

[SQUEAKING] [RUSTLING] [CLICKING]

HAZHIR Good to see you all. We have the predator prey model today. And actually, a lot of you submitted models. So
RAHMANDAD: that's great. We can jump in. Actually, so predator prey model, very, very classic-- of course, we had a little bit of an extended version to think about carrying capacity for the prey-- think of a deer and wolf model with vegetation. And we wanted to focus on a simple but robust model and, to the extent that you can, think about model analysis as well.

So that was the problem definition. And let me turn off the light on the presentation. That probably helps with the recording. So maybe we can get started with your reflections on things that were challenging. That's usually helpful for focusing the conversation on what might be more useful feedback as we go through the models because we have several models. And we may not have time to cover everything. So whoever likes to share, go ahead and share what were some of your major challenges as you were going through the model.

STUDENT: [INAUDIBLE]

HAZHIR So can you tell me more what was challenging with it?

RAHMANDAD:

STUDENT: Yeah. So because there are a lot of behaviors that [INAUDIBLE]

HAZHIR That's good. So what are the behaviors you were trying to get?

RAHMANDAD:

STUDENT: I wanted [INAUDIBLE] actually be a little bit lower when deer were less dense and higher when they're more dense. And they're needs to be a maximum and minimum consumption level that wolves can actually live on that has to do with that. And honestly, the difficulty [INAUDIBLE]

HAZHIR Good. I think this might have been one of the more common challenges, not only for predation, but also for the
RAHMANDAD: other side-- that is, consumption of vegetation has a challenge to it. Good. Other challenges that anybody faced in modeling?

STUDENT: The biggest challenge was I was trying to conceptualize the model based on [INAUDIBLE]

HAZHIR Yes.

RAHMANDAD:

STUDENT: But then I have this memory [INAUDIBLE] which it didn't go well.

HAZHIR OK, very interesting. So tell us more. So it's interference from the memory of classical models.

RAHMANDAD:

STUDENT: [INAUDIBLE] And I'm sorry to say that I did the whole model with a new conceptualization, a more realistic conceptualization. But then I went back to the mathematical model [INAUDIBLE] what I remembered.

HAZHIR Yes, yes. Yeah?

RAHMANDAD:

STUDENT: So the other challenge was operationalization, extreme conditions. If we don't have any deer, we shouldn't have any wolves. And that was also one of the [INAUDIBLE] wolves did not grow on their own.

HAZHIR
RAHMANDAD: Mm-hmm. Yeah, good, good. Other challenges? Jason?

STUDENT: [INAUDIBLE] I found it hard to get away from the very basic story of thinking in terms of an agent-based model of deer and wolves interacting. And at the end, what I settled on was something that [INAUDIBLE] you don't need to model those interactions directly.

HAZHIR
RAHMANDAD: So how does that thinking about those interactions make it more complex for you?

STUDENT: It distracted from what is the important features [INAUDIBLE] At the end, the rates that we wanted to compare and put together don't require these more microscopic interactions to think about.

HAZHIR
RAHMANDAD: OK. So you were starting with microscopic interaction between deer and wolves. And you couldn't get to any good solution for the more aggregate problem. Could you get to a good solution for the individual-based model? If you were building an individual-based model, do you think it would have been easier? That's interesting. We can come back to that. I'm not sure. I think, actually, the individual-based model in this case is more complex than the aggregate one. But we can see. Any other reactions?

STUDENT: Well, I just forgot the time step size was important.

HAZHIR
RAHMANDAD: So how does the time step play a role?

STUDENT: I just couldn't get it to calibrate when I set it equal to 1. And I needed to set it to 0.25.

HAZHIR
RAHMANDAD: OK, good point. So that's a very general rule, lesson that time step is actually not a construct that has a conceptual meaning in the real world for us. Real world is a continuous world. So if the behavior of your model is sensitive to time step-- that is, you change it and the behavior changes-- it means that you have too big a time step. Just bring it down further, divide it by 2, and divide it by 2 until there is no change in the behavior. That's a very simple heuristic that usually works. Oh, Anne, go ahead. And somehow, we are not seeing you. I don't know what's the problem here.

STUDENT: Are you sharing the screen? I don't think you're sharing the screen.

HAZHIR
RAHMANDAD: Ah, that might also be a problem. OK, so let's continue. Basically, we were discussing common challenges. Anything else that anybody wants to share on what was challenging in this week's modeling work for them? And then we dive deeper.

STUDENT: I'm just debating whether [INAUDIBLE].

HAZHIR So that's a generally an important question, whether you go with a single flow or separate birth and death rate.

RAHMANDAD: And in fact, separate birth and death rate from natural causes versus natural death rate from predation. So that's another good question. OK, let's look at some of the models that you have built and see how you have handled some of these complexities. So that would require opening a bunch of models. I'll just open the version B for those of you who have submitted two because we can look at them.

OK, so this is a good enough number to get us started with. So this one is for Sahin. So let's take a look. So stocks, we have the prey population, predator population, and vegetation. So I think everybody is pretty much on the same page about that. Some of you have broken it down by two age groups. The majority have not, from a quick look.

So let's look at one of them. And we can see how the ideas are working for the rest. So this is the vegetation side. And there is a regeneration rate for vegetation. The regeneration rate, I think, is fairly common and fairly standard to think of it as a fractional regeneration rate, as a first cut. So who has used a similar formulation? Basically, the stock multiplied by a fractional growth rate. Anybody has a formulation that is different and wants to share that or think about that? Let's look at your model then.

STUDENT: I mean, it's not that much more complicated. It basically just creates the idea [INAUDIBLE]

HAZHIR Very good. So that's actually an important consideration, if we want to-- so let's see. I think I had your model

RAHMANDAD: somewhere. It's just hard to know which model we are looking at.

STUDENT: [INAUDIBLE]

HAZHIR Yeah.

RAHMANDAD:

STUDENT: [INAUDIBLE]

HAZHIR So here, you have max vegetation level impacting regrowth rates. Let's take a look at this. So max vegetation

RAHMANDAD: minus vegetation divided by time step here. So you are basically-- this is actually a very different formulation. It's not problematic. So there are two different logics that you can think about when you're thinking about regeneration of vegetation.

One is that you need seeds that come from that vegetation to regenerate. The other is that, as vegetation gets denser, it competes with itself for food, for water, for sunlight. And that brings down its regrowth rate. And therefore, it cannot go above some level. So vegetation alone model without deer and nothing should not grow exponentially forever.

So this is what Jake's model accounts for more strongly. So this model says, there is a total vegetation level that is feasible to have in the area. Now, here, we don't have the area explicitly. You could have a max density multiplied by the area to give you the max vegetation level. And then you look at that versus the current level of vegetation. And you would say, vegetation is trying to grow back to that level. So this formulation does that alternative conceptualization.

I would say this is more robust for the extreme condition that we discussed-- that is, the exponential growth in vegetation forever is not going to work. It does have a downside. And that is, if you are running out of vegetation, the regeneration rate-- and therefore, the growth rate-- is actually going to collapse.

And that is a challenge in many of the actual ecosystems. When you have too much of the prey, basically, deer in this case, eating the vegetation to the bone, then they don't regenerate. And that becomes a bottleneck. Whereas, in this model, the increase in the vegetation will be always very robust when you don't have much vegetation. So this is actually a non-trivial formulation to get both of these factors play a role in your overall rate of vegetation.

STUDENT: Do you have any preference between adding max vegetation like this versus just having two variables that [INAUDIBLE] what you calculate your regeneration [INAUDIBLE]

HAZHIR
RAHMANDAD: So I'm not quite following the two options. So one is max regeneration rate or max total vegetation--

STUDENT: No, either you take it into account like this--

HAZHIR
RAHMANDAD: Yes.

STUDENT: [INAUDIBLE] there's one maximum amount of-- maximum amount of--

HAZHIR
RAHMANDAD: Shipment in that case.

STUDENT: Or like paperwork and processing just because of how many [INAUDIBLE]

HAZHIR
RAHMANDAD: So that formulation plays a more important role in the outflows. So we will get to the outflows. I think, for the inflow here, we need a formulation that actually brings both of those ideas together.

STUDENT: So we have been trying this. And I think it has a little bit of downfall because, if the max vegetation is very high-- imagine it's like here. And you have two deer only. And suddenly, the inflow at the very first beginning is very large and they grow very rapidly, whereas, if we like multiply it by the fractional growth--

HAZHIR
RAHMANDAD: Wait, wait. This is not for deer. This is for vegetation.

STUDENT: I was just giving a sensible example. But in general, in any species, if the max level is very high, so suddenly the inflow is very large compared to the exponential growth that it controls [INAUDIBLE]

HAZHIR
RAHMANDAD: Yes. And it also partly reflects whether you are using a single flow or double flows. That is, in a two-flow model, you could actually have this regeneration rate being exponential, being a direct multiplier of the stock. But then you could have an outflow that grows significantly as you have more density. And at some point, it saturates. It doesn't allow it to go up.

So that would be one way to capture the non-linearity rather than putting everything into the inflow. Now, there is some physics behind it either way that you need to capture. So one way of thinking about the physics of it is that, OK, the seeds are spread, but they don't grow. They don't even start to grow because they don't have the sunlight and the water that they need.

And so as such, the inflow is disrupted. So I need to capture it in the inflow. The other conceptualization is saying, well, the seeds are spread. They start to grow, but they die out very soon. So the outflow is much higher. You could go with either of those and still have a story to tell. Alexander?

STUDENT: So trying to capture both, I used the logistic function in the inflow. So it has the property where, if there's little then you'll have that reinforcing loop. But then it doesn't go above.

HAZHIR Yeah, that's generally a good idea. But let's actually check out your model. I don't remember.

RAHMANDAD:

STUDENT: I didn't include vegetation. I just did part A. But I did it for deer.

