

[SQUEAKING]

[RUSTLING]

[CLICKING]

HAZHIR OK, let's get started. So today, we have the Ponzi model. Basically, interestingly enough, we kind of asked **RAHMANDAD:** Yvonne and Ryan to basically work on this Ponzi model a while back as part of their PhD admission process. So they are familiar with the problem as well, although kind of the version we gave them was a little bit different but largely similar.

And the basic prompt is to build a model of the Ponzi, try to stay grounded in the case, but also think of a more generic version that can apply to other settings, Ponzi-like schemes or schemes that may not be nefarious as Ponzi was but maybe even legal as long as you are not deceiving the investors, it is fine.

So let's start with what we did in the previous sessions as well, which is what were your common challenges and take it from there. So what were some of the common challenges that you were facing as you were trying to build the Ponzi model? And folks online, please unmute yourself and chime in and don't wait for us.

AUDIENCE: I think I'm just approaching it fundamentally the wrong way, but I was having challenges with the timing of-- like if they had more trust, there's the chance that they [INAUDIBLE] they continue their lease and their payout is different. And it's more dependent on what the rate of return was promised, and maybe I was just overcomplicating it and couldn't make any sense [INAUDIBLE].

HAZHIR How to make the timing of investment versus expectations of return to work. It's an interesting question because **RAHMANDAD:** you think about it at an individual level, it becomes a messier problem that, for each individual, I was promised three months ago with a 50% return. Am I getting that or not and so on? What else?

AUDIENCE: So there's the element of trust of customers in this scene. I was a little bit unsure how visible the business is to the customers, but how much information they have from inside the business. I went with full visibility, but I wasn't very comfortable.

HAZHIR Very important question. So if they have no visibility, then the only signal anybody would get is when the Ponzi **RAHMANDAD:** fails to pay off. Then you can assume there might be investigations or people talk about it and analyze what his story is versus how viable that the story is, and they start to build some visibility into what might be happening. And the bigger than the gap between what he's saying and what he's doing creates a more serious perception problem in the public.

Good, so that's another important piece of the model how to think about it and how much do you want to model trust explicitly and how do you deal with that. Other issues, other challenges?

AUDIENCE: Mine was, [INAUDIBLE], if I'm trying to do that in an individual level or an agent-based model, and the running time is just a crazy number.

HAZHIR So it became expensive computationally. Yes. It's an interesting example. Go ahead.
RAHMANDAD:

AUDIENCE: No, I was just going to say I find it difficult to generalize the exact same model between Ponzi and Social Security, even though they're very similar. But I was trying to [INAUDIBLE] for a while and then give up.

HAZHIR OK, that's fair. Great. Any other issues? So as we go through it, we will try to touch on some of these, I think,
RAHMANDAD: more explicitly. I think these are good problems to have in mind because they are important and tough ones.

So I guess when I'm looking at the Ponzi model-- and I think actually Yvonne brought up an important question that we usually don't ask in this class that what level of aggregation in terms do you go all the way to agents? And we have been building aggregate models in the class pretty much for all of the cases, but I think also for everything, we could have gone a significant level of detail at a lower level of detail to individuals or to individual houses in the retrofit model or in the predator-prey for individual predator or prey. So you could have built very detailed models.

Part of it is that we are focused here on some of the more aggregate modeling challenges, and part of it is the computational cost. Even though the costs with increased computational powers are increasingly, for a single simulation, trivial, not really something to worry about. But as soon as you want to do more serious analysis, which includes calibration specifically, things will be different.

That is, if you need hundreds of thousands or millions of simulation for a simple calibration run, then simulation times that go beyond a second will be terribly slow. And again, your computational resources are important. So that can change some of the analysis, some of the trade-offs.

There are two other significant trade-offs that come to my mind. One is that when you go into building agent-based models, it is actually very intuitive in the sense that the concepts are concepts you deal with in the real world more closely. So we think of a person, we think of an animal, we think of a house as a very concrete object. And as soon as you are building a model of that thing, that's the first thing that comes to mind.

And sometimes, that means you may lock into that level of aggregation just out of habit, out of simplicity, rather than by thinking whether that is the right choice for your problem. And therefore, it has this kind of an attractor in the space of possibilities that you may converge to without enough thinking.

It doesn't mean it is the wrong choice. It's just that there is a risk. There is a kind of a psychological risk of locking into that level of aggregation, where it is not the right choice. And then you have basically not thought through the process correctly. So that's a subtle but, I think, actually a very important feature of this.

And to add to that, there is also kind of demand side pressures that may push you towards that level of more detailed level of aggregation. That is, people get impressed when you show them graphics with some agents moving around, or deers and wolves kind of attacking each other or whatever that might come out of an agent-based model.

It is kind of cool, and cool is going to increase demand from your audiences to see something like that, or you get applauded for showing something like that. And then that increases the kind of behavioral bias we may have towards that level of aggregation-- again, very explicit. It doesn't mean sometimes that is not the right choice, but it means there is a built in bias in favor of that level of aggregation you need to be aware of. So that's one set of questions or issues beyond computational question that we talked about.

Then there is a third set of issues. And that is about understanding where the behavior comes from. So we emphasize, even though we haven't done as much deep analysis in this course, but we emphasize building a relationship between the structure that generates the behavior and the behaviors that we are observing.

If we don't understand where the behavior is coming from, we are saying, that's just not going to cut it. You're not generating much insight. You can't tell whether it is a bug or a feature of the simulation that you are learning from or being misled by. So you really need to make that connection.

And basically, when you have thousands and thousands or millions of interacting objects in an obscure code, obscure in the sense of you can't delve deeper as easily to see what are the causal chains that create those macro outcomes you care about, it is much, much harder, even with the kind of deterministic, differential equation models with just a handful of state variables, the stock variables. We have a hard time explaining them.

I think we have seen a few of those examples in this class, but more generally, this is a pretty hard problem, and that is why it is a very active research area to just-- what are the best ways to make the connection between the structure and behavior in the much, much simpler case of deterministic models with few interacting parts. Now you go to a stochastic models with orders of magnitude more interacting parts. The problem becomes really hard. I mean, I'm not saying it is hopeless, but there is not even a serious literature trying to do that in a systematic way.

So then you end up with all these kind of very detailed models that people say, yes, this is the behavior. And then you say, why is that behavior happening? And they say, oh, it is an emergent behavior. What do you mean by emergent? Basically, it means I don't understand what's happening. So not that emergence doesn't exist as a construct of microlevel interactions can create counterintuitive macrolevel outcomes, but as a way to push aside understanding, that is a bad concept to use.

So those are three of the reasons I'm usually trying to push myself to think hard at the beginning, what's the right choice in terms of the level of aggregation, rather than going down the trajectory of building a very detailed model and then being disappointed for any of these reasons that we discussed.

So very generally, when is it a good idea to use an ABM? I've heard something that if you want to look at the distribution of something across regions, it's good.

So distributions are potentially important. So you need more details to get the distributions well. And there are interactions-- for example, network structure, how it impacts outcomes. To capture that, you really need to go one level deeper. And sometimes that is really important. In diffusion models, when you're looking at local diffusion, sexually-transmitted diseases, they go through local networks. Modeling those at the network level adds value.

Now that once you understand how it works at that level, you could actually build more aggregate models that capture the features of that more detailed model as well. So sometimes, it is actually a way to discover what is the mechanism that you need to capture at a higher level of aggregation.

So many of the kind of good agent-based modelers kind go between these levels because they say, sometimes, I need to actually go very deep, build all of these things, and then I'll discover that there are two or three core mechanisms that need to be accounted for in the more aggregate version of the model. Then I can build a more aggregate version of the model that has those features. But at the beginning, I didn't know that. So that's another way.

There is a kind of another possibility, and that is when policy levers are very customized at the kind of level of individual. And you need to see the impact of those policy levers that are kind of differentiated based on very detailed characteristics of actors. So that's another area. So those are some examples.

AUDIENCE: Just a general questions, which I think is related. Then how do people understand the behavior of ABMs because, surely, some people do it [INAUDIBLE]?

HAZHIR Yeah, so I think the best practice is they try their best at the level of agent to make sure it is very robust. So all
RAHMANDAD: of the extreme condition thinking that we bring to our equations, they run through that for the individual agent. And so that with the range of inputs that it can get, it behaves meaningfully and reasonably. So that way, you make sure the model is generally robust, and at least you are not looking at noise or kind of error in code or something, but it doesn't fully explain what is generating the behavior.

Then it is basically we went through the kind of hypothesis testing by cutting loops. So you have a hypothesis for what is generating things. And then you cut the loops, and you see whether that behavior goes away. You can do that type of deep analysis with an ABM. That is, whenever you see a behavior, you should come up with a hypothesis for explaining it. Then go through cutting that mechanism and seeing whether that behavior goes away. So that scientific method of testing applies to ABMs as well as any other model as well as bigger problems, other problems. So that's the channel typically.

What happens in practice is that it is harder to do, so you usually end up with models that are dynamically simpler-- that is, they have fewer feedback mechanisms and interacting parts, conceptually interacting parts, but then much more detail within the components that you have. So you may have agents that are people interacting. And in our setting, it might normally-- in an aggregate model, you may have just a reinforcing loop of a Bass diffusion model.

And you say, OK, this is a very simple model. Let me add this and that and that. In that ABM formulation, you may stick with that, but then you start to say, OK, what is the tail of distribution of adoption doing to the rest, or how it is changing the dynamics. And then you would incrementally add complexity and understand. So those are ways to do it. But none of those are kind of algorithmic automated ways of doing deep analysis.

So going back to the question of-- I would be typically looking at when I'm looking at the Ponzi model, so this is about investment. So you have people who are investing in Ponzi or in whatever the scheme you're talking about. There is money that they bring in and then is paid to people.

So there's kind of the stock of money that matters. And then there is liabilities, what Ponzi owes people. And these two are conceptually different, even though they are close to each other. And if you don't make that conceptual distinction, and you try to mix them together, things don't work out. So that's one of the first places that sometimes people have trouble with this problem.

Then there are the dynamics of investor acquisition and people leaving the scheme and what drives that. I think kind of there are basic formulations you can use from standard methods like Bass diffusion model with word of mouth, with saturation. But then there is this risk and reward considerations. How do you decide what is the infectivity or what is the adoption rate at any point in time or the withdrawal rate at any point in time?

And that goes to the question of reputation that has some brought up. How do you make good formulations for reputation that are operational? You can connect it to risk and reward very closely. They are well-balanced in the sense that when you formulate the model for initial condition, whatever is kind of ponzi's initial condition, it works with that.

But then you can think about the extreme when everybody now trusts Ponzi or the other extreme when everybody has lost trust in him and his scheme, the model should behave. That trust formulation or reputation formulation should work well. And then all kinds of robustness to extreme conditions are important as well. So those are questions when I'm entering, looking at these models in the back of my mind. I'm just kind of highlighting what is happening.

So now is a good time to switch to the models. And I think we have a dozen of them. So let me bring them. Let's go through a few of them. So let's first look at how the stocks and flows are structured across different ones. So this is, in a way, the first session we have a more open-ended problem. So I expect a lot more variance in this one compared to the previous ones because the previous ones we had a lot of detail in how the problem was defined already.

So this is by Ross. And here, we have investors stocks and flows, so new investors and investors at maturity. And I assume, by investors at maturity, Ross means people who are basically have invested with Ponzi. They have reached maturity of their notes. They should either get their money back or reinvest into Ponzi. And that's kind of what it is structured here. Conceptually, those are good categories to have.

There is a question-- do you need a stock for investors at maturity, or it is fast enough that process that this is really the flow of people leaving or going through maturing? And then at that point, they split. They decide which path they take. It is an interesting question, which I think, in the Ponzi's case, this is also a good idea, potentially, to have that explicitly, but it may be a bit of an overkill.

