANDAD: Yeah, so one possibility is policymakers. And so how many vintages do you have in your model for this?

AUDIENCE: So we have from before 1950 all the way till 2050. And then it's divided into 10-year cohorts because that's how we could get data on housing efficiency in areas.

HAZHIR
RAHMANDAD: OK, very good. So note, first of all the level of aggregation is kind of 10-year cohorts of buildings. So buildings built in 1950s, 1960s, 1970s, and so on, are like-- this is for a policy application. And partly driven by the availability of data. If they had more detailed data, maybe they would have broken it down into smaller cohorts as well.

For our purpose, we are doing this more for learning purposes and at a higher level of aggregation. So I'm telling you to use only three cohorts, or three stocks, basically, of buildings of different ages. So this should keep your life easier. But it is conceptually very similar to the kind of a model that can go to the policy level. You just add more cohorts.

Then as you build the model, try to keep it in equilibrium. That is, given a rate of-- I'm not sure how equilibriumy this model is. So if you have retrofits happening-- that is, you're investing in retrofits-- by definition, the energy efficiency is improving for each of the stocks over time. So it may not be even meaningful to have an equilibrium.

So make sure it is in equilibrium without retrofits happening, because with retrofits happening, it becomes infeasible to have equilibrium unless you have saturated-- that is, you have basically used all the retrofits possible, and then it is not having an impact anymore.

AUDIENCE: Just making sure I understand how this works. So the vintage effect, that's the characteristic of the building. It's not like how I was [INAUDIBLE] is like, if they're built in the 1950s. Depending on the time span of the model, do they age? Do the buildings age into different layers of vintage, or is it just with them by birth?

HAZHIR Yeah, so this is actually a good question, and a bit of a tricky one. That is, you can have three stocks, let's say, **RAHMANDAD:** for three groupings of buildings. So then you can have buildings come into it in the 1950s, and then other buildings coming in 1960s, other buildings coming in 1970s, and then they age and go through this.

So at any point in time, depending on varying the simulation time you are, you may have 1950s buildings in here, or in here, or in here. So that becomes-- and then how do you track that their characteristics carry over with them? So the tracking of the characteristics is through the coflows. Basically, that's what the coflows do for you.

And yes, that's how you would do it. In this case, just have three groupings. So it is not that this stock is always the 1950s buildings in the way we are building it in this model. You could do it differently. That is, you don't change these buildings, their cohorts. So you would have actually two layers.

One is the age, so going to the right is the age of the building. And then the other one is the cohort that you have. That this, this is the cohort of 1950s is aging this way. And the cohort of 1960s-- that is the more correct way, or more-- but it is a little bit too complex for our purpose. How did you guys do it?

AUDIENCE: We had separate stocks for every single cohort. And then we track-- sorry?

AUDIENCE: And age them, too, through time?

AUDIENCE: What do you mean?

AUDIENCE: Like a specific cohort, if the span of your model is going for 30 years, will they deteriorate? Their condition deteriorate?

AUDIENCE: No, we actually don't include that, because I guess we could, and we actually do track the different cohorts' age. Like that is a coflow. But we don't-- that has no effect on the rest of the model because-- I actually never discussed that. But I think just data wise, it would be almost impossible to see those effects. Getting efficiency data, even for 2020 was hard, much less getting a time series of it.

HAZHIR Yeah. So this is an important architectural choice, so I want everybody to follow what we are talking about. So **RAHMANDAD:** there are two different ways. One is to capture the aging of buildings and think of cohorts to be happening with that aging. That is, I'm feeding the 1950s cohorts into the first set of stocks. 10 years later, I'm feeding the 1960s cohort in the first set, and this first set is now going to the second set of stocks, and so on.

The other is to say, no, I have different stocks and flows for each of these cohorts. So for the 1950s houses, I feed them. And then when I get to the 1960s in my simulation, I go to the second set of buildings, feed that group. And then when I go to the 1970s, I feed the third group.

And then if I care about aging separately, I would have separate stocks and flows for 1950s homes that are 10 years old, and then 20 year old, and 30 year old, and so on. So Alexander's approach in their work has been separate cohorts, but don't worry about aging. There is probably some deterioration, demolishing, long term that you just have.

AUDIENCE: We have demolition.

HAZHIR Yeah, so you have demolition. But do you have a higher-order delay for that?

RAHMANDAD:

AUDIENCE: No.

HAZHIR No. So you just have the first-order draining of that.

RAHMANDAD:

AUDIENCE: Yeah, we don't have any hierarchy.

HAZHIR That's OK.

RAHMANDAD:

AUDIENCE: And then obviously, like we use subscripts for everything.

HAZHIR Yeah.

RAHMANDAD:

AUDIENCE: And so like--

AUDIENCE: So [INAUDIBLE] can look at it.

AUDIENCE: That's right, yeah.

AUDIENCE: Because if you don't, then it's--

AUDIENCE: Then it's-- yeah, we have whatever eight or nine cohorts.

HAZHIR Honestly, you could go with either of these choices. I think the impact is minimal in our purposes. But I like to

RAHMANDAD: have the continuity over aging process to be separated so that it kind of you practice the coflow structures' nuances, a little bit more with that structure. So what that means is feed the model the 1950s building rate, and the 1960s building rate, and so on.

AUDIENCE: One aging chain that goes through the different lineages?