HAZHIR OK, so we can take a look at the model you have for deer and see how that is playing a role in the formulation.

RAHMANDAD:

STUDENT: So I called it the natural change in prey population. So that's supposed to capture-- it's not only births, but it's also supposed to be deaths.

HAZHIR So this is really a single flow version, net flow version. Now, net flow, this model is simple, but it is not trivial. And

RAHMANDAD: you will see that many of you had a challenge getting the inflows and outflows robust. So when you combine inflow and outflow, it is very risky. It becomes even harder because you need to keep in mind what is happening both for the inflow and outflow.

So this is prey multiplied by their natural birth rate. This is the part that is the positive growth rate. And then here, you have this whole thing multiplied by a factor. This factor is net positive as long as prey is below its carrying capacity. And once prey exceeds its carrying capacity, that ratio becomes above 1. 1 minus that ratio becomes negative.

Now, you have a net loss rate. So this is how Alexander has formulated it. And it works well. I think this does OK in creating a population that grows exponentially, gets to a saturation point, and stays there for a single species without influence from anything else. So this is a meaningful, decent formulation if you are doing a single flow version of it.

My worry with this formulation is that the mechanics of birth and death are missing. So you don't as easily tell a story. It is more a mathematical function that works OK and gives you what you want and is perhaps equivalent to another model. That is, if you had the model with natural birth rate multiplied by prey as inflow, this model would then have an outflow, which is prey to the power of 2 divided by carrying capacity multiplied by natural birth rate.

Now, that is not a very intuitive outflow because where does the prey to the power of 2 come from? It comes from a meaningful idea that too much density leads to more killing of the prey. So as such, you can tell a story. And mathematically, it works very nicely because then it saturates at carrying capacity.

STUDENT: I spent like 45 minutes on this yesterday. If you did the math if I do the math, basically what it is it's kind of odd, but basically the outflow is essentially the prey population divided by some reference lifespan-- so what the lifespan of prey is at the carrying capacity-- multiplied by-- I think it's the ratio of capacity to-- yeah, multiplied by the ratio of capacity to prey.

HAZHIR
RAHMANDAD: The prey should be in the numerator, not the denominator.

STUDENT: As in like prey's in the numerator, and then the denominator, the lifespan. It's a first order draining process.

HAZHIR
RAHMANDAD: Yes, yes. It will be a first order. So basically, the natural birth rate of prey will be 1 over the lifespan in that version, in the outflow conceptualization. You're right. OK. So this is one way to do it. And it would work OK in this case. And I just forget. Any other formulations for vegetation growth rate?

STUDENT: [INAUDIBLE] I have one mistake. But I only did it for the inflow. So I realized that's wrong [INAUDIBLE]

HAZHIR
RAHMANDAD: OK, so let's take a look at your model to see what we had. So this is the carrying capacity. And your vegetation growth has a max vegetation and vegetation growth rate.

STUDENT: It shouldn't be growth rate times max vegetation, but just max vegetation.

HAZHIR
RAHMANDAD: Yes. So if you remove this max vegetation multiplication, because that one doesn't actually work, then you have vegetation growth rate-- so this has units problems as well or maybe not. So if the growth rate is one per year and you have max vegetation, then the units will be fine because the second part is dimensionless.

STUDENT: Yeah, that should be just the vegetation.

HAZHIR
RAHMANDAD: So it should be the vegetation, if you want to-- yeah. And if you make it vegetation, then it becomes very similar to the formulation that Alexander had.

STUDENT: But then it's wrong still. I mean, based on what you're saying, it's still wrong because then it can go negative.

HAZHIR
RAHMANDAD: So this can go negative. So if you don't have-- I mean, I don't know where-- oh, this is the stock, your carrying capacity. So it is not going to go negative, if you don't have any other inflows. So if this is the only inflow to the stock, the stock will grow until it reaches the max vegetation.

But as soon as it reaches max vegetation, this will shut off. It becomes 0. And therefore, it cannot go above that max vegetation. In fact, even if it goes above max vegetation, this becomes an outflow for the vegetation. It kills vegetation and, as such, brings it back to the max. So it is not problematic, in the sense of going negative, if you think of it as a two-way flow, not as a one-way flow.

STUDENT: Yeah, that's what I meant, that it can only work if it's a net flow [INAUDIBLE]

HAZHIR
RAHMANDAD: Yes. But as an inflow, it is still doing an OK job because here you have the consumption of vegetation by deer. You don't have the natural death of vegetation as the grass gets dry and dies after a while. So that part is actually already built into here. So it is a two-way flow in your model as well.

OK. So you got an idea. So basically, yes, exponential growth type with only vegetation multiplied by a natural growth rate is a possibility. But the possibility has the downside of becoming too strong. So you do want to think about the impact of carrying capacity of the land for vegetation. One way to do it is similar to what Alexander did, which made me think of it as a two-way flow.

Another is to have, basically, a consumption or a natural death rate of vegetation that will grow as vegetation becomes denser. You need to have one of those two. And because we have the same story with the deer, we can go through the formulations in the deer population and, in fact, with the wolves, that will give more info on how you have been thinking about it.

So let's go back to-- oh, we can go to any of the models for-- let's just pick another one. So this one this is Alisa. OK, so Ali has just simple names-- n_0 , n_1 , n_2 -- which I suspect this is-- this is the wolf, predator. And then you have the deer as n_1 and n_2 being the grass.

So here, you have the death rate that is coming from the-- the birth rate that is coming from just regeneration. And then you have the death rate that is-- let's look at this equation. So I'm a little bit-- so just looking at this graph, I can't follow what the logic of that equation is because there are four, five different-- six arrows going into that equation. So the logic will be complex. You are combining six different things into a single flow. Not a good idea.

Break down the logic. It's clearer for anybody to look at it, but also will be clearer for you because you can then open up each of those pathways and see how it is impacting your overall flow variable. So that's just a simple process thing. Don't drop in 100 things together. Now, we have n_2 -- that is the grass-- multiplied by its natural fractional death rate plus n_2 divided by normal n . What is normal n ?

STUDENT: It's a unit correction.

HAZHIR OK. So unit corrections are risky things to use. That is, if you're just adding something to fix your units, it means
RAHMANDAD: you're in trouble. Your logic is not working well. Almost always, there is a way to fix the units based on the mechanisms rather than adding just ad hoc parameters. So think of those as fault factors you shouldn't use.

There are situations-- like, if you're raising something to a power in Vensim, it gives you an error no matter what, if you have a unit for that element, and then you want to make it dimensionless to not get it. So there are very rare cases you need fudge factors, but don't use them. But this is ultimately some-- maybe even a value of 1. But you have the grass multiplied by grass carrying capacity. And then this thing is multiplied by grass itself, ultimately. So I'm not following that logic. Can you tell what it is?

STUDENT: The first thing that you pointed out is natural death rate. Second one is kind of like a carrying capacity if they reach a limit. So it slows--

HAZHIR It slows it further?

RAHMANDAD:

STUDENT: Yes, it slows it further, and then kills grass.

HAZHIR But why is it multiplied by carrying capacity n_2 ?

RAHMANDAD:

STUDENT: I was thinking of a factor that--

HAZHIR Yes, but don't think about factors in this abstract way. Factors can be any number, any mathematical number.

RAHMANDAD: And then you have infinite possibilities. And you can't narrow down into something meaningful. So actually, carrying capacity is a meaningful concept.

So if you have n_2 divided by carrying capacity of n_2 -- that is, the total grass that possible-- that ratio becomes basically how close you are to carrying capacity. And then you are saying, as we get closer to that ratio of 1, then the natural death rate goes up and, in this case, linearly. So it can become double-- well, not double. Then it has to be multiplied by some fractional--

STUDENT: Right. [INAUDIBLE] same as the fractional death rate. That's why I did multiplication communication.

HAZHIR OK. And then you have n_y divided by normal n multiplied by impact of predator n_1 . So basically, you are looking

RAHMANDAD: here at the deer eating the grass. But you're multiplying that by n_2 . Why is that?

STUDENT: So here's the thing. If we don't have any grass, we shouldn't have any outflow.

HAZHIR Very good. So everybody has had a version of this thinking, I think, most of you have been thinking. And then the

RAHMANDAD: way you have implemented-- many of you-- I had a very quick look at some of the models-- was this idea that you multiply by the grass so that you capture the fact that, if you don't have grass, the consumption of grass should go down, which goes up to some point to address that need. But what kind of a problem does it create, if you just multiply the grass, in this case, by the deer by some constant as the outflow, which is what this formulation is doing? Katherine?

STUDENT: It doesn't account for the appetite being constrained. So if you have infinite grass, then the deer are going to be eating so much grass [INAUDIBLE]

HAZHIR Yes, very good. So that's a very big robustness problem. And it's not really just a robustness problem. It is really

RAHMANDAD: the physics is gotten wrong. If you have infinite grass and one deer, that single deer will be consuming infinite grass. That is not going to happen. So there is a serious constraint on how much grass can be consumed by the deer that this type of multiplicative constraint does not address. Does that make sense?

So let's actually look at that in a few other formulations, because this is really at the heart of the tougher formulations in this model. So let's look at some other random models. Whose is this one? This is Oriana's. So this is the vegetation. And you have--

STUDENT: [INAUDIBLE]

HAZHIR Yeah, we could go and look at the prey one. So you can see that the logic is actually very similar. So you can look

RAHMANDAD: at either of the prey or vegetation. And the underlying structure shouldn't be that different. So here, the version you have is prey--

STUDENT: This is like the prey that died because of the predators.

HAZHIR Yes, yes. And then the prey predator death rate is-- yes?

RAHMANDAD:

STUDENT: [INAUDIBLE]

HAZHIR It depends on the predator and prey required per prey per day-- predator per day. So this is the constraint that

RAHMANDAD: Kathy mentioned that, OK, you want to have some amount of consumption based on how many predators you have. So this is a good indicator. This is the desired consumption of prey by the predator. So far so good.