Then you have the principle that is owed, principle invest, and interest owed at maturity. So that's how much these people are owed. And then you have money at hand. And then you can, for accounting purposes, look at the money that was collected.

So I like these stocks and flows. You have people. You have money that-- liabilities. This is money owed and then actual money. Let's see how others have this. So this is what we just opened.

This one is Jason's. So Jason has also peoples, but this is closer to assignment 4, so people who are not interested, then potential investors, new investors, mature investors, and former investors. And then you have new funds and mature funds which go with mature investors, if I'm understanding correctly. But what does the meaning of the new versus mature investors mean here?

AUDIENCE: The thought was, in reality, the money that everyone brings in ages with them as they go through the fund, but their returns will only mature if investors earn returns, which are coming from the total funds that we're taking from the new people total fund divided by the mature population.

HAZHIR So what is the definition of mature investor versus new investor? I just want to understand what's the distinction.

RAHMANDAD: Is it the people-- so mature investors are similar to Ross's, people whose investment has reached maturity?

AUDIENCE: Yeah.

HAZHIR OK, got it.

RAHMANDAD:

AUDIENCE: After a year, you're not earning anything. And then--

HAZHIR It's actually 45 days or something when they get into the maturity. But then these guys can't reinvest, so they

RAHMANDAD: don't go back. So that's the assumption in this model.

This one is from Alban. So Alban also has an-- oh, actually, the previous one. Does it have money separated from what is owed? So this is money or what-- they are the liabilities?

AUDIENCE: I don't have the liability explicitly--

HAZHIR So that probably will create some problems, although we should look at it to be sure. Then Alban has this

RAHMANDAD: structure with investors that are potential investors active, waiting to be paid. So this is very similar to Ross's structure. And then investors that have left. Then you have the liability or debt, and you have the cash in the scheme. So this is kind of similar architecture that we saw with Ross's model. And I think it's a good set of options to have.

This one is search-ins. And we have investors potential versus those that are current investors. Then you have money that is owed, and then you have money available. So this is separating cash and liability, and then total defaulted payment. So you're capturing something that can be traced into basically trust reputation based on defaults. So that's one way to capture the trust dynamics. So that's this model.

AUDIENCE: We have Hesam's here with cash, liability, and so Hesam is formulating people rather than people as how much money they can bring into the scheme. And that's OK. I think that's also--

HAZHIR I didn't consider [INAUDIBLE] maturity period. That's something I thought investors can cash their money and

RAHMANDAD: profit--

At any point in time, yeah. That's not too bad. I think it could work fine for as-- it's a good first order approximation. So we could talk about that. So this is kind of the fairly simple but has similar ideas. And as long as you are in the recruited funds, you are reinvesting potentially. So the reinvestment is not separated, if I understand correctly.

Then we have Cathy's. So we have the investors, those who are not willing to invest, those who are potential investors, and those who are the current investors. There is a flow back to not willing. There is a flow back to not willing from most of the stocks. And then reinvesting can be captured here as well. Then you have cash. Then you have trust. You don't have liabilities. You have something about failure. This is just accounting, not really having any feedback into the rest of the system. So the trust is the more important part, if I understand correctly. OK, good. So this is also missing the liability part, which could prove important.

This one-- Gerardo's model has delay. I don't know what that delay is about. So it's the investment goes into this and then comes out and-- we need to look at it more closely to understand that. But it seems to be money that is-- what's that?

AUDIENCE: [INAUDIBLE] structure to capture the effect of the debt visibility on the credibility of the investor. So it's a really bad name, delay, but this is the sense. And there is another page just for the [INAUDIBLE].

HAZHIR I see. I see. So this is more about the credibility, this part. And then you have the money at hand, available
RAHMANDAD: liquidity. That's the money at hand. I don't know if you have the-- and then you have the Bass diffusion type for investors versus potential investors. And then [INAUDIBLE] has nothing. Great.

And let's see next one. This one is Jake's. So we have investors that are scheme seem susceptible, yes, scheme susceptible population. This is-- can go into investors and then go into formally invested. And then if they're unpaid, they wouldn't go back. Then you have the money and--

AUDIENCE: The way I thought about-- I mean, my liabilities is a simpler one, where it's just desired money in terms of if I know somebody soon I'm going to have to pay out, then I go and hastily try to recruit two more to or multiple more to Venmo.

HAZHIR I see. So we can revisit kind of the formulations there in the next round. And then I think this is the last one, Ali's.
RAHMANDAD:

AUDIENCE: [INAUDIBLE]

HAZHIR So this is similar to the sum-- funds, liabilities, and cash. So you guys coordinate with each other. But that's all
RAHMANDAD: right. That's all right.

So now kind of big picture-- we had these three, four categories of people. You could break it down based on whether they are potential investors from scratch or unwilling and then become potential and then become investors. I think capturing unwilling population is an interesting idea. I think a lot of investments, people start with being very skeptical.

But you could also capture that skepticism through some mechanism about trust. That is, everybody is a potential investor is just that they are very skeptical at the beginning. That is, the priors, the initial perception, if you hear somebody is promising 50% return in 45 days is like, this is a scam, I'm not going to look at it even or think about it. So there are these two different ways.

But if you are going with the second way of using trust to model that, you need a machinery for modeling trust, that probably needs a stock variable somewhere there. So that it changes over time. That is, you need to put an initial value for the trust level or kind of expected return or something along those lines.

Then we have the liability and the cash. Those are pretty physical, clear as well. Then the fourth stock and flow is about trust. Now whether you capture it as explicit stock and flow or not is a choice. And I think most of you have simpler versions based on return rate of whether you are able to pay back-- your formulating that.

Now let's start doing deep dives into perhaps the liability and cash part. It is kind of instructive because they're not super complicated, but they have enough interesting pieces to make it worth taking a look at them. So this is Ali's. We can just look at this one as a starter point.

So when you recruit investors, you are issuing liability. And this is in the units of people-- oh, sorry, funds because you guys are modeling it as funds. So that simplifies your life that way. So the liability goes there and the same amount goes into your incoming cash. So that's straightforward.

Then, at the end of the period, so you have a delivery time-- I'm sorry, let me see. How does the exit happen? So you have people who are-- so there is a indicated--

AUDIENCE: You have a confidence in the business, which is characterized by [INAUDIBLE].

HAZHIR Yes. So you are assuming-- so this is actually an important assumption here, that this confidence is cash divided
RAHMANDAD: by a liability plus some small number so that you don't have overflow when you get--

AUDIENCE: People know that how much cash--

HAZHIR Yes, so the assumption here is very important. So there are two assumptions that are important. The first one is
RAHMANDAD: the bigger one is that people know what is the cash that Ponzi has. That is usually not a good assumption. I mean, we could debate, but most of the businesses don't have all of their detail financials visible, at least not the ones that are not publicly traded.

Now, publicly-traded companies have a lot more requirements for visibility. And part of it is exactly these dynamics. That is, once you remove that visibility, the balancing feedback loops go away, and things can get really messy.

So I would be OK as long as you understand that you're making a strong assumption here and have a way to get out of that strong assumption. That is, you can say, OK, I have this switch that says how much this confidence matters versus some other version of confidence that is not reliant on this cash to liability ratio. I mean, you could even have that ratio raised to some power. And if that power is 0, basically, you get to there is no effect from this side or something along those lines.

So right now, if liability is much bigger than the cash, then indicated withdrawal goes up. The way it is formulated is liability multiplied by deserting factor multiplied by $1 - \text{confidence}$. So deserting factor is just a factor that takes care of the units. So 5% of people leave every day as soon as-- assuming that there is no confidence, looking back at this formulation.

But let's be a little bit-- this is cash divided by liability. So if cash divided by liability-- then as long as cash exceeds liability, this number should be above 1 which leads to negative values for $1 - \text{confidence}$. And negative values for $1 - \text{confidence}$ are not a good idea because, then, your settling liability thingy will go negative.

AUDIENCE: So the assumption is that this is a scheme.

HAZHIR No, so that's not-- yeah, but you shouldn't build assumptions like that hidden inside equations. So the key thing

RAHMANDAD: here is let's go with the first assumption that people can see the confidence or can see the cash and can calculate the confidence. Still, the confidence should not go above 1 in the next part of the equations. Maximum confidence is 1.

So as such, you want this to be minimum of 1 and this ratio rather than the ratio alone. Otherwise, you create the risk of settling liability going negative. And then that breaks down the whole model. And yes, in practice, this model cash is always below liability unless you have situations where you are investing, actually, or you bring your own money into it. And both of those are not impossible.

Now what I like about this is how simple it is. That is, you're just saying a fraction of people will leave the scheme if they are not confident that things are working and then as soon as they leave-- I mean, as soon as the confidence goes down.

Now, deliverable cash is kind of a maximum rate that you can impose on how much withdrawal you could have. And how are you accounting for-- so you're accounting for growth in liability using this compounding rate. So basically, interest rate goes into compounding interest. And so this is also a nice, simple way of doing it. Now you have this--

AUDIENCE: The reason for this is that the Ponzi person is getting caught. So the liability cannot go grow forever.

HAZHIR So that's actually an interesting point.

RAHMANDAD:

AUDIENCE: [INAUDIBLE]

HAZHIR Yeah, I don't like the way you have formulated it by just cash bigger than one or some kind of ad hoc thing. So

RAHMANDAD: even if you want to put some kind of a switch that says scheme going bust, and then if scheme going bust has not happened, you continue with this accumulation of liability, otherwise you wouldn't, rather than build it inside the equation, it's the same idea, but it's kind of being built into the equation that doesn't work.

AUDIENCE: So this is also saying that a confidence is when people never withdraw.

HAZHIR It is saying that if confidence is 1, people will never withdraw. Exactly. So is that a good assumption? Probably

RAHMANDAD: not. That is, people would not leave their assets for eternity with the scammers. At some point, they want to use that investment. So as such, the formulation for indicated withdrawal perhaps should have a scaling factor that goes from a maximum withdrawal rate when there is no confidence to a minimum withdrawal rate. That is when everybody thinks this is very legit, no risk. But at some point, I would like to take out my money. And so there is a kind of a maximum residence time and a minimum residence time, which are the 1 over those deserting factors.

AUDIENCE: This is another thing-- I remember you telling us that every relationship in the model should have some controlling parameter. So in this example, if I were to do this formulation, the other issues aside, I think I would be trying to have some of sensitivity of withdrawal and confidence or something--

HAZHIR So let's separate the two things from each other. Not every equation needs a controlling parameter. Some

RAHMANDAD: equations are just mathematical operations that bring things together, so that's not the case.

Then there is the question of, OK, sensitivity to confidence or something like that. Would that be helpful? I could go either way. So there are two different ways to formulate. So Alexander's question is an important one that is, here, we have 1 minus confidence. So once confidence is defined, this equation is deterministic and is driving things.

So I could do two things. I could add a parameter here that says the sensitivity of withdrawal to confidence-- so if you think of this one minus confidence as a switch that moves between those two extremes of maximum withdrawal rate and minimum withdrawal rate, it could be a linear thing, but it is still could be 1 minus confidence raised to some power, and that power can regulate how nonlinear that response is.

Or I can build that nonlinearity or that response, whatever I call it, inside the confidence equation itself. That is, in the equation for confidence, I could make it to change nonlinearly rather than just being the ratio of the two things to be nonlinear.

I think both of them are feasible, meaningful ways of doing things. In this equation, I would probably do the latter. That is, I would build that inside the confidence equation so that when I look at that confidence variable, it is already informing me about how strong this mechanism is kicking in because, otherwise, I'm calling it confidence. But it is some half-baked variable that is not really operating in my model, and therefore, it's not very informative to double-click on that and look at its graphs.