HAZHIR Yeah, and just have three different stocks, basically. So that's kind of the key architectural choice at the

RAHMANDAD: beginning for vintage versus aging. And I think kind of you're combining them together into a single stock and flow chain.

Then there is the question of what other stocks you would need to capture what we are trying to do. And here, I want you to actually go and deal with that challenge yourself. But a hint is that if you need to-- if you need memory for something that in the system is changing, you will need some stock to keep track of that memory.

If something is not changing, then it is a parameter. So that distinction is key. There are things that are parameters that are not changing. And then there are things that need to change. And you need to be able to calculate-- to observe them, and that they will call for stocks. See how that kind of works for you.

Then the last part is thinking about performance tests for this model. That is, when do you have a model that you are kind of satisfied with based on the objectives of this exercise? So what are some of the tests that come to your mind? Any takers?

AUDIENCE: Extreme condition tests.

HAZHIR Yeah, so what would be a good extreme condition test in this case? So maybe before even extreme conditions,
RAHMANDAD: think about what are the inputs and outputs in this structure? So what are the inputs? What are the outputs?

AUDIENCE: Seeing what would happen if you built a million buildings [INAUDIBLE].

HAZHIR OK. So building of new buildings. That's a potential input. Yep. What else?
RAHMANDAD:

AUDIENCE: You can think about like you're raising the ceiling of efficiency.

HAZHIR Yep. Investment in efficiency technology.
RAHMANDAD:

AUDIENCE: Right.

HAZHIR So introduction of new technologies. What else?
RAHMANDAD:

AUDIENCE: Investment in [? retrofits. ?]

HAZHIR Investment in retrofits. So those three, I think are the biggest inputs for this module. And this would be a module
RAHMANDAD: that you actually can use in a lot of different models, that are kind of the core ideas, very much applicable to a lot of other settings. Then let's think about outputs. What do you care about?

AUDIENCE: Total energy consumption. Total energy consumption.

HAZHIR Yeah, so total energy consumption. And then per square feet energy consumption, because you also may care
RAHMANDAD: about the normalized one if you are changing the inflow of the stocks.

AUDIENCE: Is that what we're calling efficiency per square foot consumption?

HAZHIR Yeah, basically, or its reverse of efficiency [INAUDIBLE]. Anything else?
RAHMANDAD:

AUDIENCE: Demolitions or--

HAZHIR Perhaps, but they're probably not that critical. Another thing might be, I think, relevant here is efficiency of
RAHMANDAD: investments in retrofits. What does that mean? It means I'm investing \$1, how much gain in energy efficiency I'm getting out of that. That's kind of marginal utility or benefit is key for decisions that you will build into these types of models. You guys also deal with that, I think, somewhere in your model.

AUDIENCE: I'm trying to think. We do-- We actually don't talk about retrofit technology all that much.

HAZHIR So this is not about the technology. This is about the decisions for each cohort or each group, whether to invest

RAHMANDAD: in retrofit or not.

AUDIENCE: So we do-- yeah, so we do have an endogenous like optimal efficiency.

HAZHIR Exactly. So that's what I'm saying, that if you want to calculate optimal efficiency investments, you need to be

RAHMANDAD: able to see the marginal value. Or if you are investing this how, much you are getting out of it to then inform whether you increase your investment or decrease your investment.

AUDIENCE: So by efficiency, do you essentially mean more like profitability?

HAZHIR Yeah. So basically, it turns into I invest this amount of dollar, I change my efficiency in units of energy per square

RAHMANDAD: foot by this much. Then you multiply that by the price of energy, and so on, and your total energy consumption, and you can do that. Now, you guys don't need to do any of those. I'm just saying why this is an important output to track, not that you need to do the calculations for optimal energy.

AUDIENCE: One quick clarifying question. To reap the benefits of investment in efficiency, we require retrofit, right?

HAZHIR Yes, some level of retrofit.

RAHMANDAD:

AUDIENCE: I guess-- so just for conceptual clarity. So if retrofit happens at time t of a given level, that efficiency, that persists. And then in the future retrofit happening at t plus n --

HAZHIR Yes.

RAHMANDAD:

AUDIENCE: [INAUDIBLE]

HAZHIR Yes. So the retrofit, when it happens, it brings up the efficiency or brings down the consumption of energy per

RAHMANDAD: square foot.

AUDIENCE: Just wanted to understand how those two are connected.

HAZHIR Yeah. OK. I think those are probably a good coverage of the key inputs and outputs. So then in thinking about

RAHMANDAD: extreme conditions, probably if you are investing in efficiency, you don't want-- if you're investing in retrofits, you don't want your energy consumption to go up. That's kind of simple basic things like that.

They should be trivial, but your model should also not fail on things like that. You don't want energy efficiency to go below the minimum feasible given the current vintage, whatever you have. So that's another consideration.

So energy efficiency remain above minimum for vintage. If you invest in retrofits, energy consumption per square foot should not increase. And I should just make this a little bit smaller. There are other things that you can think of on the extreme condition tests? OK. This is a starting point.

So is the task clear? You're building a model of retrofits that hopefully captures the requirements and is robust, and you test it with a few of these inputs and outputs. Try to put the model in equilibrium when there is no investment in retrofits. When there is investment in retrofits, by definition, it wouldn't be in equilibrium.

And when there is no change in technology. But then, you should change the technology and you should change the retrofit levels or investment levels. Great. Any questions?

OK. So anybody online-- any questions that you guys have? OK. Thanks so much, folks. And see you next week.