But in the next part when you multiply that by the prey population, you get two problems. First of all, you get the same equation that Ali had. Basically, it is the equivalent equation. The second problem is a unit problem. So what is the unit problem? This should be in deer per wolf per day or per month or whatever is the unit. Multiplied by the wolf, it becomes deer per month.

Now, it is in the right unit for outflow. You multiply that by the deer. You would not get the right unit as well. So that's the challenge. And it probably shows up in the units as well. OK, let's look at some more formulations. I think-- let's see if we have any of the folks online their models we can check out. Let's look at Gerardo's model.

So Gerardo has two stocks for each of the-- or at least-- yes, for these two groups and also for vegetation. Two stocks are fine. Nothing wrong about it. It adds to your headache for keeping track of the fact that, OK, yes, the baby prey could be eaten as well. So do you want to capture that or not? They are actually easy prey.

And so in this version, that is not captured. So for now, let's assume this is not a problem. Let's not worry about the baby prey and their fate. Assume they all grow to maturity. And if you make that assumption, this average maturity time works fine. Now, we have starving. We have death rate from average prey lifetime and eaten prey, which is from the predators.

So this is an interesting question. Do we need all of the three pathways? Or do we need fewer? So I like to have two, separating the hunting and harvesting from natural death. The separation between natural deaths and starving death is harder to pin down. It feels to me they are too close to each other. But sometimes, it may be helpful.

OK. For now, let's focus on the eaten prey because those two-- that adds another layer of complexity. So eaten prey in Gerardo's equation is based on desired prey-- desired prey are coming from predators. So this is the same equation that Oriana also had, predators multiplied by their average consumption. This is the desired. That's good. This is what we want to have.

Then let's go to the equation. This looks more complex now. So this is max of 0 and minimum of desired prey versus prey divided by feasible time to eat the prey. So is the logic clear? Who wants to take a stab at explaining this logic, what's happening here?

STUDENT: [INAUDIBLE]

HAZHIR So basically, the max of 0 is saying it shouldn't go negative. Now, this should not go negative anyway. So this is

RAHMANDAD: another kind of a stylistic idea to keep in mind. That is, the second part of the equation should be robust. It should not go negative no matter what.

Putting this max 0 does two things that are not necessarily good. One is that it makes your equation a bit too complex. So when I'm looking at it, I have to process that and make sense of it. The second thing is that, for whatever reason, if your other equations lead to a negative value because they were not correctly formulated, this hides that error.

So you actually don't want to add these max 0 thingies everywhere. As long as you have taught through why the equation shouldn't go negative, let the equation do its work. Don't add another layer of protection. OK, so simpler solution would be just the minimum.

So now, this minimum, it is either what the prey wants to eat or what the-- sorry, what the predators want to eat or what the prey can offer. And that's what the prey can offer. Here, it's formulated very simply as number of prey divided by the minimum time to harvest or something-- that is, minimum time to find prey to eat or something like that. This is actually a pretty good formulation. This is a good starting point. This is perhaps the simplest formulation you can use. So if you want-- anybody who has a critique for this formulation? Alexander?

STUDENT: So just the Min creates this sharp behavior. And so this is essentially the same as the shipment formulation. And so for that, you typically use a soft [INAUDIBLE]

HAZHIR So the critique that Alexander said is that this creates a very discontinuous behavior. That is, as long as prey
RAHMANDAD: divided by feasible time to eat prey is even slightly above desired prey rate, then you get the desired. Whereas in reality, in nature, preying on prey is kind of a very engaging task that very much depends on the density of the population.

So it is not that it is hard to get it, and then it becomes completely abundant. So abundance emerges more continuously. So that softmax, which we have different ways of formulating it, could be useful. Is that clear, the core idea? But this is a better formulation than the other ones we have seen because it doesn't have the problem that, now, if we have infinite prey, the consumption of prey by a single predator goes to infinity. It would not happen.

STUDENT: I was wondering if you do not [INAUDIBLE] Does that resolve some of your issues? I know, fundamentally, I guess the infectivity model isn't the same as prey eating, but you can imagine the prey going from alive to the stomachs of predators. It's like their contact frequency is how often they see each other [INAUDIBLE]

HAZHIR Yes. So the part that physically distracts from that conceptualization is that you see all these lions seeing all deer
RAHMANDAD: around them, but they are not hungry at the time. So they don't go and eat them. That's the part. Whereas the virus says, oh, if there is somebody, I'll catch them. It doesn't wait for you. It doesn't have any satiation. So the satiation is not captured in the Bass diffusion model. And that makes it not a good formulation.

STUDENT: What about the fact that there are different populations? Because in Bass diffusion, it's one population just changing states. But these are different [? kind of things. ?]

HAZHIR Yeah, so--

RAHMANDAD:

STUDENT: [INAUDIBLE]

HAZHIR So if you had a kind of an animal that was just into killing their prey regardless of their hunger--

RAHMANDAD:

STUDENT: [INAUDIBLE]

HAZHIR Yeah, so like humans going and shooting animals without eating them, then you get closer to a multiplicative
RAHMANDAD: function that, as soon as you have an interaction, an interaction comes with the multiplication of the density of the two species. So in that sense, it gets closer to Bass diffusion or to that multiplying the two populations. Now, it is not a Bass diffusion model because of what you said. It's two different species. But the rate of decline for one is a function of both. That's the idea.

STUDENT: So I didn't do this, but something-- I didn't do this formulation. But something I was thinking about a lot was something about the effectiveness of predators. Maybe they're really bad at hunting or something like that. How would this formulation capture that? Or how would you include that?

HAZHIR The effectiveness of the predators in hunting?

RAHMANDAD:

STUDENT: Yeah.

HAZHIR So they have a desired consumption. But then there is a separate concept that you are saying is about their
RAHMANDAD: effectiveness that may bound how much they can consume. So that, basically, would come in this formulation from that softmax. The current one that Gerardo has wouldn't capture it.

But a softmax formulation would say, OK, you can grow your consumption. But even if you have quite a bit of prey around, you don't get to your desired level. You need to have a huge amount of prey to be able to get to your desired level. Actually, maybe I should-- give me a second. Let's see if it works.

Let's say here we have the max consumption possible from how many deer you have divided by desired. It's very hard to do this by hand, but that's what it is. And here is basically what fraction of the desired you get to consume. So what Gerard has is, at the point 1 and 1, we go the 45 degree line up to here. And from this point on, it's just a straight line. So this is Gerardo's equation.

Now, Alexander's point was that you need a softer approach towards the end. So this becomes the alternative, that you have the softmax. And then the point that I was making, in response to Alexander's other comment about efficiency of predator in prey is that the slope of this curve could be different. So you could have this curve. Or you may have something that more looks like this.

Oops, it goes down-- goes up like this. So the less efficient predators are suffering unless there is a lot more prey in the landscape. So that's one way to capture it, not the only way. OK. Do we have--

STUDENT: [INAUDIBLE]

HAZHIR What do you mean as an alternative to the mean formulation? I used a lot of-- I don't know a better alternative.

RAHMANDAD: Practically, we use a lookup function. OK, great. So let's actually-- then this is very much aligned with what we are right now discussing.

So how can you solve this problem? There are two general ways-- analytical functions or table functions. Table function is pretty straightforward. You have this max divided by desired as the input to your table function. And you get the shape that you want. You plug it in.

That's nice and clean. System dynamicists are comfortable with it. Outside of system dynamics, people would say, what's that? That's too many degrees of freedom. That's confusing. I can't follow it. Give me an equation. So then you need to have at least an analytical alternative for other types of audiences. So anybody has a feel for how you can get the flexibility for these different curves with an analytical function that does all of the things we want? Jason, what would be your suggestion?

STUDENT: The one I landed on was input divided by 1 plus the input. To get flexibility, you could raise that whole quotient to some exponential power [INAUDIBLE]

HAZHIR OK. So let's call this x for simplicity.

RAHMANDAD:

STUDENT: So it's x plus 1 over-- or x over--

HAZHIR So your formulation is x --

RAHMANDAD:

STUDENT: Divided by 1 plus x .

HAZHIR Divided by $1 + x$.

RAHMANDAD:

STUDENT: Right, because it gives you the property that when x is 0 you get the 0, 0 because the numerator is 0.

HAZHIR OK.

RAHMANDAD:

STUDENT: And then it will approach--

HAZHIR It would approach 1.

RAHMANDAD:

STUDENT: It approaches 1 as x goes to infinity.

HAZHIR OK.

RAHMANDAD:

STUDENT: The problem is it's kind of a slow approach to--

HAZHIR Yes.

RAHMANDAD:

STUDENT: So you could speed that up.

HAZHIR Yes.

RAHMANDAD:

STUDENT: Speed that up by putting an exponent around the whole thing.

HAZHIR So you raise this to some larger power?

RAHMANDAD:

STUDENT: Yeah.

HAZHIR OK. Are you following? This is exactly the type of thinking you want to do. That is, OK, I want something that

RAHMANDAD: starts from 0, goes to 1. x over $1 + x$ does that. So that's nice. But then it doesn't approach fast enough. And it doesn't have any degrees of freedom right now. You don't have any parameter associated with it. So you can't get different saturation speeds. So this is not yet doing well. So now, Jason's next step was, oh, maybe we can do this, raise this to the power of α .

And then that perhaps gives us something a little bit more flexible. Remember that, between 0 and 1, x values between 0 and 1 is a very active region for this input. So you want a formulation that works in that region. So if you have, let's say, close to 1-- so if you get 1, in this case, this becomes 0.5 raised to the power of α . Well, it is important. So if it is 0.5, what do you want it to be? Do you want it to go above 0.5 or do you want it to go below 0.5?

STUDENT: You want it to be below.