So that would be my solution, which means I would make the confidence equation have that-- if you want to have sensitivity raised to the power of some sensitivity, take care of it there. And then when you come here, you just use that concept. Then you can look at the confidence variable directly.

AUDIENCE: So your suggestion is to have, I guess, exactly one of the two.

HAZHIR Yes, but not both of them. So why not both of them? Because in this case, there is no-- the intermediate
RAHMANDAD: construct that you're generating, so let's say, we made this confidence and it had its own parameter. And then in the next step, we had a separate parameter. So we had two parameters.

There are serial parameters that are just compounding on each other. We are not taking anything else from this confidence to go in another direction that has an impact. We don't have observables on it. So you basically have redundant parameters, and redundant parameters will cause havoc later on when you want to estimate the model, when you want to understand the model. So it doesn't help.

AUDIENCE: I'm just wondering if you can clarify what this liability construct means [INAUDIBLE] supposed to mean.

HAZHIR So it is supposed to mean how much Ponzi owes people based on how people think they are owed, or based on
RAHMANDAD: the contacts because they are writing contacts to pay these people back.

AUDIENCE: So it's how much people think they are owed?

HAZHIR Yes. Well, how much people are owed, let's say that way. Let's keep it that way. Why are you confused with the
RAHMANDAD: distinction between those two?

AUDIENCE: Because when you make an investment, you're assuming some risk. So why should I--

HAZHIR So yes, risk-- let's separate it. So risk is about investments that don't have promised rates of return. This has a fixed promise rate of return. Now, risk in the sense that it is part of the business-- that is, if it doesn't pay. I'm not cheated. I invested in the stocks, and the stocks go up and come down. And therefore, even if it is average 8% per year over the last two decades, I may be unlucky and get minus 20%, and I can't complain. This is not that type of investment. So it was an investment that says 45 days, 50% return, we will guarantee that.

AUDIENCE: And then, so with that, how is this capturing that dynamic?

HAZHIR So the way this is capturing it is a bit tricky. So what is the problem with this formulation in terms of Ponzi's stories? It is a nice kind of formulation, keeping things very simple. But what is the shortcoming in connecting it with Ponzi's story?

AUDIENCE: [INAUDIBLE] maturity period.

HAZHIR Basically, it doesn't have the maturity period. So what does that mean in this case?

RAHMANDAD:

AUDIENCE: You have to be ready today to--

HAZHIR It means as soon as you're matured, what will happen in this model? It is reinvested back into Ponzi. Now that reinvestment in the real world didn't happen automatically. Ponzi was not telling you, oh, if you didn't come and get your money at day 45, I will reinvest all of your investment, and it will grow exponentially forever.

He was saying, no, you have to come get your money, then come back and reinvest. If you want to do that, you're welcome. But that creates this kind of group of people who have matured their investments but have not yet decided whether to reinvest or not. And that's why the formulation that Ross and Albon and a couple others had makes this a little bit more explicit and, as such, adds some additional value for the context of Ponzi's story.

Is the distinction clear in this case? That's pretty important in the actual story because, then, you can actually create situations where the bottleneck is the payment processing system, and that may or may not help you. I mean, sometimes, it can even help you because I'm paying everybody with my fastest speed. It's just that I just have one processor who's paying people, so I'm not failing to pay people. It's just a slow process. Yes, wait here. There is a three day line to get your money back, and there is another three day line to invest it back. And you can do both of those.

So capturing all of that complexity requires a lot more structure, but we want to be clear what is missing when you are going with this simple formulation.

But otherwise, this is a pretty nice, decent formulation. So as soon as you settle the liability, basically, you are paid or you withdraw the money, that money goes out of cash. Note that cash availability is a constraint on that. So this is kind of a maximum delivery rate based on availability of cash. So if you are not settling liabilities here because delivery is constrained by cash, now things start to look bad for Ponzi in this case.

Now I'm not seeing how that is reflected. That is, your measure of confidence here is only getting feedback from this ratio of cash to liability. The day that Ponzi is not paying people because he doesn't have cash, that is a very different day from the day before when he was paying people whatever they were asking for. Whereas in this model, confidence is only this kind of gradual decline based on the ratio of cash to liability.

AUDIENCE: One thing that happens though is, because we will pay that liability out. So if he's short on cash, whatever cash there is will go out in that one day. So cash drops to 0. Confidence drops to 0. It's not exactly as it should be, but it [INAUDIBLE].

HAZHIR So you're right. In that sense-- and you could also capture it using some additional nonlinearity in the definition of
RAHMANDAD: confidence that you have a cliff-like behavior that as long as cash to liability is above 0.1 or 0.2, nothing is happening because nobody is really seeing it. So raising it to a small power, like raising that ratio to power of 0.3 or 0.2, will give you that behavior that you have a cliff like end, but otherwise, things are fine. And that would do a decent job.

So this is kind of good to think about it, that how does this confidence formulation address this failure mode. So I'm OK with it, but with the caveat that then you need to make sure this delivery time is small enough that feedback is not lost.

AUDIENCE: One more formulation question.

HAZHIR Yes.

RAHMANDAD:

AUDIENCE: [INAUDIBLE] do compounding interest [INAUDIBLE] cash?

[INTERPOSING VOICES]

AUDIENCE: Oh, this is the--

HAZHIR Yes, that's just that if cash-- yes, so that's not really doing much. And then he has his own investments. So this is
RAHMANDAD: nice to have what the actual investment of Ponzi might be. So if there was a legitimate business, like Social Security, you could have that investment going into it. That's fine.

And then because they have formulated it with liabilities, once you're getting funds out, you're getting funds out, and you're getting investors out. And that's fine. There's nothing strange happening there. And then these people, after a while, may decide to return. So this part of the formulation of people returning without any concern for confidence or anything else is a little bit kind of not a great alternative.

So the way I would think of it is that people would return only if the confidence level is high, and if you have that confidence already defined, you can use the same construct for informing whether people return to the pool of potential investors.

There is also nothing here about the rate of return impacting the choices to join. So the recruitment factor that you have here in your Bass diffusion type of a model is kind of just a free parameter, whereas in practice, it should be tightly coupled to the promised interest rate and to the confidence. So both of those are important inputs into the recruitment factor.

AUDIENCE: [INAUDIBLE]

HAZHIR Oh, you have feedback from confidence.

RAHMANDAD:

AUDIENCE: But you're right, not from the interest rate.

HAZHIR Yes, but not from the interest rate.

RAHMANDAD:

AUDIENCE: Reality should be some of measure of the fund's interest rate and the investment interest rates [INAUDIBLE] comparison [INAUDIBLE].

HAZHIR Yeah.

RAHMANDAD:

AUDIENCE: You can't go [INAUDIBLE].

AUDIENCE: So just to clarify, when you're saying the promised interest rate, people are comparing it to the reference rate, so if it's higher than reference.

HAZHIR Yes, exactly.

RAHMANDAD:

AUDIENCE: --like market administrators.

HAZHIR Yeah, so the way I would probably formulate it, I would think of the confidence-- I would even name it as this is

RAHMANDAD: the risk of losing my money. So if that risk is 0, then I believe the promised interest rate. And if that risk is 1, they might then my expected return rate is minus 100% or kind of equivalent to that.

And then I will formulate that risk of losing my money. I try to formulate it very robustly and then calculate the expected return rate using that, perhaps with some additional sensitivity that, OK, people are risk averse so that they don't just look at the expected return, but there is kind of a risk-adjusted expected return. So you can add volatility cost to it as well.

Then that becomes kind of the key driver. And that's what I meant by when you're talking about confidence in this model or trust or reputation, building a very operational version of it really helps. So in this case, that operational version would be something like, what is the probability that I'll lose my money? It will be between 0 and 1. You can then use it in calculation of expected return into risk-adjusted expected return. And then those risk-adjusted expected returns can make up for attractiveness of this scheme compared to the alternative schemes.

And then the Bass diffusion model-- remember, what's the idea of Bass diffusion model? It is about diffusion of information. So it is about people knowing about this risk-adjusted return rate. If the risk-adjusted return rate is below the market return rate, then nobody will adopt it. People will become aware of it. So the Bass diffusion model, what it does makes people aware, but then their decision whether to invest or not could actually come from the true-- or the market perceived risk-adjusted return.

AUDIENCE: So how would you do this? How would you do this risk-adjusted expected return? So you have the expected return and that makes sense. You just multiply the probability. You just take it to some power--

HAZHIR Yeah, so I guess, in the kind of finance literature, they just have a volatility factor that says it is your risk-- so
RAHMANDAD: there is a linear-- so minus some alpha multiplied by a standard deviation of the return that you expect becomes your kind of risk-adjusted return. So it is the mean minus some beta by standard deviation of the return rate for that asset class. So you can use something similar.

But here, you are not calculating the standard deviation. So you're calculating the risk that your money will be lost. You need to then go from there to something like a standard deviation of the volatility or kind of a measure of volatility in return. I haven't gone through making those calculations. You could--

AUDIENCE: Is volatility really what matters here, or is it rather that there's a nonlinear response to losses that losing \$1 is much worse than gaining [INAUDIBLE].

HAZHIR So there are really two different things. So the underlying reason why people are sensitive to risk is the same
RAHMANDAD: psychological story that losing \$1 is worse than gaining \$1 or more utility loss than gaining \$1. That's why people are sensitive to volatility.

So the underlying mechanism is the same. But how you translate this probability of loss to an equivalent volatility is not obvious because it's a question about the distribution of the interests or the kind of return that you would expect.

I would not go through building a whole machinery for that distribution because there are a bunch of assumptions that go into it, but ultimately, it is just something like a parameter. So I would just have a parameter that translates this probability into its impact on my expected return.

So you could think about promised versus kind of full loss, which is minus 100% or per sum, that 45 days, as the range. And then you are scaling between the two. So that probability is a number between 0 and 1. So raise it to a power and do this-- switch between the two. That should do OK. I think that would be a decent first cut.

AUDIENCE: Question-- it wasn't an easy decision for me to can go with confidence or trust or any measure of that [INAUDIBLE]. Do we need to start with that or [INAUDIBLE]?

HAZHIR So it's a very good question. And here, you have stocks in the sense that you are using the liability and cash,
RAHMANDAD: which are both stocks. So the inertia in the payments is showing up in these two stocks. So that was why I didn't actually bring up this as a problem that you don't have inertia.

If you were getting it, for example, from this flow rate of actual payment versus desired payment, I would have signaled that as another issue to think about. But more generally, there is a question of spread of news about whatever the signal is and that is not captured in this model. You are assuming cash is known today, and not only that, people are responding to it right away. Not a great assumption. So an explicit delay process for perception, spread of the information becomes relevant as well. Yeah?

AUDIENCE: So I know I'm talking about this [INAUDIBLE], but even if expected return is low-- I'm just thinking about this-- so long as that recruitment factor or whatever this infectivity is positive, then everyone will eventually join, just over a longer period of time, right?

HAZHIR Yes.

RAHMANDAD:

AUDIENCE: So long as it's positive. And so that seems a little odd.

HAZHIR So why is it odd? So if this is the best investment available in a marketplace, why is it odd that everybody would
RAHMANDAD: try it?

AUDIENCE: Well, I guess-- sorry.

AUDIENCE: [INAUDIBLE]

AUDIENCE: I guess I was thinking that in the case that it's not-- I was thinking about of it being some ratio to the-- yeah, I guess I was thinking of it as being as it being some ratio to some reference to a term and that if the ratio is small, so it is worse than the reference--

HAZHIR Oh, as soon as it is worse, it shouldn't then create-- and that was the point about this effect on your recruitment
RAHMANDAD: factor. Again, with the caveat that the Bass diffusion is kind of conflating two different things-- the diffusion of information about this thing that exists and the decision to actually become an investor. These two are separate from each other.