HAZHIR Yes, because if it is above 0.5, it is going to violate another constraint here, that you cannot exceed the y equals

RAHMANDAD: x line because the y equals x line-- that is, the output equals the input-- means your max divided by desired-- that is what you get out of this-- will need to be multiplied by the desired.

So if it is y equals x line, then max divided by desired is going to be multiplied by the desired. This gives you the max. Now, any value above one above x equals y -- if this is y , this is our output--

STUDENT: Would give you more than desired.

HAZHIR This will lead to more than max, not more than desired. So this is the problem. So if you call this thing y , then you

RAHMANDAD: are going to multiply y by the desired. And if you multiply y by the desired-- let's, for a second, think of a situation where y equals x . And what is x ? We see it on the right-hand side of the screen. It is max over desired.

So that means these two lead to max over desired multiplied by desired is the total. Let's call this z . This is the consumption. So that means the consumption will be equal to what? Max. So this is saying that whatever is the output of this function should not go over the line y equals x . So that's another constraint that we have, which we didn't think about before this, for making a robust formulation.

STUDENT: That is where the constraint comes from this [INAUDIBLE].

HAZHIR Yes.

RAHMANDAD:

STUDENT: [INAUDIBLE]

HAZHIR 0.5. So I said, if x is 0.5, x divided by $1 + x$ -- oh sorry, if x is 1--

RAHMANDAD:

STUDENT: [INAUDIBLE]

HAZHIR It's 0.5, yes. So it is below. So it is fine for that one. You're right.

RAHMANDAD:

STUDENT: Right. The problem is, though, if you want anything stronger, you wind up with stuff above it.

HAZHIR At some point, it could go above. That is, at the very beginning, at the slope of-- at the starting point, the slope is
RAHMANDAD: right now x at the very beginning. And so that gives a 45-degree line at the beginning. So this, without α , will never go above 45-degree line. But as soon as you add alphas that are below 1, for example, to go up faster, then it goes above the 45-degree line.

So that's not a big problem. You can solve that. And the way to solve that-- so actually, first of all, you need alphas that are smaller than 1 for a sharper increase because that gets you closer to 1 more quickly. And then you can have, basically, the minimum of x -- this is y equals x and this function that you just came up with, so x divided by $1 + x$ to the power of α . This does the job OK.

STUDENT: But it still has some pretty sharp-- or it will have some point where--

HAZHIR Yes, there will be a discontinuity.

RAHMANDAD:

STUDENT: There will be a kink in there.

HAZHIR There will be a kink, yes. And actually, it is not very easy to come up with a formulation that doesn't have the
RAHMANDAD: kink. You can, but you basically need to build a much more complex formulation. So this is one version. You could do other ones. So you could use an exponential approach to the line rather than this polynomial approach. So if you want to do the exponential approach, you still need the mean of x and something. But that something could be $1 - e^{-\alpha x}$. So if x is a large number, $1 - e^{-\alpha x}$ becomes--

STUDENT: [INAUDIBLE]

HAZHIR If x is a large number-- that is, we are on the right-hand side of this graph-- this e to the power of minus
RAHMANDAD: something becomes 0. And therefore, you get to 1 as the total. So you're exponentially getting close to that. You're approaching that curve. So that's an alternative. I'm not sure which one is better. To see which one is better, you actually need to graph them for different values of α and see which ones are qualitatively closer to what you want. Alexander?

STUDENT: So I mean, if you were going to have a kink in this anyway no matter what, why not just do what Gerardo did?

HAZHIR Because this gives you the flexibility to capture the other factor that you talked about, which is important in this
RAHMANDAD: case. So that's, I would say, the primary thing. And the other thing is that it doesn't completely remove the effect. It just changes the speed of the effect, whereas the min function completely removes the effect. It has two very different regions. So it shuts off some feedback loop, whereas this version leaves the feedback loop on, but weakens it over time. So as such, the kink is not as strong. The impact is not as strong.

STUDENT: More of a process question-- I can see where you would come up with this if you're starting from the point of I'm going [INAUDIBLE] But if you start from the point of hunting effectiveness [INAUDIBLE] it takes a while to land here on your own.

HAZHIR So I think the logic to get gets you here very quickly is I have predators, they want to eat prey, they have a
RAHMANDAD: desired value. So that is the driver. That's like the demand in an inventory model. But then if I don't have enough prey, I need to bring down their ability, their fulfillment ratio, their ability to get the prey they want. So I just need a formulation to capture that.

That's the fast route to get to close to this step. Then you need to go back into your memory of, OK, what are the ways to capture these consumption of something out of a stock that is satiated on the one hand by the demand and on the other hand by the availability of the stock. And that's where, if you remember your assignment 6 from 873, you would be able to do that fairly straightforward using that formulation.

STUDENT: So you start from the story design.

HAZHIR Yes. Hassan?
RAHMANDAD:

STUDENT: Can we use a second [INAUDIBLE]? Then for that formulation, we actually know where the alpha is that will create [INAUDIBLE] So we can limit the alpha so that it will never pass [INAUDIBLE]

HAZHIR So yes, but you want it to also start at the slope of 45 degrees.
RAHMANDAD:

STUDENT: Yeah, so the second half of it. So we'll put the inflection point at 0.

HAZHIR Yes, but the sigmoid at the second half at the inflection point of the slope is flexible.
RAHMANDAD:

STUDENT: Yeah, but we know where that limit of that slope is.

HAZHIR Yes, yes. We know limit of the slope. But I'm saying that you actually want it to be at the max consumption rate
RAHMANDAD: at the very beginning. So this is also an interesting but likely, reasonable approach that basically, if people are starving or the predators are starving and there is only so much prey, they will try to get as much as they can. That's the idea.

STUDENT: [INAUDIBLE] there's just one value that fits these limits. Then we don't have any control over it.

HAZHIR Yes, exactly. So this min function actually brings that second constraint into your equation, but has the kink that
RAHMANDAD: Alexander highlighted.

STUDENT: Could you explain what you were saying a minute ago about weakening the balancing feedback loop?

HAZHIR Yes. So here, we have one balancing feedback loop that is more deer, more consumption, and therefore less
RAHMANDAD: deer. So that's the first order control that we need to avoid the stock going to 0. So that feedback loop in the min function, in Gerardo's version, would only operate where the x is between 0 and 1, OK? And as soon as it is above 1, that feedback loop goes away because, in this formulation, the predator gets as much prey as it wants. And therefore, that feedback loop is not active anymore.

Now, this is not fundamentally wrong. Those feedback loops saturate at some point and are not active anymore. But making it a very quick shift between those two domains, for all kinds of analysis, becomes a little bit less easy to trace why this is happening. Even in some optimizations, it becomes helpful to have those feedbacks active so that it pushes the optimization engine to find the right balance. Otherwise, that parameter never works.

STUDENT: So OK. When we have this kink-- I see. So then when we have this kink to the left, when we do this thing, then it's always active.

HAZHIR It is always active, exactly.

RAHMANDAD:

STUDENT: Yeah, that makes a lot of sense.

HAZHIR And in fact, in a table function, you don't have it always active because, ultimately, you put a value that says

RAHMANDAD: maybe at level x equals 3 it saturates. So table functions are not, in this case, necessarily the best tool. You define it and you can't control it as easily. You have to go and manually change it. So I like the analytical function for all of those reasons.

Good. So if you build this part, this can go both into the consumption of prey and the consumption of the vegetation by the prey. So both of the stocks can be constrained by a mechanism like this, by a formulation like this. And that will basically do the kink thing. But Gerardo's formulation is a decent starting point. It is not fundamental-- it doesn't have any fundamental problems. It's just has this non-trivial jump.

OK. So now that we saw that, let's actually look a little bit more at Gerardo's formulation and see if there is anything else we want to discuss there. And Gerardo, we haven't heard you. We just have been discussing your model.

STUDENT: I was out of time this week. And I think that there are a lot of things wrong in this model.

HAZHIR But actually, you had the better formulation so far for the most challenging part. So that's good.

RAHMANDAD:

STUDENT: [INAUDIBLE]

HAZHIR Yeah.

RAHMANDAD:

STUDENT: Jason was suggesting doesn't necessarily have a max or a minutes baked into it.

HAZHIR Yes, let's look at it.

RAHMANDAD:

STUDENT: [INAUDIBLE] current consumption over 1 plus a max consumption.

HAZHIR Where am I? So this is your--

RAHMANDAD:

STUDENT: [INAUDIBLE]

HAZHIR Yes, let's go for the deer.

RAHMANDAD:

STUDENT: [INAUDIBLE]

HAZHIR OK. So for the deer, we have the effective deer lifespan. And then we have predation of mature deer. And you do

RAHMANDAD: have the predation of juvenile deer, which is good. If you have juvenile deer, they will be eaten.

STUDENT: I think current consumption is kind of the one.

HAZHIR Yes. So let's look at the current consumption.

RAHMANDAD:

STUDENT: So I have that idea of there's a success rate in hunting, which admittedly is just static here. But it is made into something that is like [INAUDIBLE]

HAZHIR Yeah, so let's go to the current consumption and then-- OK. So attack rate multiplied by hunt success rate-- those

RAHMANDAD: two together are always together. So think of that as just a single parameter. So it makes the world a little bit easier to make sense of.

STUDENT: But you could change it.

HAZHIR Yes, you could change it, absolutely. And if you made one of them dynamic, you can see the value of having the

RAHMANDAD: parameters. So one parameter multiplied by the deer population raised to the power of 2. So the idea is that, as the deer population grows, the consumption rate can go up divided by 1 plus some time constant.

STUDENT: It's not a time constant. So if you look at time to consume here, it's basically setting the maximum amount of deer consumed per day.

HAZHIR Yeah. So basically, you have the numerator divided by 1 plus some parameter multiplied by that numerator. So

RAHMANDAD: let's think about that formulation. So x divided by $1 + \alpha x$.

STUDENT: And α is the max. α is like 1 over the max.

HAZHIR OK. So that formulation, if x is 0, it goes to 0. That is, if you don't have any deer, there is no consumption per

RAHMANDAD: predator. If x is a very large number, then this converges towards 1 over α . So if α is then your 1 over maximum rate that you want, you get what you want out of this formulation. So tell me about where the power 2 came from.

STUDENT: The power 2-- so I originally wanted something that depended on density a lot and depended on basically interactions between wolves and deer and their success rate. And then through a lot of back and forth thinking about that specifically, I originally just had deer. And then I was thinking back through, admittedly with AI, this particular idea of I want it to depend more on [INAUDIBLE]

HAZHIR Yes.

RAHMANDAD:

STUDENT: And so it suggested the idea [INAUDIBLE]

HAZHIR Yeah. So basically, you used a more mathematical way of trying to get to a function that does what you want it to

RAHMANDAD: do. That is, when the density is low, you don't get as many deer hunted per predator. And as the density goes up, you get more and more, but then it saturates. And this does all of those things. So what are the problems with this? So the first problem is that the units don't match. And the reason the units don't match is that time to consume--

STUDENT: Well, I called it time to consume.

HAZHIR It doesn't matter. Whatever is the unit, because it is 1 plus this thing, 1 is dimensionless.

RAHMANDAD:

STUDENT: Yeah. So is that term, the second term.

HAZHIR Ah.

RAHMANDAD:

STUDENT: So the numerator is like consumption. And then time consuming deer cancels out that consumption.

HAZHIR I see, I see, I see. Then the units can work. You're right, you're right. OK. So then the main problem that remains

RAHMANDAD: is just it is hard to consume this equation in terms of the micro-mechanisms. And if we go through the actual exact equation and we compare it with the one that we designed, the question of whether the 45-degree line constraint gets violated here or not, with an alternative conceptualization that thinks of deer availability as-- or max consumption rate of deer as another constraint-- that I don't know how it will pan out here.

That is, in this function of $1 \text{ over } 1 \text{ plus } \alpha x \text{-- or } x^2 \text{ divided by } 1 \text{ plus } \alpha x^2$, whether you may go over the 45-degree line in the range that we are considering. It's not something I can mentally do. We would have to write it down. And that's why having a story that breaks down the logic will help a little bit more with getting good formulations.

STUDENT: I mean--

HAZHIR But this does the job. So this is good because it does the things that we want. It shuts off consumption when

RAHMANDAD: there is no deer. It allows for max consumption per predator or desired consumption per predator when we have plenty of deer. It has the softmax story. It has a free parameter, that $\alpha = 1$. So it has all of those characteristics. So it is good on all of those fronts.

STUDENT: But I guess my thing is, isn't there a more reasonable story to--

HAZHIR To match this?

RAHMANDAD:

STUDENT: To this than to something that has a kink in it?

HAZHIR So this one, again because we haven't done the analysis with the 45-degree line, may violate that. I don't have

RAHMANDAD: the intuition right now.

STUDENT: What would it mean practically [INAUDIBLE]

HAZHIR So if it violates it, it means that at some levels of prey-- the rate of consumption of prey exceeds the maximum

RAHMANDAD: rate that, based on random interactions, is even feasible to happen in that-- with the density of deer that is available. So that becomes-- so let's say it takes some time to prey any deer. There is a literally physical time that you need. And it takes some time to find the deer. You add those two together. There is a minimum time to catch a deer, so to speak. So if you have very few deer, the total predation should not exceed that. Whether that will happen here or not we cannot tell from the equation.

STUDENT: I think that might be what my attack rate is because it's like--

HAZHIR It is a parameter. I'm just saying-- so the attack rate right now is trying to capture that idea. But because you

RAHMANDAD: have just deer to the power of 2 playing something, it is not clear how it actually ends up downstream when you get the total consumption. But it does the job. In fact, so this one actually doesn't have the flexibility to change the slope of how fast you converge. You just change the height of where the total consumption per predator converges to.

If you want to make that flexible, you would play with the power here. So you can go with power of 2 or 3 or whatever. So that becomes the alpha equivalent for the other formulation we saw. OK, this is a nice alternative to think through. And thanks for bringing that up [? and ?] never. OK. Anybody else wants to look at their formulation for this part? So Kathy also wants-- let's go to Kathy's. Let me see. OK, good.

STUDENT: So it's a different analytical formulation. It has the same problem, where it needs to be-- it goes above the 45-degree line. So that--

HAZHIR Yeah, so let's start looking at it step by step. So you have also separated the starvation from natural deaths. And

RAHMANDAD: as I said, it's a little bit-- we can get to that equation as well. So I think that's a separate equation than eating of the deer. So let's focus on this one. And this has four things going into it. So I have to open it and see what it's doing. So minimum effective RDD and deer. What is RDD?

STUDENT: Relative deer density, so how many deer over some reference, like full density amount of deer.

HAZHIR OK, multiplied by indicated deer eaten versus deer divided by minimum time to eat deer. So this is similar to

RAHMANDAD: what we had with Gerardo's formulation. And this first part is a little bit more involved. So let's look at what that is doing. So indicated deer eaten comes from wolf multiplied by wolf appetite. So this is the desired, OK.

And then this relative RDD is adding another constraint on consumption that is-- OK. So let's look at this equation and make sense of it. So it is max of 0. OK. It shouldn't go negative. Some parameter A multiplied by 1 minus e to the power of minus B relative deer density. So if deer density is very high, this thing turns into-- the exponent goes to a large negative number. e to the power of that goes to 0. 1 minus that becomes 1. It goes to A.

But in this case-- OK. So that's for high deer densities. This goes to a number A. And for lower deer densities, it can go down to 0, basically. So if it is 0 deer density, that e to the power of blah, blah becomes 1. So this thing is-- so it's very similar to the equation that I wrote for the other part. And this is saying per predator. So what's the unit for A? All of these are dimensionless. So why do you need A here then?

STUDENT: So A is 1 currently. I was thinking though that there is kind of a gluttony effect. And maybe I want want it to go to 1.2 or something like that.

HAZHIR But why not capture that already? You see, human learning is very slow. OK, so why don't you capture that

RAHMANDAD: involved appetite? Because ultimately, this is going to be multiplied by appetite. So it is just one degree of freedom that is not well-defined and doesn't do anything extra for you. So don't add that type of a--

STUDENT: Yeah, I was thinking of wolf appetite as the equilibrium. But if they have 100 deer in front of them, maybe they'll eat extra, I guess, over and above that. But right, it doesn't need to be there.

HAZHIR Yeah, because whatever it does, it will ultimately be multiplied by this. A. It will never-- and then you don't need

RAHMANDAD: the max with 0 thing as well, I think.

STUDENT: That's true. I removed it when we were talking.

HAZHIR Yeah, you don't need that. And then yeah, beta does the thing for slowing it down. So yeah, this should work as

RAHMANDAD: well. So this way, you have the 45-degree constraint going through the min function. And you have the other constraints kicking in through limiting how many deer the wolf can eat. So this works fine. I think that's good. OK. Let's start looking at the starvation equations or let's take a break.

STUDENT: Break.

HAZHIR Yeah, I think it's a good time to take a break, 10 minutes.

RAHMANDAD:

So this is Jason's model. Let's take a look at a couple of items. So we had the predation of deer from desired consumption. And then you have the desired fulfillment ratio. So this is the formulation we talked about in following the same idea as the shipment from inventory.

So there is a max feasible consumption, deer divided by minimum time to capture, versus the desired consumption that gives a prey sufficiency. Let's just make sure we know the formulation. This is the max divided by desired. OK. It's very similar to what we just discussed. And then this goes into a fulfillment ratio that, right now, we talked about it. You highlighted that. And we said how you can actually overcome some of the challenges with this formulation to get it to do what you want to do. And then you get the deaths just from predation.

Let's go to starvation part. So you have the general death from old age. So deer, after a while, they die. That's very standard, simple. You take care of it that way. Then you're saying a fraction of the deer is at risk of dying from starvation. So to calculate that-- I'm not following the units here. Oh, you have deer also here. OK. Where did the deer come from?

STUDENT: It's the first order. So there's some minimum--

HAZHIR Oh, yeah. There is deer inside here.

RAHMANDAD:

STUDENT: So if they don't eat for two weeks, they'll die. And then that flow gets closer and closer to 0, depending on how satiated they are.

HAZHIR OK, so you're saying deer divided by deer time to starve is the death rate from starvation. If satiation is 1, then

RAHMANDAD: nothing happens.

STUDENT: Right.

HAZHIR So that's how you close this valve off, basically.

RAHMANDAD:

STUDENT: Yeah.

HAZHIR OK, that's fine. I think the risk I see with that is the deer satiation equation. It needs to be turned on only in

RAHMANDAD: relatively extreme conditions or cases. That is, you have significant lack of malnutrition. Otherwise, it starts killing deer quite rapidly.

STUDENT: [INAUDIBLE]

HAZHIR But in this formulation, we can't see that, because it's the calculation of deer satiation, that determines it. And in

RAHMANDAD: this case, you have just a grass fulfillment ratio, which is the ratio of what the grass is available for the desired consumption for the deer.

STUDENT: Yeah, [INAUDIBLE]

HAZHIR That's good. So this is actually a very important piece. And I was looking at some of the other models. Some of

RAHMANDAD: them didn't follow this idea. So note here, the grass fulfillment ratio-- that is, what fraction of the desired consumption by deer is actually happening because of the availability of grass-- is having an impact on the grass consumption.

But it also feeds back into deer starvation. It is very important to use the same concept, rather than defining this concept in another way, in another part of the model, because the underlying concept is deer does not get to eat grass or does get to eat grass. It's the same thing. It's the two sides of the same coin.