In the current version, they are combined into a single action that is recruiting of investors whereas, in the real world, it is when a lot of people are investing with Ponzi, a lot of people will learn about this investment opportunity, and then they have a separate gate for deciding whether to invest in it or not. And that separate gate is informed by-- so I would probably, if I wanted to capture this separately, I would have, rather than this going to the recruited funds, it goes to considering Ponzi for investment or something like that.

AUDIENCE: [INAUDIBLE] exposed.

HAZHIR Exposed, yeah. So that's kind of similar to the exposed state in the SCIR.

RAHMANDAD:

AUDIENCE: [INAUDIBLE]

HAZHIR Sure.

RAHMANDAD:

AUDIENCE: --trust and confidence. So if you believe [INAUDIBLE] when I wanted to show this [INAUDIBLE] and then suddenly, [INAUDIBLE].

HAZHIR Good. So if you have a stock, the signal that is going into it, whether this is the current version of the confidence
RAHMANDAD: that this model has or some alternative would have a sudden shift, but then, yes, the input into that delay function would have a sudden shift because-- and you would like it to have. The sudden shift comes from the fact that you don't have money to pay people or some other way.

So for example, you may have people investigating you. Journalists were writing about Ponzi and investigating what is this guy doing. And that was actually the trigger mostly. So a couple of articles that came in Boston Globe and other-- I'm not sure if it was Boston Globe, but in a couple of news outlets were the triggers to runs on Ponzi. One of them, he managed to control the other one. He ultimately couldn't with some delay.

So in that sense, there is the information coming in that is changing the trustworthiness of this investment very quickly, either because he's not paying people or because somebody is investigating them. Then there is the spread of that information in the population. And a simple delay has a kind of a delay in it, so people wouldn't know about it overnight.

But in these cases, the real world of perception is very asymmetric. The adjustment time for that delay, if the confidence is increasing, people hear about it but more slowly because it's like, yeah, I actually haven't invested in Ponzi, and I got my money back. That's an interesting kind of-- I'm not sure if I will go and talk about it with a lot of people because now that's a good investment for me. I don't want others to saturate that investment opportunity. I'm more likely to just keep it to myself.

But as soon as somebody is not paid, everybody starts shouting, oh, my cousin has invested with Ponzi. Let me call them and tell them or just go and bring them to the line because, otherwise, they wouldn't make their money. This thing is collapsing. So that asymmetry you can capture in the delay function for perception. So you would have a delay time that is very small when things are going down. But when things are going up, it would be a longer delay time. So that would be a little bit more advanced formulation but pretty standard. So is what I'm saying clear because-- yeah?

AUDIENCE: [INAUDIBLE]

HAZHIR Sure. So the switch would be basically the flow for the delay is desired minus current level of a stock divided by a
RAHMANDAD: time constant. So now, rather than dividing it by just the time constant, you divide it by if then else.

AUDIENCE: [INAUDIBLE]

HAZHIR Yes, desired is bigger than the stock T1. Otherwise, T2. That's all. So that was a good point. And I think that
RAHMANDAD: asymmetric delay may work well. So we kind of beat this model enough. Let's go to some random other one. So I don't know-- whose is this one. So Alban's, so let's take a look at Alban's model.

So here, we have potential investors who invest who are waiting to be paid who are not interested in this scheme. The rate of becoming-- so with a payback time of, in this case, three months. Was it three months or 45 days? I don't remember. Was it three months?

AUDIENCE: 45 days.

AUDIENCE: It was 45 days.

AUDIENCE: [INAUDIBLE]

HAZHIR Yes, 45 days, 50%. So this is good because you get the investors going to this stock, and then they decide it's
RAHMANDAD: operationally a little bit cleaner to follow and kind of also avoids the problem with that. But of course, it adds one level of one additional stock. The Bass diffusion model idea is, again, pretty standard here, except that the adoption fraction also has some effective expected return rate, and expected return rate is coming from claimed interest rate of the scheme and confidence in the scheme.

So this is very close to what we talked about right now. And we could take a look at how it is formulated. It is claimed interest rate multiplied by confidence in the scheme. It's largely unit-consistent. That's good. And then the confidence scheme is coming from that fulfillment ratio. So you may remember, we talked about one way to capture it is looking at how much he's paying people versus what people were expecting to be paid.

And as soon as that ratio starts to fail, then you have a very strong signal. So that fulfillment ratio is a number between 0 and 1 that he raises to a power for this sensitivity of confidence. And what should this sensitivity of confidence to fulfillment ratio be? Should it be below 1 or above 1? So that fulfillment ratio is a number between 0 and 1 that says what fraction of what people are asking for you are paying. Would you put the sensitivity of confidence to fulfillment rate below 1 or above 1? That's the question. So think about it for a second. And why?

AUDIENCE: [INAUDIBLE]

HAZHIR So you would say below. Why?

RAHMANDAD:

AUDIENCE: Well, not really because--

HAZHIR So if I'm paying only 50% of what people are asking for, if I go with a below 1 number, then the indicated
RAHMANDAD: confidence will be 0.9. If you actually want it to be close-- if I pay even 90% of what people are owed, that's usually signal enough that this thing is going to be bankrupt in 24 hours. Everybody will be attacking the doors of Ponzi. So that means the confidence or indicated confidence should now go down to 0.01 or something, some very small number. So that requires a much-- I mean, above 1 number. So that's to think about this.

And then, to his credit, Alban has the asymmetric adjustment here as well. So basically, he's accounting for that difference, which is very nice. It's exactly the formulation we just talked about [INAUDIBLE]. So that's good. And then this confidence, the parts that are feeding into adoption, it's an interesting part because this needs to go to 0, essentially, as soon as confidence starts to go down.

So then Alban also has the effect of expected interest rate on fraction returning, which is good. I like to use the same effect here because, conceptually, it is coming from the same underlying cost-benefit analysis, so creating a separate channel for it would not be adding much value, and it introduces more free parameters. So this is fine. Let's see. Yes?

AUDIENCE: So I know a few of the submissions have this idea of investors who are active investors who are waiting. But it seems like the only residence time in the w here is the minimum processing time. Couldn't you just have combined the two stocks? I guess I'm just asking what the purpose of it is.

HAZHIR So the purpose of this stock?

RAHMANDAD:

AUDIENCE: Yeah.

HAZHIR Yes, you could combine it because it is a short period of time. And that short period of time is coming from--

RAHMANDAD:

AUDIENCE: [INAUDIBLE]

HAZHIR Minimum money processing time. Well, part of it is coming from that. Yes, that's true. That's the main driver. So
RAHMANDAD: the time constant in that stock is fairly short and, therefore, there shouldn't be many people left in that stock. And then you go back to the formulation we saw with Ali's model, which I think is fine. I don't think there is any problem with using that.

The only thing is that this allows you to track people a little bit more concretely in this intermediate state, that they want to be paid, but they are not accruing additional liability. So that's the core issue. So that way, he can basically say, OK, I will pay. As soon as somebody gives me money, I'll add 50% to it or 100% to it depending on what time. So he's using one third per month. Why is it per month? So this is actually interesting. I am not following what is happening.

AUDIENCE: Alban, [INAUDIBLE].

HAZHIR Oh, Alban, go ahead.

RAHMANDAD:

AUDIENCE: Oh, I was just trying to say that [INAUDIBLE] to start because the word of mouth is only coming from an active investor, once you are waiting in the line, you're probably waiting for get paid. And if the fulfillment ratio is not 1, then you are waiting in line to get there, you probably don't [INAUDIBLE].

HAZHIR So that's another reason that might be a good reason. But now there is an interesting piece here. If I understand
RAHMANDAD: it correctly, Alban, when you have debt in the scheme, that debt continues to grow until you are paid and you get out of-- you're cashed out, correct?

AUDIENCE: Yes.

HAZHIR So at least in Ponzi's story, that's not how it works. Basically, as soon as the term has finished, you are not
RAHMANDAD: accruing interest. So the formulation for that inflow-- so that was why I was a bit confused was it can be even simpler. You don't need the debt in the scheme multiplied by claimed interest rate. You just need a claimed interest rate that is a fraction of the cash in.

So it would be 1 plus that fraction. So if it is 50% over 45 days, it would be 1.5 multiplied by cashed in. And then that becomes how much he owes people as soon as they are going in. You don't accumulate this forever. That would do a better job of accounting here than the current version because here, people who are investors there waiting to be paid continue to accrue liability or Ponzi continues to accrue liability. So that's a minor--

AUDIENCE: Yeah, I misunderstood the [? contract. ?]

HAZHIR No, that's fine. That's otherwise pretty good. Let's see. Then there is the max cash out that should control the
RAHMANDAD: actual cash out, which does. And then this creates the debt fulfillment ratio, which impacts actual getting paid. And then that impacts people who leave. And what fraction return depends on effect of expected interest rate.

Now I think this is all good. There is a subtle thing here, pretty subtle, but it might be interesting to think through it. So let's assume that fulfillment ratio overnight falls from 1 to 0. Basically, this is the night that Ponzi goes bust. So the confidence in the scheme starts to go down very rapidly. But it actually starts at a high level.

So the fraction returning to Ponzi will actually be 1 at the very beginning of it, or a large number. Most of them will return to put their money back. But at that very point in time, these are people who are not being paid. That is, the actual getting paid is much smaller. So for a period, because of the delay in confidence in the scheme, you have people who are not paid but still are going back and reinvesting in Ponzi. And that's just not how the things would actually work.

So I probably would have this fraction returning to A to be bounded by the debt fulfillment ratio as well. That is, it shouldn't go above that or something like that. Is that clear? Yeah, go ahead.

AUDIENCE: Well, so I know we've kind of been talking a lot about the confidence idea, but in my head, system confidence is always going to be either 1 or 0 basically because that's basically what the debt [INAUDIBLE] rate is going to be.

Either people are getting paid or one person didn't and now it's all canceled, and Ponzi probably won't even talk to new investors because he knows, eventually, somebody's going to come from him, so he's going to get out of there. So it feels like the confidence matters less on the system confidence and more just on your exposure to-- an individual's exposure to the idea that this can work and has worked.

HAZHIR So that's a good observation on the downside. That is, as soon as there is no payment to even a single person,
RAHMANDAD: things will unravel very quickly. But on the upside that is people building confidence into it, the reason at the very beginning, these schemes don't go to the roof is that most people are skeptical. That is, their initial assessment is that it's probably a scheme. We haven't seen the evidence. Maybe it takes a couple of years.

But we are not-- I'm not going to invest in it. So that building of confidence, it starts from a small number. That is, their priors is that 90% this is just a scam. But after six months, I'm seeing my friends getting paid back. I would say, well, maybe it is not 90%. Now, it is 60% in my assessment, and I haven't seen any negative signal. As soon as I see a negative signal, hell breaks loose. Yeah?

AUDIENCE: [INAUDIBLE] also have average investment [INAUDIBLE]?

HAZHIR We could, but then we are kind of adding a mechanism that is already can be captured in the number of
RAHMANDAD: investors. So in a way, I think what Ali and Hesam did, which was how much money can be invested, takes care of that to some extent, although it has its own kind of storytelling problems, that money doesn't talk with money, it is people who talk with each other.

But I'm OK with keeping it simpler this way rather than making the amount of investment endogenous, even though, in practice, that was part of the story. That is, people would go, yeah, I just put a dollar here and see what happens. But next time, if it worked, I would put \$10 or whatever was the scale of the investment.

AUDIENCE: The idea [INAUDIBLE].

HAZHIR Go ahead.

RAHMANDAD:

AUDIENCE: On the idea of the [INAUDIBLE] I still feel like that's not necessarily a system confidence thing. I am becoming more confident because I have been exposed to more and more of my friends who have been successful. So that isn't that going to be, on the individual level, dependent on percentage exposure of my friends who have done it?

HAZHIR

RAHMANDAD:

So partly yes, but partly depends on information diffusion mechanism you hypothesize is operating in that environment. So is it only my friends? Do I see news items about Ponzi and this scheme that might be actually working? Do I hear from people talking about it in the corner streets? So the more you expand that, the more generic for the whole population, you can talk about.

Now the Bass diffusion model is essentially doing that. That is, it is saying people talk about it with their friends, and if they have experienced it, they will talk about it more. So you could basically think of this Bass diffusion model to be doing that for you with the caveat that I mentioned, that Bass diffusion model takes people to the state of I am now considering. And then there is a process of conversion that may take some time and depends on whether that expected rate of return is high enough at that point in time, based on their perception.

So I think this is a pretty good model, Alban. I haven't checked every equation. So let's actually do a simulation here to just see what's the baseline behavior in the model. So we have investors. So more and more people are investing. But at some point, it stops. And the reason it stops is that the confidence crashes because it has a very short delay. So you see that the confidence goes up slowly as we discussed and then crashes very quickly. This asymmetry delay does the job nicely here.

And then the cash in this scheme, it is crashing. It's an interesting double dip here. So what is happening that is creating that double dip? So part of it is cash out being slowed down, I think. And then part of it is cash in being shut down. So if we look at what is driving it, so cashing slows down for a little bit and then crashes. Cash out starts to go down because of the constraint on how fast you pay.

So max cash out-- 13 million. Yes, this is exactly when the max cash out starts to become the binding constraint. So for a few periods, max cash out keeps the stock of cash to go down even faster. But then there is no inflow. And then it goes to 0. That all makes sense.

And then you have investors that are waiting to be paid. So when he can't pay, basically, people wait there to be paid. And that actually creates an interesting reinforcing loop because then you have desired to be getting paid and, therefore, you have bigger desired cash out. I mean that strengthens the rapid adjustment at that point in time because you can settle your accounts basically. I think the behavior makes sense in this case.

One thing that would have been nice to have here is to have two things. One is to have actual return of the investment, if Ponzi was actually making money or investing this in some meaningful investment that had a rate of return here. And then comparison of expected rate of return of the scheme versus some outside option for people for their investments so that they decide whether to-- so this adoption fraction comes from that. It is kind of built into this max effect and sensitivity factors. But having an explicit external investment option would make it nicer. But this is pretty good.

So maybe we should take a break, 10-minutes. We can come back and look at a few more models and then go to the next few weeks assignment.

So maybe for this part, let's get volunteers. Who wants us to look at their model? Cathy? Sure. Let's see. This is Cathy.

AUDIENCE:

I think? Yeah. So I don't have liabilities, and then I think I heard you say-- so I'm driving the kind of indicated payout to current payout from the flows.

HAZHIR So let's actually look at that because let's see whether it works or what are the challenges. So basically, you

RAHMANDAD: have cash on hand. And it is coming from investment and average investment amount and something else. It's investment or reinvesting also. That's fine.

AUDIENCE: Oh, and I also found a bug at 908 in my investors stock formulation. Could you click into the equation for that really--

HAZHIR So in the investor stock formulation.

RAHMANDAD:

AUDIENCE: It should be minus reinvesting plus reinvesting. I think when it autopopulated it-- it should be both minus and plus.

HAZHIR Sorry, what do you mean?

RAHMANDAD:

AUDIENCE: Like minus reinvest plus reinvest.

AUDIENCE: It goes out and then it goes back in?

AUDIENCE: Yeah.

HAZHIR Ah. So what it basically means is that it doesn't have any impact.

RAHMANDAD:

AUDIENCE: Exactly. So I wanted to ask about that. I wanted this capturing thing to happen just for face validity reasons. And then [INAUDIBLE] overwrites it every time I open the formula. And so that indicates that it's not that good. And I think that the better way to do it is this two-flow maturing investor thing.

HAZHIR And basically, that's what you-- it forces you to do for good reasons because, otherwise, it's very confusing for

RAHMANDAD: people looking at it. And then it really doesn't have any impact on that stock. So basically, you would have an outflow and an inflow. And then the outflow informs the inflow directly.

AUDIENCE: Exactly. So these are discrete packets of, like, you have your 45 days. I guess, the way I was thinking about it is. You just need not investors and investors. And they can just cycle between each other. The whole reinvesting thing is not at all-- the actual individual, whether they [INAUDIBLE] doesn't matter. It has no influence.

HAZHIR So you would put them back into the noninvestors or potential investors, and then they can come back and

RAHMANDAD: decide. That's right. You could do that. I don't think that is wrong. The only thing is that in different models, people have implemented it differently. But the investment decision is not just a decision to invest. In most of these models, it's also being informed about this investment opportunity. So people are using Bass diffusion model in most of these to drive that investment flow.

So if you put them into potential investors, now there is a kind of additional process of being informed about this opportunity that doesn't apply to these people who are reinvesting. If we had it into those kind of an intermediate stage of people who know about this opportunity but have to decide right now whether to invest in it or not, then it would be a better fit. So you go into there and then. So that's that.

So now, here, the inflow is clear. The outflow is the more interesting thing. So you have the minimum time to pay that puts the upper bound-- so it's a minimum of cash on hand divided minimum time to pay and indicated payout. I'm OK with this, but perhaps a good idea to separate that max feasible payment or something as a separate variable. It's mathematically equivalent but a little bit cleaner.

And then the indicated payout is coming from how many people are cashing out. And then the amount owed at payout, which is just a promised fraction expanded. So right now, the payout is happening. That is investors are leaving. But if there is no money, they are not paid, correct?

AUDIENCE: Yes.

HAZHIR So how do you account, accounting-wise, about how much money is lost in this scam? That is, when all is said
RAHMANDAD: and done, how do you calculate how much money was owed and never paid?

AUDIENCE: So those are the scamages, the scam damages, so that's just the indicated payouts minus the payouts.

HAZHIR Got it. So yes, then this works. That is basically what you have here, this additional stock of scamages is doing
RAHMANDAD: what you could also do with the liabilities. But it is the gap between the cash and the liability that is-- not the cash and liabilities. What he had to pay and he didn't.

AUDIENCE: I suppose I just didn't have a-- because investors can't observe liabilities, I like what Jake said about liabilities being a pressure on the scheme leader to go recruit more people. I didn't think about that. But other than that, I didn't have a good feedback story about what are liabilities doing outside of--

HAZHIR Accounting.

RAHMANDAD:

AUDIENCE: --accounting.

HAZHIR Yeah, so I think that's fair. And liabilities are really-- their first-order effect in this model is actually accounting. So
RAHMANDAD: that's why once you have it in this way, it does that accounting job. Now the reason this is working partly is that you are getting everybody out at the 45 days. They are not any more investors. They are people who had invested and had to be paid at that time. And if they weren't paid, bad luck. They are not willing to invest or some other category.

Now the reinvesting part is tricky here because it is not-- because you have the cash out, and cash out is coming from what? This is from the payout time and total investors.

AUDIENCE: So investors are leaving. And then some fraction decide to reinvest.

HAZHIR I see. I see.

RAHMANDAD:

AUDIENCE: And those who don't just cash out. And I assume the baseline fraction deciding to reinvest is 0.9. And then that decrements by losing trust.

HAZHIR Yeah, so you have these people who are reinvesting in this way of thinking, assuming that this reinvesting
RAHMANDAD: basically doesn't do anything to the stock that is on the flow. That flow has no effect.

AUDIENCE: I think the assumption is that they have to go see their money. They can't just say, I guess-- there has to be some proof that their return existed.

HAZHIR Yes. And the way they do it in this case is-- so you are getting the indicated payout from cash out. So this is only
RAHMANDAD: people who decide not to reinvest.

AUDIENCE: No, it's also coming from reinvesting as well.

HAZHIR Oh, OK. It is coming from reinvesting. Got it. And you need that because you get to reinvesting to also impact
RAHMANDAD: investments. Yes. So this is actually the right way of doing it in the case of Ponzi because they had to go and get the money and then go back and invest it. So I think that's fine. In fact, all you need to do is to have this reinvesting without having it impact the investors. That's basically what is happening. I mean, you just don't need this flow variable to be a flow variable. It just is an auxiliary variable.

AUDIENCE: [INAUDIBLE]

HAZHIR And then your model will be fine on that front. Population growth-- it's an overkill for this model.
RAHMANDAD:

AUDIENCE: Oh, I was wondering-- so the last question was how do you get it to be sustainable over time. And population growth--

HAZHIR I see. I see. So you were thinking that population growth could be a driver of that. So that's interesting. The first-
RAHMANDAD: order answer is, well, you also need to have return on your cash. That is, you need to grow the pot of money. That's important. So that is currently missing but could be added.

AUDIENCE: I suppose I was assuming it was purely a scam, [INAUDIBLE].

HAZHIR Yes, then it is the slow growth. That's the only way is that within a finite time horizon, you might be able to
RAHMANDAD: sustain it if you are not too attractive. So perhaps Madoff was closer to this model because he sustained it for decades, if I remember correctly, more than 20 years. And that was basically, yeah, this is not an unbelievable rate of return. He was just kind of a little bit more stable than the market and a little bit better than the average investment opportunity in the market.

AUDIENCE: And he never had to-- or he didn't often have to show someone their money.

HAZHIR Yeah, so that's also part of it. But nobody is showing all the money in the account. Whoever wanted to be paid
RAHMANDAD: back, he would pay them back. So in that sense, he was paying things back. But then he would reinvest. So whether this-- so right now, you have reinvesting going both into the indicated payout and investment.

You could say, for Madoff, he didn't use either of these channels. That is, it didn't go into the indicated payout, but it also didn't go into reinvestment. And there will be a bit of a difference because your max cash out becomes more quickly binding if you have both of them in the indicated payout. But not a big deal.

AUDIENCE: [INAUDIBLE] could have actually gone on for a long time if there wasn't an exogenous shock to [INAUDIBLE].

HAZHIR Yeah, basically.
RAHMANDAD:

AUDIENCE: [INAUDIBLE]

HAZHIR Yeah, exactly. It could have continued.

RAHMANDAD:

AUDIENCE: He might have started as early as the 1970s [INAUDIBLE].

HAZHIR Oh my god. OK.

RAHMANDAD:

AUDIENCE: So you could wait a few minutes longer [INAUDIBLE]. But what's the point of the scamages? Is that just because it's just a [INAUDIBLE]?

HAZHIR Yeah, it is, but it is important for accounting because that's one of the key outcomes in this case. And then let's

RAHMANDAD: look at the trust equations. So trust is coming fraction. This indicated trust, and indicated trust is coming, effective fraction of obligations met. So this is kind of perceived fraction of obligations met. So you have this perception delay here. What's the idea of having this intermediate perception delay and then the trust, again, adjusting with some time constant?

AUDIENCE: It could do without. I think the idea, though, is just that people don't get paid, and then it takes some time for them to tell people.

HAZHIR Yeah, but that is also inside the trust and investment slow adjustment that the reason that is adjusting as a stock

RAHMANDAD: with-- and nicely, you have also the asymmetric adjustment is basically that, that it takes time for people to learn about it. And so I wouldn't have a separate delay process here. It's kind of too much overlap. Now you have an S-shaped function.

AUDIENCE: Yeah, so actually the function that's active right now is an exponential reactivity parameter-- the bottom.

HAZHIR This last part. So this is similar to what Alban had as well. So then that--

RAHMANDAD:

AUDIENCE: I want it just to fall very quickly.

HAZHIR Yeah, so that reactivity parameter would be-- yeah, so you have a large value so that it goes quickly to 0 if you're

RAHMANDAD: not meeting obligations. Good. So that's working fine. I'm not looking at the logistic one. Yeah, this should work fine, I think.