STUDENT: It's easy to see it, if you scroll down to the wolves. It's the same thing, but I just broke it down into more steps-- actual deer consumption, and then wolf satiation, i.e. it's the fraction that they ate divided by what they desired, which is equivalent to the desired ratio. But that story, that is [INAUDIBLE]

HAZHIR Yes. So both of those give the same answer mathematically. But the version down here is saying, OK, this is the

RAHMANDAD: desired consumption versus actual consumption. That ratio gives satiation. And it is a good practice to spell out those steps because if, for whatever reason, you add another mechanism that changes the actual consumption, for example, it would not be captured in that fulfillment ratio alone. It will be captured in something else. So it is fine to add this extra variable, even though it is ultimately the same as desired fulfillment ratio. Is the logic here clear what we are talking about?

STUDENT: Yeah.

HAZHIR If you feel anything is, OK, I'm not following, raise your hands. I wouldn't know. But we would pass it too quickly,

RAHMANDAD: then. OK, so the only place that I'm a little bit unsure is that this wolf satiation should be the same thing that goes into here and get 1 minus that satiation.

STUDENT: [INAUDIBLE]

HAZHIR No, I know. But I'm saying, it may be too strong a feedback of starvation because time to starve is relatively

RAHMANDAD: short. And then basically, OK, biologically, what happens? If you don't get to eat what you desire to eat, you get thinner. And your desire for eating actually goes down because your metabolic rate goes down. And then you reduce your activity level. And you do all these things.

STUDENT: [INAUDIBLE]

HAZHIR So there are all these balancing loops that you wouldn't start dying right away. In fact, for anybody like us, we

RAHMANDAD: may actually live for a while without eating at 100% of our desire-- 2,200 calories or whatever it is. If I bring that down to 2,000 calories, it doesn't mean that I'll start to die. It just means I'll probably equilibrate at 2,000 calories. Even if you bring it down to 1,500, you may equilibrate at that and not die.

Now, if you bring it to 1,000, then you will probably die after maybe a few months or something. So that's the thinking that gives me the feeling, OK, this wolf satiation shouldn't directly go into the death. It needs to be transformed somehow. And the transformation should make it keep it closer to 1. That is, if the satiation is 0.7, I should push it up to 0.95 or something like that. You see the logic why-- the way to think about this? So then how would you do that? So what is the right or easy way to do this with the current formulation that Jason has? Any suggestions? Anybody?

STUDENT: I mean, when you have wolf satiation [INAUDIBLE]

HAZHIR Yeah, yeah. Well, not desired consumption. Wolf satiation-- OK, so the story I was telling

RAHMANDAD:

STUDENT: [INAUDIBLE]

HAZHIR Included a feedback into desired consumption, you're right. But that's too much. That's messy. So that was not

RAHMANDAD: what I was asking. I was asking for a simple formulation that gets wolf satiation and gives you effective wolf satiation on death rate. Basically, right now, Jason is equating wolf satiation-- or 1 minus wolf satiation with death effect.

STUDENT: [INAUDIBLE] [$? \times ?$] the wolf satiation to some exponent.

HAZHIR OK, so what should the exponent range be? So let's say we have wolf satiation to the power of alpha. Should the

RAHMANDAD: alpha be small or big?

STUDENT: [INAUDIBLE]

HAZHIR It should be smaller than 1. So if alpha is 0, you shot off this feedback loop. That is, basically, there is no

RAHMANDAD: starvation because anything to the power of 0 is still 1. And therefore, this thing goes away. If alpha is 1, it is the current formulation. So somewhere between that range is where you want alpha to be. And that's a good way of formulating this because wolf satiation will never go above 1 the way it is formulated here because it is always actual divided by desired. So at max, it will be 1. So I'm fine with that range. It will all work. Is the idea clear?

So that's a good formulation. I think, with that correction, this does a decent job. We talked about the issue of whether we want to combine them or separate them. Now that I'm seeing how you have separated them-- and I think some of the others have done a similar thing-- I like it. It is fine. I don't have anything against it because the logic of natural death from old age is easy to capture using this lifespan. And then you can separately capture it here. So this is fine. Ali?

STUDENT: [INAUDIBLE] I had this issue also. And I found it very last minute. So I think, if you don't have any vegetation, deer would grow because there is no control on the [? earthworms. ?] So what I'm trying to say is that the outflow from vegetation should be the inflow to the deer. [INAUDIBLE] of the vegetation should be the inflow to the birth rate of the deer.

HAZHIR
RAHMANDAD: OK, very good point. So now, this is an important biological question for us who are not biologists and have no clue about how deer life spans are. So the question is, if people are-- or deer are starving, would they have babies? And the short answer is not as many. Basically, the stress of starvation brings down their birth rate. So that is another way to potentially have a feedback into the system. So that definitely is a logical, biologically sound idea.

Now, there is a question, do I even want to do that? And the reason for asking that question is, this outflow is pretty strong. And you have control over it. And it is proportional to the size of the deer. Basically, you have deer divided by some time constant and so on. So you can say I'm capturing those unborn babies as death. So you are a pro-life person or something.

So that's one way of making the argument that I don't need to separately capture it because then I would add some other parameter and some other structure. And I'm perfectly fine with that recognition, and then saying I'm not doing it. But it's good to have that recognition. OK, great.

So I think this is working fine with the couple of ideas that we mentioned. We didn't have the-- yeah, so the grass is growing exponentially here. It doesn't have any balancing loop. So if you leave the grass unchecked with the deer, then it grows exponentially. And that is a problem that this formulation doesn't capture.

Now, again, for that, you could have a grass death separately. Or you could use Alexander's or some alternative version that uses the single flow for grass and saturates that. Great. Let's see. Anybody else want to bring a question from their models? I think we have really discussed the key issues. So we can open it to any models. Alexander, do you want to take a look? Sure.

STUDENT: About the predation rate and the death rate. So I completely-- I guess I implicitly assumed that animals have insatiable appetites. So I forgot about the whole capacity thing and things like that. And I focused a lot on the hunting efficiency.

HAZHIR
RAHMANDAD: Uh-huh.

STUDENT: And so basically, what I wanted to capture was that, as there's fewer deer, wolves can't hunt as well, can't hunt as well. And then their efficiency maxes out at some point. So that's the top half of the model.

HAZHIR OK, so let's look at it. So you have the hunting efficiency multiplied by the number of the predators. So this

RAHMANDAD: hunting efficiency is what we already have seen in various formats. And that comes from some reference multiplied by some effective availability of prey. And that effect is a fraction of carrying capacity, so prey divided by carrying capacity of prey.

It should be fine. So that number, if the model works well on other fronts, would always be between 0 and 1. But you can never be sure because you can actually start from beyond carrying capacity. You can throw in too many prey in a natural environment. And then effective prey population and efficiency is--

STUDENT: So it's similar to what Jason was suggesting earlier. It's x over $1 + x$ to some alpha. And then I put that 2 in the beginning so it's such that, when that fraction is equal to 1, then the efficiency is [INAUDIBLE] So without the 2, and let's say the sensitivity is 1, then if the fraction is 1, then this whole effect would be $1/2$. And so I put in the 2 so that way, when the fraction is 1 [INAUDIBLE]

HAZHIR Yeah.

RAHMANDAD:

STUDENT: So I just wanted to bring this up because I approached this differently than a lot of other people. And I just wanted to know what the issue is.

HAZHIR Yeah, so let's think about it. So we already have talked a little bit about some of these. So basically, this one is

RAHMANDAD: saying, at carrying capacity levels of prey population, we give predators what they want. And higher levels wouldn't happen because this fraction of prey carrying capacity should not go above 1.

So anything below that will lead to less consumption per predator. And how much less depends on this sensitivity parameter. You're currently putting it at 2. So this leads to a relatively sharp reduction. That is, so the table function-ish view is more like this.

This is 0 to 1. It goes quickly up, which means it is very sensitive. If you don't have enough deer, the amount of consumption goes down non-linearly. So you wouldn't have a problem with hitting the 45-degree line in our other version of the formulation, as long as your sensitivity parameter is above 1.

We do have the issue that maybe that is too harsh, maybe-- that is, to predators. That is, they could consume more. So we said that, at the 0 level of prey, the 45-degree line in the other conception is a potentially good reference curve.

And the argument there was that, if there is prey to be eaten and there is plenty of predators, they'll be eating the prey with the maximum rate feasible that comes from prey divided by number of-- minimum time to consume prey. So that was the argument for targeting the 45-degree line as a reference line.

Now, you may argue that is not the case. That is, the search process for the prey scales non-linearly with its density. That is, if the density is very small, it doesn't go down the rate of consumption-- linearly, it goes non-linearly. And I think your equation, Jake, also had that alternative way of justifying it.

So I want us to be clear about what the formulations are doing in both of these cases. In your case, I'm not sure because we haven't connected it fully to that formulation. In this case, it is making the assumption that the non-linearity in preyability is significant. And therefore, even if there is some prey, they would not be hunted because they can't be found. They go into some holes and something. I don't know. And that's OK, as long as you know that's the assumption you're making.

STUDENT: So yeah, I was thinking-- I just want to bring this up because [INAUDIBLE] I was thinking about the search process as opposed to the [INAUDIBLE]

HAZHIR Yes, but you also have basically the desire. It is built into your parameter reference hunting efficiency. That's
RAHMANDAD: basically the desired parameter in your setup. And its units should be in deer per wolf per month. So it is really just the desire.

STUDENT: OK. So I think, in that case, my mistake was is that I-- OK. I think my mistake, in that case, was that I didn't realize that. I wasn't thinking about [INAUDIBLE]. So then when I talk about the effect of hunting on the wolf death rate in the lower half, I'll have a separate parameter, which is the required prey per predator. And now, I see that they're [INAUDIBLE]

HAZHIR Yes. Conceptually, these two are very similar to each other. And basically, you shouldn't be creating yet another
RAHMANDAD: process. Now, this yet another process will give you a number that is very comparable probably with the parameters you have chosen. And then you have another sensitivity thingy here. So maybe, ultimately, it doesn't matter, in terms of degrees of freedom. You have plenty of degrees of freedom to get the behavior you want. But it reduces the clarity of the formulation.