Well, we didn't look at-- so this trust, where does it impact things? One is on adoption fraction. So if trust goes down, you're bringing the adoption fraction down linearly with trust. So we have one nonlinearity for this reactivity parameter that then reflects in trust. Yeah, that's fine.

AUDIENCE: So I think if I were to make an edit, I would also have it affect time to forget and just blow that to infinity or something-- or very high.

HAZHIR Time to forget. Forget what?

RAHMANDAD:

AUDIENCE: So people become not willing to invest, but then eventually--

HAZHIR Oh yes, so you don't have this decide not to invest is impacted by trust. But becoming willing to invest is not. And

RAHMANDAD: therefore, they would continue to become interested.

AUDIENCE: Yeah, so I think that's--

AUDIENCE: Why do we even need that?

HAZHIR Why do I need what?

RAHMANDAD:

AUDIENCE: Forgetting. The people who get scammed are susceptible to get scammed again. And I think it was another thing that I added thinking about how could this go on.

HAZHIR So it is not just time to forget. So we need to be a little bit careful about what exactly is happening conceptually.

RAHMANDAD: So potential investors are people who are willing to invest in Ponzi and then not willing to invest are people who are not interested in Ponzi.

And let's for a minute forget about this deciding not to invest part, so just the inflow from not willing to invest to potential investors. That flow is capturing the idea that some people had no interest in Ponzi at the very beginning. And then, over time, they become interested and they're willing to consider it in some sense. But they don't know about it, correct? Because they haven't heard about it.

AUDIENCE: I assumed--

HAZHIR But because the potential investors--

RAHMANDAD:

AUDIENCE: --are coming out of investors as well, then they know about what happened.

HAZHIR Yeah, so that's why I'm saying this is kind of conceptually a little bit unclear what is happening here. So the not

RAHMANDAD: willing to invest right now includes people who know about Ponzi but are not willing to invest, and people who don't know about Ponzi and are not willing to invest just because they are not into financial investment, for example. So which of the two?

AUDIENCE: I think it's everybody knows about Ponzi.

HAZHIR So that's your assumption, that everybody knows about Ponzi. But then if everybody knows about Ponzi, why do

RAHMANDAD: you have Bass diffusion model?

AUDIENCE: No, so that everybody-- sorry, everybody would be willing to invest if they knew about it. I think my initial people not willing to invest is 0. So everybody who's in that not willing to invest stock is either a potential investor or somebody who would or somebody who has.

HAZHIR Oh, OK. So these are people knowing about Ponzi but not willing to invest. So at the beginning this is 0. That

RAHMANDAD: makes it internally consistent. So this is different from assignment for people not interested in VCRs because the price was too high. It is different. OK, good.

So in that case, then just making this time to forget dependent on trust and investment would, I think, take care of the people forgetting about this.

Good. So if we run this, now if I run this right now, because of this reinvesting going into investors, things may not work well.

AUDIENCE: I think it works if you do plus reinvesting minus [INAUDIBLE].

HAZHIR Or doing this this way. Ah, because we have the other model open. So we get the collapse, as we expected. And

RAHMANDAD: the reason is that the fraction of obligation met collapses. So that fraction is one. And how come that fraction goes to 50%? Because in the very long term, you just can't pay 50% of what he's getting. I mean--

AUDIENCE: He can pay back what he receives.

HAZHIR And then fraction deciding to reinvest in the long term is not zero. It's kind of 6% reinvest, even though they are

RAHMANDAD: not making any money. So kind of long-term things are not that meaningful here because, basically, once you have crashed, you have crashed in these. But it is still informative about how the underlying formulations are working. So in that sense, it is not a bad idea to look at those. I think pretty much everything is in the meaningful space and is working decently.

Now you have a relatively large total population that basically people come into the population, many of them just leave into not willing to invest. Some of them go in at the very long term, get burnt, and then that way, they come back. So that's not also unrealistic completely. And then Ponzi doesn't have much money all the time. It's kind of hand to mouth. OK, cool.

AUDIENCE: [INAUDIBLE]

HAZHIR Yes, Anne, go ahead. Somehow we are not hearing you, even though you are not mute.

RAHMANDAD:

AUDIENCE: Maybe your microphone.

AUDIENCE: Let me know.

HAZHIR Now it is good. Thank you.

RAHMANDAD:

AUDIENCE: So I used your reason for reinvesting would be [INAUDIBLE].

HAZHIR Yes. So the formulation that Cathy has here says people-- so basically, people who are reinvesting, they get out

RAHMANDAD: of the stock and instantaneously come back into the stock. So as such, the net effect on the stock of investors is 0 because you come out and then you come back in. So that's why it is not an actual flow in the mathematical sense, even though, conceptually, it is both an inflow and outflow.

AUDIENCE: Oh, I have another question.

HAZHIR Sure. No, no, that's good.

RAHMANDAD:

AUDIENCE: So I was thinking about the relationship between investors and cash. I think, in a lot of the models, when investors leave or there's one when investors leave, which is they're no longer contributing cash to the Ponzi scheme. But what about investors who vary the cash that they reinvest at different times depending on trust? So maybe early on were reinvesting all of the money that you made plus principal back into the scheme. But over time, you slowly reduced the amount of cash that you reinvested. How would we capture something like that?

HAZHIR Yes, so basically, in all of the formulations we have seen so far, although kind of Ali and Hasem have a little bit of
RAHMANDAD: a different version that kind of combines the two effects, but in most of the formulations we have an amount that each person-- average investment amount that's kind of a fixed parameter.

Now you could have a couple alternative versions of it. One is that the average investment amount can change over time. Now if we do that, and that's something also, Cathy, for you to keep in mind, if you do that, then your formulation starts to fail because then the amount that is indicated payment is not based on the current average investment amount, but based on the delayed version of it. Then you would need either the liability or some hack to account for how those average investment amounts are changing.

And then back to your question, Anne, there is also the reinvestment, what fraction of the money that you get back do you want to reinvest. Now that is less important in the sense that you already have a fraction deciding to reinvest. This is fraction of people. But for each person, you're assuming it is 0 or 1, that is either reinvest or reinvest.

Now, if you had this intermediate range of I am reinvesting only 50% of what I got, it is OK to think of them as half a person who reinvested fully and half a person who didn't reinvest at all, and you get the same result. So in that sense, having a continuous fraction, deciding to reinvest, takes care of that issue by and large. That is, it gives you enough flexibility on that front.

Where the difference is more salient is the average investment amount that may change over time as people build confidence or lose confidence. And it's more in the building confidence part. That is, at the beginning, people don't have confidence about these things, so they are willing to test it with the small amounts of money to see, maybe it works.

Like Bitcoin-- I mean, when it started, people would just put a few dollars to see what this thing is. It's a matter of curiosity. But then if, six months later, they see that they made a lot of money, they are willing to put more money and then more money and more money, and now becomes \$1 trillion market, even though it doesn't have any meaningful return. It's a different story.

In that sense, if you wanted to capture that, I think there is definitely a way to do that. You can basically use the thrust to drive the average investment amount, and it cannot go beyond whatever is people's typical saving or ability to invest. To be honest, in these investment schemes-- now Ponzi is a bit of an old-school, physical get cash, go to the bank, get slips of paper back.

But these days, you probably want to break the market into different sectors. So small investors are very different story than large investors, and there are these kind of mega whales of investors.

And probably you separate the model for those categories because their sources of information, their depth of knowledge about what you do or don't do is different. If somebody is going to spend or invest \$100 million with you, they'll definitely spend quite a bit of money figuring out whether you are a trustable. But that's beyond our problem definition for sure. Others? Anybody else wants us to look at their model? Everybody is shy?

AUDIENCE: A couple of questions.

HAZHIR Sure.

RAHMANDAD:

AUDIENCE: I was missing some big parts of the story, so I didn't have any notion of promises or liabilities. Instead, I had return rates, reported returns, so the investment manager returns money, and then the reports of those returns feed back to give you new investors. I guess--

HAZHIR OK, let's look at what you're doing here. So this is your model. So you have new investors. These are people who

RAHMANDAD: basically have invested, and their investment has not matured. After a while, they become matured. When you are matured, you want to leave. I mean, lifetime is killing them. It's a bit hard, but you stay--

AUDIENCE: For some time.

HAZHIR For some time. But what's the idea of this some time? Five years you're staying in this stock. So what's the point

RAHMANDAD: of being in this stock for five years?

AUDIENCE: Basically, at some point, you decide you want to get out.

HAZHIR No, but get out of what? This is a mature investor. If I understood you correctly, when we first mentioned it, you

RAHMANDAD: said these are people whose investment has already matured. That means they are not earning any interest. So why would I-- oh, you are.

AUDIENCE: Yeah, so it takes a minute before they can start. I was [INAUDIBLE] on the story. For the first year, they're just waiting, and then, at maturity, they start accruing.

HAZHIR Ah, so the new investors are the ones who haven't yet--

RAHMANDAD:

AUDIENCE: They're not accruing any interest.

HAZHIR Accrued interest. So how are you capturing that? So funds are going into this. This is money that investors put.

RAHMANDAD: Maturing-- OK, this goes to the next step. Then I'm accruing interest. Where does that get accounted for?

AUDIENCE: All that's happening is we're taking the total pot of money, [INAUDIBLE] new and old funds and spreading it over everyone in the old population. So I'm in the old population. I came in with some initial investment. And my returns are equal to whatever is currently in the total pot divided by whatever I invested.

HAZHIR Wait, that's not how your return is actually calculated.

RAHMANDAD:

AUDIENCE: [INAUDIBLE]

HAZHIR So what you are being paid is the average money that is associated with you. So you have an aging chain--

RAHMANDAD:

AUDIENCE: No, you're--

HAZHIR I'm just looking at the formulation for fund depletion. It is investor exit multiplied by average old member

RAHMANDAD: balance. And average old member balance--

AUDIENCE: It's not going into the right spot.

HAZHIR Yeah, so this part, the actual money being dispersed, is based on how much you invested and it accrues no

RAHMANDAD: interest. Then you have this other thingy here, which is reported old account balance, where you are calculating something different, which is total fund divided by mature investors. And that number will be larger than average old member balance. And then you are calling that reported balance. But you're not taking that out of the funds. So that's two errors-- 18 out of 20.

AUDIENCE: That's not bad. No, it's worse than that. The main question I have is about the Bass diffusion part. I know this is wrong. So I did one more into the VCR model, where becoming interested is driven by the return relative to the reference. And then it's a normal information diffusion. But mostly everyone else seems good. The investment probability depends on the return. So my question is this--

HAZHIR So this is actually an important difference. So this is the Bass diffusion model. Once you are in the potential

RAHMANDAD: investors, you will be investing basically. And that's kind of what you are formulating. So if, today, you have learned that Ponzi is a scam and is not trustable-- now you don't have that part very clearly accounted for here. But let's assume you had it-- still, these potential investors will be adopting with the same--

AUDIENCE: Unless they flow back into--

HAZHIR How do they flow back based on the indicated billing? So that may work. So max indicated willing to invest minus

RAHMANDAD: actual willing to invest versus minus potential investor divided by time to become interested. I mean, you adopted this from the homework assignment, and this is kind of a [INAUDIBLE] formulation that we put in that assignment to make it simple enough that we can give it to students.

But it's not the perfect one because, basically, it's saying the time to become interested is symmetrical. That is, the time to become interested and time to lose interest. Whereas in this case, if you are moving backwards, the chances are it is a much faster move backwards than the forward part. But putting that aside--

AUDIENCE: Just the choice of where to--

HAZHIR Yes, but putting that aside, you still will have people-- so today, everybody knows this is a scam, but you have

RAHMANDAD: 100,000 people in potential investors. You will continue to turn them into new investors even though none of them should. So that is not-- so you do need the impact on the actual investor transition. And the other parts, you already know some of the challenges.

AUDIENCE: It's not the right story.

HAZHIR OK, cool. Any other one? This is Cathy.

RAHMANDAD:

AUDIENCE: Mine is different in that it just captures-- in that it doesn't capture a lot of what you're talking about, but it does have the pull.

HAZHIR Yeah, let's take a look. Let's take a look. So this is Jake's. So we have the different categories of investors, so
RAHMANDAD: scheme susceptible group. This goes grows with population growth rate. They may decide to join--

AUDIENCE: That's just going to be [INAUDIBLE].

HAZHIR So you have a collapsed function. So as long as it has not collapsed, it is minimum of this kind of maximum rate
RAHMANDAD: and then desired people rate. So a couple of items here that comes to my mind just quickly-- so first, this expression here, even though it is kind of unit consistent and does the job for you, it is using time step inside formulations. And using time step inside formulations is not a good idea.

And the reason it is not a Good idea is that it is conceptually doesn't exist in the real world. There is nothing called time step. It is fully a mathematical artifact needed for numerical integration. So you don't want to use those concepts inside things that have a meaning inside the model. In this case, the meaning is that the scheme-susceptible population cannot be depleted instantaneously.

Or it can-- actually, with this one in [INAUDIBLE] it can, but you cannot exceed a maximum rate of depleting these people. So just have minimum time to deplete the scheme susceptible population as a parameter. Don't use time step. So that's standard.

And then the other one is the collapse thingy. I'm a little bit worried what is happening inside the formulation for it. Until we go into it, I don't know. Just the term collapse sounds a very switchy thing, and I'm wondering whether you wanted it to be a very kind of 0-1 switch, or is it something that you want to work for you continuously? If it is continuous--

AUDIENCE: It's 0.

HAZHIR And then these are people. So ultimately, this is actually more interesting. Part is what is the desired people rate.
RAHMANDAD: And the desired people rate is coming from desired money and average investor contribution. So basically, you have something about desired money, which is a complex formulation. So it's current investors divided by promised return time multiplied by average investor contribution multiplied by promised return factor. So basically, you're saying if I need to pay people \$1,000 based on their maturing rate, I need \$1,000 of new investment. So that's my desired money. Is that the idea?

AUDIENCE: Yeah. So Ponzi is going to recognize, I have to pay out twice, so I'm going to go get two people.

HAZHIR Yeah, so that's an interesting story that is basically Ponzi's proactively trying to go and recruit investors, which
RAHMANDAD: perhaps happens. Now, I don't know the story of Ponzi, how much that played a role because probably the word of mouth type of things were much more important driver of investment acquisition. But in Madoff's story, perhaps this was important as well, I don't know, or some of the other schemes that are more slow burning, the active engagement and recruitment of investors may be important.

AUDIENCE: My thought was maybe that even if there was a lot of demand to invest, Ponzi would recognize if everybody invests at once, that's going to be a problem for me.

HAZHIR Yeah, that's an interesting idea. So basically, you're saying Ponzi keeps the new investment at an optimal level of

RAHMANDAD: sorts that is close to what he needs to just pay back the maturing investors. So that way--

AUDIENCE: To prove that he can do it because he can go to anybody and call 50 people that they know and say this works.

HAZHIR That's interesting. So I like that. But the way I would capture that idea, if I wanted to incorporate that idea in the

RAHMANDAD: model, would be to have this act as a gate for getting new investors when they have been convinced of becoming a new investors. So I would have the Bass diffusion model to transform people from not interested to interested.

And then, from that interested into actually getting their money into the scheme, it is gated by Ponzi accepting their money, essentially, and Ponzi can, at that point, put an upper bound on that. He cannot control the lower bound if there is nobody willing to invest because now he has lost his reputation-- he goes bust. That's an interesting idea. I think it does make sense for the kind of designing a good scam. Good that is working, not good for the human utility functions.

Current investors-- and then they have the promised rate of return. They try to cash out, but sometimes, they don't succeed. And you have the time step going here to limit the money out as well. So that's a little bit of kludgy as well. So you want that. And then the ratio of--

AUDIENCE: I think, originally, they were all promised return time rather than time stuff, but at some point, I switched one and then I got it.

HAZHIR And then if you are not paying what people are asking for, this is what will happen here? Anything happens in

RAHMANDAD: your model?

AUDIENCE: So the second that you have-- the switch for collapse variable is going to be the one that you want to look at, which is above-- switch for collapse, yeah. Collapse and switch collapse are the exact same thing. Earlier, I iteratively built it [INAUDIBLE] there. So the second bit--

HAZHIR So basically, you either don't have the money coverage that is 1 or you have some unpaid population, you kind of

RAHMANDAD: kill everything in the operation. You have the collapse, which in this case means nobody is accepting investor equity. I mean, Ponzi is trying, but nobody is paying him.

AUDIENCE: So the second you don't pay one person, suddenly, it's over.

HAZHIR OK. So I think this is kind of working for you because it grows exponentially as long as Ponzi is able to bring

RAHMANDAD: people in, but then, at some point, it collapses. So it's an interesting story here. So let's look at what is happening here. So there is the exponential growth phase. Then there is a kind of a slowdown phase. So what is driving slowdown?

AUDIENCE: The slowdown phase starts when the game-susceptible population just is equal to the-- it becomes equal to the growth in population rather than having some excess.

HAZHIR Yes. So for a while, population growth becomes the bounding factor.

RAHMANDAD:

AUDIENCE: And then you run out of cash that you would saved up.

HAZHIR I see, so you had saved up. But how did you save up? Because you were recruiting just enough to pay back these

RAHMANDAD: people. How is it that you save money?

AUDIENCE: I think it's because your reinvestors who are convinced, oh, this-- basically everyone who invests is going to reinvest in this case, so you always have their cash in the system apart from the time that it takes them to actually--

HAZHIR Yeah, so you have the reinvestment process also here. And that allows you to accumulate money. So you're

RAHMANDAD: bringing in new investors even though you actually don't necessarily need all of those new investors at the beginning of the time.

AUDIENCE: Correct.

HAZHIR Got it. It's interesting. It has kind of these challenges we talked about, but it has an interesting feature that other

RAHMANDAD: models didn't have, and that is kind of the pool from Ponzi side for getting money or kind of putting a gate on how much money he is trying to get. Cool. So I think this was pretty good. Across the models, there were several that had most of the components in a good shape.

I think we could go through more, but I want to get to the other parts of the conversation because next week and the week after, we don't have class. It's Sloan innovation period, and then it is spring break at MIT. So we would be meeting up April 4, something like that. So that's a bit of a time. So for that longer time, we have the next session-- for next session, we have the market growth model.

Now this is basically adopted from Jay Forrester's famous market growth paper. Don't look at that model so that you build it from scratch on your own. And it is not replicating exactly what the market growth model does. It has a couple other additional pieces to it. This is also like Ponzi, a little bit kind of broader boundary. You might formulate things a bit differently.

So it's about a company that is selling a durable product in a limited market, but a large market. It is in a monopolistic setting, so you don't need to worry about competition. And you're kind of looking at the market growth from inception all the way to maturity. And this is a product that is complex and life-long kind of durable products. So they sell it through direct sales force. This is not a Bass diffusion type of a word of mouth. People hear about it and come and order a million dollar item. It's kind of a much more structured go and sell it type.

And the firm's heuristic is to allocate a fraction of its revenue to Salesforce for selling things. And also, if they can't-- if they don't have product, they can't sell. Basically, their delivery delay increases. They make it to order. That is, there is a backlog. When they get orders, they will make the products. It's not that they have an inventory of the product. And if they can't deliver on time, they promise a delivery delay. If they can't deliver on time, customers become less interested to buy from them. So the success rate of their sales force in selling the product goes down if they are not delivering on time.

Their manufacturing operations have-- it takes time to build the facilities. And the signal they look at to decide to expand manufacturing capacity is the delivery delay. So they use that to expand their manufacturing facilities, and those facilities stay around for a while.

But they also depreciate, ultimately. You want to build a model of the dynamics in this setting, kind of the historical case that you are trying to explain is this is a very good market in the sense that these guys have a monopolistic setup, they have the market, and they have a good product in general, but they have seen kind of ups and downs in their growth. They have been growing, but have been seeing ups and downs in it. So that's the reference mode of sorts.

But you also want to understand what is driving it and what are the alternative policies that may help them avoid the ups and downs. So that's the big picture of the case.

Now, this is, again, a model that I'm explaining the components of it-- in a way, I'm telling you the mechanisms to model. You didn't need to go and do the research to decide what are the mechanisms. So as such, it is a simplified modeling task compared to most real-world problems, but it has more moving components than some of the previous work.

So we could think about what are the questions that come to your mind, thinking about this formulation right now or this problem definition right now. What are some of the things you say, OK, I don't know how to think about it or how to model it. We could have maybe five or seven minutes of conversation around questions, and then we can switch to your problem definitions for your personal problems, which some of you have already shared.

AUDIENCE: Maybe the depreciation of factories [INAUDIBLE].

HAZHIR Yeah. So it's a very important part of the model to build the production facilities as well. And there is kind of two
RAHMANDAD: or three pieces to it to do it well. One is it takes time to build things. So you have to separate work in progress factories that are kind of under construction versus factories that are constructed.

Then there is the depreciation of factories that are already constructed. That depreciation, perhaps a simple delay function works, but a simple delay function with higher order delays. If you have a first order delay, it's kind of weird because production facilities don't start depreciating the day they are built, so accounting for that would be important.

Then going back to your question about I have a factory. Does it mean it is producing at its full capacity or not? The answer is it doesn't need to be producing at full capacity. If you don't have demand, it shouldn't be because then you would be in a different business model with inventory. And even there, you wouldn't produce when there is no customer. So accounting for that capacity utilization-- so there is a capacity utilization feedback that you need to keep in mind.

AUDIENCE: [INAUDIBLE] if we're running at full capacity [INAUDIBLE].

HAZHIR That's an interesting observation. Don't worry about that. In practice, it is not a very clear-cut case. So for
RAHMANDAD: simpler equipment, whether it is running or not does make a difference in the depreciation time, but not running something actually can lead to depreciation as well. If you leave your car in the garage, it actually depreciates as fast as if not faster. A lot of the parts may rot in place and rust in place. So that's the argument.

AUDIENCE: Same thing with planes.

HAZHIR Good question. Other thoughts, questions in how you think about this model? You have a little bit more time, but
RAHMANDAD: you also have your personal projects to define.

AUDIENCE: [? Oscillation? ?]

HAZHIR You don't have to expect that, but you would try to see whether you can get oscillations from it. And what are the
RAHMANDAD: conditions under which oscillations happen? And on the bonus side, this one kind of in the original model and the version that is your baseline, the fraction of revenue that goes to the sales force is a fixed number.

| can endogenize that based on marginal productivity of the sales force or how useful is to have sales force at any point in time. So that adds a separate feedback mechanism that is interesting and may change some of the dynamics. I actually haven't built that version myself, so I'm curious to see how you do it. I mean, if you had time, that's a good expansion.

AUDIENCE: Endogenize [INAUDIBLE]?

HAZHIR Yeah, endogenous. Endogenize is something about your sales force size or budget or-- sales force size is
RAHMANDAD: endogenous. Sales force budget as a fraction of total revenue is not. Good. Go ahead.

AUDIENCE: So when you manufacturing operation with the backlog, that means that they only produce what customers order.

HAZHIR Exactly. So when customers order, it goes into a backlog. That drives desired delivery. That desired delivery is
RAHMANDAD: fulfilled based on production but is bounded by your production capacity. And if you have plenty of capacity, you will produce all that is desired, and you will have excess capacity that is not used. If you are bounded by capacity, you are delivering as much as you can, but you are not fulfilling the customer's demand. So that's the balancing act.