STUDENT: I guess I was thinking maybe there's a predator who really wants to eat a lot of prey, but is really bad at it. And that's why I had-- I mean, unironically. And that's why I had separate-- it kind of ran through my mind. And that was my haphazard justification. But yeah, so I see the-- and then I also see how the hunting adequacy, which is just prey hunted divided by required, is similar to that effect.

HAZHIR Yeah. So basically, this sensitivity parameter can take care of the efficiency consideration. And then the desired
RAHMANDAD: parameter should mean the desired.

STUDENT: Yeah, so I see how it's very similar.

HAZHIR Yeah.
RAHMANDAD:

STUDENT: And then I guess I just wanted to bring up as well how it affects it. So I had predator deaths be the same. And then what I had is that this effect, which we discussed the issues with, basically changes how long hunters live with logistics so that hunter lifespan is weighted.

HAZHIR Yeah. So now, basically, you have the version with a single outflow rather than two separate outflows. So we saw
RAHMANDAD: Jason's two outflow. Let's look at this single outflow version and see how it works. So you have the hunter lifespan. And you are just reducing the lifespan. So that's a very natural, good alternative because, even if you're dying from starvation, you're dying after a while. So your lifespan has ended.

So as such, this is also a good formulation, the basic starting point. Now, we have to get what is the hunter lifespan. You have the minimum hunter lifespan and the maximum hunter lifespan. This is the natural. The maximum is the natural if you have all of the food you need. And then the minimum is the starvation time horizon that Jason was talking about. And then this is some kind of a weighted average. So this effect multiplied by maximum, 1 minus effect multiplied by minimum, this is fine. I like that way of shifting between those two life spans.

Then the key question is, how does this effect get formulated? It needs to be between 0 and 1 because that switch requires you to be between 0 and 1. Otherwise, it doesn't work. So you have hunting adequacy, which is how much you eat versus how much you are requiring. Again, keep in mind that this required value should probably be just hunting efficiency multiplied by predator or something. But that put aside, this is also a decent input into the formulation. And this hunting adequacy should not go above 1, if it is correctly formulated.

Now, in this version, it may go above 1 depending on the parameters chosen. That is, if the required prey per predators is smaller than this hunting efficiency parameter-- and it's probably not the case in Alexander's So this is 2. That one was 1.5. So it wouldn't go above 1. But technically, it could if somebody comes and plays with the parameters. Everybody is following what we are saying? So you see, on the top, hunting efficiency is multiplied, after some adjustments, by the number of predators. That's the actual prey hunted.

In the bottom, hunting required as the predators multiplied by this required prey. So this gives a different number than the maximum up there. And if the maximum up there is bigger than this number, the ratio of hunting adequacy exceeds 1. And if that exceeds 1, things get messed up because now this has-- this ratio should not exceed 1. Now, in this formulation, maybe it is kept between 0 and 1 using the exponential. So let's look at this equation.

STUDENT: [INAUDIBLE]

HAZHIR Yeah, so this is a logistic one that says e to the power of some α hunting adequacy minus 1 divided by same
RAHMANDAD: thing plus 1.

STUDENT: Yeah, I also screwed up the parentheses, I realized here.

HAZHIR OK.

RAHMANDAD:

STUDENT: The entire denominator is taken to the exponent. But my intent was to do logistic.

HAZHIR Yes, you need the parameter-- sorry you need another--

RAHMANDAD:

STUDENT: [INAUDIBLE] I think the second--

HAZHIR Is extra? No, this is fine, e to the power of whatever you have up there divided by e to the power of sensitivity. I

RAHMANDAD: think it is fine. Maybe this one is--

STUDENT: Yeah, that one.

HAZHIR This one is misplaced, yes. So this should be here. OK. But the basic idea works well here. So it is saying, if
RAHMANDAD: hunting adequacy is 1-- that is, you have plenty, that you have what you wanted-- then e to the power of 0 divided by-- so you get a value of 0.5 out of this expression. Is that what you wanted to get?

STUDENT: I was thinking of hunting adequacy as basically this is like-- I was thinking of it more of like this is some minimal level. And that if you go below this level, your lifespan is going to go really down.

HAZHIR OK. So this is an important consideration. So Alexander talked about it. And he tried to anchor the operating
RAHMANDAD: point of this relationship to the value of hunting adequacy equals to 1 means some lifespan that is the middle of the starving time and the full lifetime.

That is not intuitive for me. Hearing that, I would say, OK, I can follow your logic. But if hunting adequacy-- the way I think of the concept is, if it is 1, then I should get my full lifespan. Why do you kill me halfway? So it is not wrong. Mathematically, it probably works OK. But conceptually, it is harder to justify it.

There is also, technically, you want your operating point for your model, when you are normalizing your model and trying to set your baseline equilibrium level, not to be somewhere in the middle of these time constants or things like that. It just becomes much harder to put the model in equilibrium and get it to behave well. You want it to be somewhere in a more normal range.

Here, more normal means you have your adequate life. The deer is plentiful, probably. And grass is plentiful or something like that. Now, this is a complex model. Putting it into equilibrium is not trivial. So you may need to think more about it. And I haven't looked at it. So this works, but has those characteristics. Great. It was good to look at this. But the idea of a single flow with this type of feedback all works, all meaningful. Also, the formulation here is a kind of S-shape when you have the logistic.

STUDENT: And that was kind of intentional. My point was saying that, when you're crossing over that threshold of what you need, your lifespan improves quite a lot in that short little range.

HAZHIR Yes. So You were making that very responsive. So I would have probably thought of hunting adequacy to keep it
RAHMANDAD: between 0 and 1, and then normalize around the value of 1-- that is, make that the equilibrium region or the baseline region.

And then you can have a sigmoid function if you think there is non-linearity, which there might be. Yeah, if I have 90% of what I want to eat, my life span wouldn't come down as fast. And then there is a point that it does come down very fast. And then it doesn't matter if I'm getting 10% or 20%. I'm going to die. So that is fine. I think the S-shaped logic here does have a good biological support for it.

STUDENT: But you were saying that the anchor should be on the ends of these distributions?

HAZHIR Not always. So in this case, it is hard because, when you put it to the end of the distribution, it wouldn't saturate.
RAHMANDAD: So you have to go a lot, to a lot of deer. And a lot of deer may not give you equilibrium on the other parts of the model. So that's why I'm saying putting this model into equilibrium is not actually always easy. You need to-- so in terms of the explaining and the logic, it is easier to put your anchor at the point where you're getting what you want. In terms of getting the analytical equilibrium, it may or may not be. You have to write it and see if you can get [INAUDIBLE] Good. Anybody else wants to look at their model? Anything else that is here that we want to look at?

STUDENT: [INAUDIBLE]

HAZHIR Yes.

RAHMANDAD:

STUDENT: In my mind there's a requirement [INAUDIBLE]. What was the bit about hunting efficiency should actually be--

HAZHIR Oh, yes. So here, the hunting efficiency is a parameter that is basically saying how many deer per wolf per month

RAHMANDAD: are being hunted. OK. And then there is the required prey per predator. That is too how many deer per wolf per month is desired to be hunted. And the two are conceptually very close to each other.

Now, Alexander had this idea in mind that I want-- maybe my hunters are not good hunters. And in fact, in this version, then they should die in some way. I'm not simulating it. There are other reasons for it. They're not going extinct. But if they can't even hunt what they desire in their best case, then they are in big trouble. That's challenging for them.

But I was saying, it is better not to separate these two parameters, to have a single one that is desired prey per predator. That desired will determine both the required hunting and hunting efficiency. That's the same thing, basically, except that hunting efficiency also has the effect of prey population and efficiency.

STUDENT: [INAUDIBLE] I need 15 deer a year to survive. But at this base rate, I'm going to eat 25.

HAZHIR Yes. So that difference comes from effective prey population. So that is captured here. But otherwise, it should

RAHMANDAD: be the same. OK. [? Albert? ?]

STUDENT: [INAUDIBLE]

HAZHIR So we have the predator, prey. And let's look at a few things. So consumption rate of prey comes from

RAHMANDAD: consumption per predator. Sorry, consumption is per predator. And that comes from normal consumption per predator and effect of density on consumption. So effect of density on consumption comes from density of prey versus a reference density, and then normalized by sensitivity of consumption rate to density. So we can see this. So this is if y density equals reference-- so let's think of the reference density of prey to be maximum carrying capacity. Is that what you had in mind, [? Albert? ?]

STUDENT: No.

HAZHIR No. So what is that?

RAHMANDAD:

STUDENT: I was anchoring from the prey death rate. So there's a death rate that is if you have ample food, that's a fraction of death rate. And that implies the normal consumption rate, so the desired consumption rate. And that anchored the reference density. That is, in that density, you will have that consumption rate.

HAZHIR But this is not anchored to anything else. This is just a number reference by density. So it's just how many prey
RAHMANDAD: per unit of land you can have.

STUDENT: So in that kind of density, then the predator can have catch how many--

HAZHIR Yes, they can get, how many deer they want to get. So that's fine. But if this is not the maximum for predator
RAHMANDAD: population-- for prey population, sorry. It's not the maximum density for prey population, then this ratio of y divided by reference y --

STUDENT: Can go beyond--

HAZHIR Can go beyond 1. And then that ratio raised to this sensitivity power can go beyond 1, which means that
RAHMANDAD: consumption per predator can go above the normal consumption per predator. And that you don't want to happen.

STUDENT: I was in the similar realm with Kathy that, if you have more density, then you might go above your normal consumption.