AUDIENCE: [INAUDIBLE] don't have any measure of quality of the product?

HAZHIR Yeah, so right now, we are assuming the quality of product is either exogenous or fixed or whatever. It's not part
RAHMANDAD: of the--

AUDIENCE: [INAUDIBLE]

HAZHIR Yes, for now, assume price is also fixed.

RAHMANDAD:

AUDIENCE: --impacting the customers is this [INAUDIBLE].

HAZHIR Only the delivery delay that is changing the customer's utility in this case.

RAHMANDAD:

AUDIENCE: [INAUDIBLE]

HAZHIR Delivery delay as a proxy for price?

RAHMANDAD:

AUDIENCE: Because it decreases demand.

HAZHIR Yeah, so it is a feedback. It is not price feedback because price has its own impact on your budget. So price is
RAHMANDAD: completely ignorable. You need it in the machinery of your machine, your model. And you need a very simple accounting sector to think about, OK, costs and benefits and net gains and so on.

AUDIENCE: So delivery delay signals the firm to expand their production capacity.

HAZHIR That's correct.

RAHMANDAD:

AUDIENCE: That means there's always interested customers.

HAZHIR Yes, so this is-- now the size of the market is limited. So it's not an infinite size market. And it is a durable good.

RAHMANDAD: So once you buy it, you don't need it for a while. So the maximum demand is constrained. But that maximum demand is materializing. What fraction of it you can materialize depends on your sales force partly. So your sales force is going and getting these people.

AUDIENCE: But does the demand change because it's monopolistic, so there's no competition?

HAZHIR So it does change because of the durable nature of the product. So that's the constraint. There is also, of course,

RAHMANDAD: your sales force effectiveness. So if your sales force-- you don't have anybody to sell, you wouldn't sell anything.

AUDIENCE: So the delivery date delay doesn't at all impact the customers [INAUDIBLE].

HAZHIR They do. It does. It changes the effectiveness of your sales force. So that's how it affects it. The writeup has these

RAHMANDAD: details in it. So this is a shorter version than what is in the writeup.

For the last 10-15 minutes, I wanted to go back to the personal projects, and basically, my plan is to read through what you have submitted or will be submitting in the next day or two so that I can give you some feedback and you can adjust it, rework it, get to the level that you feel comfortable, you know how to move forward.

Now, so far, as you have seen, we have defined the problems. If you remember in the memo note that I had and also briefly in the first session we discussed, there are different ways to define a problem. You may start from a reference mode that you are trying to explain and understand. You may start from a set of mechanisms that you are trying to model. You may start from policy levers that you want to then find. What are the mechanisms, what that need to be captured to measure the reference [INAUDIBLE].

In this class, so far, in all of our assignments, we have gone with the second approach of giving you the mechanisms and asking you to build models of those mechanisms. In some ways, this is the simpler way of doing things. That is, basically, I have already removed a lot of things that you could consider. I've kind of given you, these are the mechanisms you want to have. Ignore everything else so that we can have more focused conversation about formulations and so on.

It may not be easy for you to get to this type of a problem definition because whatever you're interested in, you don't know what is it that you need to focus on. So as you think about it, first of all, do your best in defining what is important or interesting to model in terms of mechanisms to capture the problem that might be more motivated by a reference mode or by some policy questions by some pain point. And then I'll try to help you with that process.

So whatever you have written down or are writing down for this first draft, I'll look at it and try to turn it into something that is a bit more closer to mechanisms so that you don't need to go through the process of clarifying a problem and the feedback mechanisms that will need to be in the model versus those that are not to be in the model.

But that is not an easy task. I'm just highlighting it because that's, in some ways, the more vaguely defined, harder part of modeling, going from a broad problem definition into what are the mechanisms that need to go into a simulation model. So I want to be helpful on that.

But maybe for the few minutes that we have, if any of you want to say, OK, I felt confused because I was thinking about it and I had no clue how to do it, or this is the problem space, but I don't know what to focus on, we can have a little bit of a conversation on a couple of cases, I think, with this limited time we have. That's what I wanted to talk about before we close.

AUDIENCE: So just to clarify, talking about going from problem of interest to mechanism?

HAZHIR Yes, exactly. I mean, whatever you had-- I haven't read through the ones that were submitted, so I don't know
RAHMANDAD: what is in there, but if anybody wants to bring it up as a case because they felt confused or didn't know, I'm happy to have that conversation. Go for it.

AUDIENCE: What I was thinking of doing was talking about how does the fashion resale industry affect brand recognition for luxury brands. So it kind of reminded me of the car case for GM. It was the used cars, but the difference is there's no leasing program in luxury. So my idea was you have luxury brands that want to gain, to some extent, brand recognition within their clientele, but for luxury brands, they don't want the wrong clientele using it because it ruins their brand image.

So there is a level of, well, they want the-- you could argue, like in Supreme's case, the resale market really helped build up like the clout and how attractive it was. But then you can also see high luxury brands like Hermes having a lot of people wear Birkins, especially people who aren't like their ideal clientele is tanking their luxury status. And so then I was trying to figure out where does that line start between resellers and not resellers because there's a lot of different types of resellers. That's kind of my--

HAZHIR So this is a very interesting problem. I think it's a good problem in the sense that it is dynamic. It kind of has
RAHMANDAD: important trade-offs. It has stocks and flows that you can imagine. So good space.

I don't know enough about the topic, but from just that 30-second introduction that you gave, it feels to me you may need to have two categories of customers based on their willingness to pay something that might be the one piece of the story. You may care about your product's life in the sense of some quality versus how long it will last trade-off, and something about price that you assign.

So for a product, you decide on the price, and you decide on that quality measure. And you might bring down the price and bring down the quality. You may get more of the low willingness to pay customer sector segment, which is much bigger customer segment. And you may still have a good margin on that, but then that may erode your attractiveness for the high brand group.

So think about a model where you have adopter, nonadopter for two groups of customers, and then you have something about their attractiveness for each of these groups that comes from your price, your quality, and your exclusivity, or how much the other group is using it. So for the low willingness to pay customers, having high willingness to pay customers becomes a factor of attractiveness, but the reverse is actually negative. So that becomes kind of the heart of your model. I would say that works well. And don't go anywhere beyond that. That's already pretty--

AUDIENCE: I guess my question for when you're optimizing where should this brand position themselves is it's not profitability because, obviously, a Macy's is much more profitable than a Gucci. But they choose to stay in that space because they value something about it, whether it's the status or the artistic value that they perceive that they're gaining from being there. So I don't--

HAZHIR That's an interesting observation. So don't worry about optimizing in that sense, but worry about whether they
RAHMANDAD: can thrive in their desired market segment. That is, if they are targeting the high customers, can they thrive there and still have a foothold in the other market, under what conditions? So keep it to that level of I'm interested in this question rather than optimizing for the whole market. I think that's a good problem. It would work out well. Good. Anybody else?

AUDIENCE: I was thinking about there's a contrast-- basically, book sales are up but reading rates are down, and my hypothesis for why that is that the reason for buying a book is less so for the human capital of consuming it but more so for the signaling and cultural and social benefits that you get from having it and carrying it around and having it on your shelf when people come over, et cetera.

And yeah, so I think it's more so like a simple paradox explanation with some reinforcing dynamics around social value of book makes it so that reading the book doesn't matter, makes it harder and less important to you to read books. And so that's how we get these sales increasing, reading--

HAZHIR So just for me to understand the sales of physical books are up or sales of books, audio and physical together
RAHMANDAD: are, but not physical--

AUDIENCE: All formats sales are up.

HAZHIR So each format alone is also growing. Physical books also have been growing.
RAHMANDAD:

AUDIENCE: Physical books are growing. Audio-- yeah.

HAZHIR OK, so that's number one. Then you're saying reading of it is actually declining. Is it reading across all--
RAHMANDAD:

AUDIENCE: So the survey question is, in the last 12-- or two survey questions. In the last 12 months, have you read a book not for work or school? And then second question is, how many books have you read in the last 12 months not for work or school? And both of those, it's like 50% of people now haven't read a book in the last year. And the average number of books has gone from 20-ish to 8.

HAZHIR Interesting. But when you say, have you read a book, people report their audiobook consumption as reading a
RAHMANDAD: book? Is that how the survey is--

AUDIENCE: I think the question hasn't been consistent over the 30 years, but I think in the recent surveys they say audiobooks count, and they should report audiobooks [INAUDIBLE].

HAZHIR Got it. Got it.
RAHMANDAD:

AUDIENCE: So I don't think this is a book format story.

HAZHIR Yes.

RAHMANDAD:

AUDIENCE: I think this is a book like a BookTok, which is like the TikTok book phenomenon that really blew up across the pandemic story.

HAZHIR So the story you are telling as this is a story of social boasting that I have some books or I read books or

RAHMANDAD: whatever, that is interesting. It's probably not that complex. There is a feedback loop of people see books as a status symbol, and that may get reified as more people use it with that idea. And that drives demand. But then what is bringing down the actual consumption of books--

AUDIENCE: [INAUDIBLE]

HAZHIR --is a different story. That is, basically, it becomes a little bit exogenous-y. What I mean by exogenous is that

RAHMANDAD: there is a mechanism that is driving one reference mode, and that is books are becoming status symbols in some circles. And then there is another mechanism that is bringing down books consumption in your story.

And that might be people are busy with other things, or there is so much of time going to social media or they are not literate enough. I don't know what, but you come up with some hypothesis, and then you can code it into your model and get it to work. But because they are kind of detached from each other, these two parts of the puzzle--

AUDIENCE: I guess I was thinking of connecting them because you're spending time talking about it, and you're spending time on social media kind of hunting what's the next thing I need to get to signal my continuing reader status. And you're spending time-- one is spending time doing that instead of actually consuming the thing that they bought.

AUDIENCE: I was wondering, is the reading rates go down, is that among everyone? I guess my question would be, among people who are actually in BookTok, if you only surveyed them, their reading rates might actually be up. So that might not be related. It might just be like, in general, people would rather watch YouTube or scroll Instagram, so reading rates are down. But among BookTok people, yes, it's a social status symbol, but they also are probably also reading books.

AUDIENCE: Yeah, no, that's a fair point. The reading rate surveys are US adults, and then they break it just by the standard things that you break down by, but it's not surveying people who are in these communities. So totally that could be a breaker here.

HAZHIR So then, I mean, it could be that Oriana's kind of hypothesis is more of a now you have these concentrated

RAHMANDAD: groups that consume a lot of books, and then you have the general population that is not consuming much. And it's kind of a question of heterogeneity more than a question of the two mechanisms you're highlighting.

Now that is actually an interesting-- that might be even more interesting for a modeling practice type of situation. That is, how do you capture this type of heterogeneity? That is, it is happening, perhaps across other product categories, that as people become more focused in their narrow niche areas, you may get more consumption of something among some groups but less among others.

So trying to actually capture that heterogeneity in a stock and flow structure with minimum complexity is pretty interesting. It becomes, again, kind of exogenously driven. It might not be feedback-rich, but see what you can tell about the feedback mechanisms that can be important. And then capturing just the stock and flow part of it that generates the heterogeneity is also interesting. So that would be my suggestion.

I think we are running out of time. So with that, if you haven't submitted your problem statement, please do. I will try to work on it on the feedback next week and get it back to you so you have a little bit more time to go back to it, edit it, and these problem definitions that I mentioned over the email, they are for you to be a little bit more connected with the problem but also have create more of diverse models that we can look at in the last couple of sessions.

If you are OK with it, we will share those for others to also make it a concrete problem that others can tackle it as a modeling challenge if they get a chance. So those are the purposes, but you don't have to share it broadly if you don't want to. OK, cool. Thank you.