HAZHIR Yeah, maybe. But even then it should saturate. That is, yeah maybe somehow you could argue for 20% more, but
RAHMANDAD: it shouldn't go to infinity. And this allows it to go to large numbers. And I'm not seeing yet-- we can look at it. But I'm not yet seeing how the prey population would remain below the reference density. And therefore, I suspect this ratio-- and therefore, this effect-- could exceed 1 very easily in this model.

So that would be perhaps the primary issue. Now, there are different solutions. And we discussed different ways of formulating it. But ultimately, this effect should saturate at 1. And then you multiply it by normal consumption. And you can call that maximum consumption so that when you multiply it by maximum, you get-- by this effect, you get maximum at the end. You can't go above that.

But the rest makes sense. And then you have consumption per predator. That creates a food scarcity effect compared to the normal consumption per predator. That's good. That's the same idea we discussed. And it is good to get it from this part of the formulation, rather than another parameter being defined for that. Then you have the effect of food on death rate of predator. So that effect is-- let's see. This is also-- so this is e to the power of some parameter multiplied by this food scarcity.

And for now, let's think about the active range of food scarcity to be mostly between 0 and 1, even though it could go above 1 in our formulation. We can look at the equation for food scarcity. OK, so maximum of 0 and normal consumption minus actual consumption divided by normal consumption. So if normal minus-- OK, I see. So 1 here means you don't get anything to eat. You don't get any of what you want.

STUDENT: Food scarcity of 1 means--

HAZHIR Yes, food scarcity of 1 means famine. And because of the max 0, even if y consumption goes above desired or
RAHMANDAD: normal, it doesn't go below 0. So it is robust on that part. So this works fine. So now, with that understanding, let's go back to the equation for effective-- so this number is between 0 and 1. 1 means significant scarcity. So then that significant scarcity is multiplied by some multiplier, e to the power of that. That can lead to large death rates. And then 1 minus fractional death rate of r with ample food, it should not go-- So I have a unit problem here that I'm not sure--

STUDENT: The 1 should be the max fractional death rate.

HAZHIR Ah, it should be max fractional death rate, yes.
RAHMANDAD:

STUDENT: Because we don't want it to go beyond 1.

HAZHIR Yes. So this is both a unit problem and a little bit of a conceptual issue because-- so fractional death rate can go
RAHMANDAD: above 1. That is, per unit of time, more than 100% can die. That is, if the unit of time is one year and you're going to die in two weeks, the fractional death rate could be 50 rather than 1.

So you don't need to worry about 1 being a-- and therefore, you need to explicitly put the max fractional death rate because its unit is 1 over year in this population because your, I think, time is year rather than month. So that's the source of confusion. You may come into this and you put 1 because you say, yes, death rate cannot go above 1. But it can, actually. If the unit is 1 over year, it is not a probability. It is per unit of time.

But if we had max fractional death rate divided by fractional death rate here, that becomes OK because now you're saying we switch to the regime where you are killing the deer with the maximum rate possible. This is 1 over-- it's related to their starvation period. This works. But it is a very clunky formulation because this exponential leads to things going up, and then getting hit by this other ceiling that you are imposing.

So it creates this weird non-linearity that I don't like. It's not doing what you really want to see in the real world. It doesn't happen that way. But it puts meaningful feedback from consumption of deer to the death of predators. OK, that was good to look at. And then the fractional death rate is straightforward. And then you have [? exogenous ?] [? deaths. ?] You have [? assumptions ?] for that. That's good. You have the natural death rate, I assume, fairly similar story down here for this. And then regeneration right now is exponential. OK, good.

So maybe we have time to look, rather than at the model, look at some model analysis. Anybody wants to look at their model and how its behavior is explained? Anybody who has given it some thought? I saw a couple of you had submitted some simulation analysis as well. So Ross, you had some?

STUDENT: I did, but my model was basically the mathematical model, so yeah.

HAZHIR OK, let's quickly look at your model. If it is too mathematical, I may not be able to-- too mathematical, in the
RAHMANDAD: sense that it is hiding the stories. No, this is Jake's. OK, this one is Ross. No, it is not too mathematical. It has prey and vegetation and so on. OK, let's run this.

STUDENT: [INAUDIBLE]

HAZHIR OK, good. Good practice. Everything starts from equilibrium. Great job. Is it analytical equilibrium or is it--
RAHMANDAD:

STUDENT: It is analytical.

HAZHIR OK, perfect.
RAHMANDAD:

STUDENT: [INAUDIBLE]

HAZHIR And what's the switch?
RAHMANDAD:

STUDENT: Oh, it's for deforestation shock. I specified for four years that's going to happen.

HAZHIR Oh OK, so you have a shock for deforestation, for example?

RAHMANDAD:

STUDENT: Yeah.

HAZHIR I see. So you can turn it on to see what happens if you take out some-- oh, OK. Huh.

RAHMANDAD:

STUDENT: [INAUDIBLE]

HAZHIR So you are turning on this shock for how long? For 48 months. Is that correct?

RAHMANDAD:

STUDENT: Yeah.

HAZHIR So let's bring it down.

RAHMANDAD:

STUDENT: It's a step.

HAZHIR So there is a pulse. So you can see the pulse in the fractional death rate of the vegetation. Basically, more

RAHMANDAD: vegetation is-- and perhaps, this model-- yeah, this model has a built-in oscillatory mode. So even if you bring it to a very small pulse, it is still would create something. But it is interesting.

So with the smaller magnitudes, it takes more time for the pulse to become stronger, yeah. But ultimately, I think it will get to a limit cycle. I mean, I don't know the formulation. We didn't go through your model's formulations. OK. But this is good.

Now, we need to actually understand what is happening in a model like this. And you don't have the start time for this. So one challenge is, with an oscillatory mode of behavior, where do you get it started? Because things are moving too rapidly. Have you gone through trying to understand it?

STUDENT: I guess I have rates underneath the graph. And I tried to look at more initial when the shock starts in the beginning.

HAZHIR When the shock starts, where do you have it? Oops.

RAHMANDAD:

STUDENT: Looking at the first 0 to 20 maybe time horizon.

HAZHIR Yes. So one of the good things to do is to reduce the time horizon because you don't want to see everything. And

RAHMANDAD: you can do that in the time axis part in Vensim. In fact, you get the shock around time 10. So we don't really care about before that. And by time 20, 30, 30-ish, we have the core dynamics unfolding. So we don't need to go above that. So let's make those as our-- what happened? End time is 30?

STUDENT: Yeah.

HAZHIR OK. Now, typically, Vensim would not update the graphs when we change this. But if we now look at the

RAHMANDAD: vegetation, it hopefully shows us, yes, between these two times. OK. So let's go through the causal tracing and see what is happening that creates these oscillations or at least come up with a hypothesis for it.

This is an important process. So we don't know what is generating the oscillations. How do we figure that out? So start with whichever you want. In this case, vegetation would be a good starting place. The vegetation comes from what? You can look at the causal trace graph. It comes from restoration and destruction.

Restoration depends on vegetation alone. And you know it is a reinforcing loop. It's not going to create oscillations for you. So you don't even need to go down the path of figuring out restoration. There is another signal to it as well. The destruction rate initially goes up. You see, this small jump up is what is triggering the initial decline in vegetation. So that is the driver of the behavior that gets us started with everything. So you would know I need to actually trace destruction rate and see where that comes from.

Now, as I'm thinking about that, I'm worried about this time horizon because this is too messy. I can't easily see. So let me limit the time even further. So rather than 30, let's go to 15. And maybe-- yeah, let's keep 9 and see how things go. So now, we are going to look at the drivers of destruction. And now, you can see a clear oscillatory mode.

So what is driving it? It is partly the vegetation. But that cannot be the main driver. It has to be prey population that is driving the oscillation because vegetation was coming from the destruction rate. So then we need to figure out why the prey population is going down. Then prey population is based on birth and death. So it's interesting.

STUDENT: In this formulation, the birth rate [INAUDIBLE]

HAZHIR Yeah, so I haven't looked at that. But my first reaction was that the shock here-- the discontinuity here is very

RAHMANDAD: sharp.

STUDENT: If the birth rate is--

HAZHIR But before we look at it-- because normally, you wouldn't-- you shouldn't be needing to think about what the

RAHMANDAD: formulations were as much as looking at these and deciding which of these two drivers, birth or death, is more likely to explain the prey population dynamics. That's the key question. And the prey population is going down. But the question is, which of these two is driving it down? Is it because birth is going down or death is going down? What's the answer?

STUDENT: Birth.

HAZHIR It should be birth because death going down doesn't lead to prey population going down. So death cannot be the

RAHMANDAD: driver. It should be the birth rate that is driving it. So now, we can click on that and see what are the drivers of that. OK. Prey birth rate is going down. It could be because of vegetation going down or could be because of the prey population going down.

This is a little bit more tricky at this point because, yes, if the prey population goes down, birth rate will also go down. So how do I know which one is driving it? Let me look at the valley, The, minimum in the prey birth rate. That is where my cursor is now on all of the graphs. That valley, where we have the minimum birth rate, is happening after the minimization of vegetation, but happening before the minimization of the prey population. You see that? That's a very important signal for which of these two is more likely to be driving the behavior. Which of the two?

STUDENT: Vegetation.

HAZHIR Vegetation. So that takes us where? Back to where we started. So what does that mean? What does it mean?

RAHMANDAD: What does it mean? It means we have a candidate hypothesis for what feedback loop is generating the oscillations here.

The candidate hypothesis is that more vegetation-- or less vegetation leads to less prey population, which leads to more vegetation-- less vegetation consumption, which leads to more vegetation, which reduces this. So this is a balancing feedback loop. It has two stocks on its way-- vegetation and prey population-- which means it is a higher order balancing feedback loop. So it can generate oscillations.

The first order balancing feedbacks cannot generate oscillations, but this one can. So this is good. You already have a good hypothesis. But remember, it is still a hypothesis. So then the question is, how can we disrupt this? Or how can we change this hypothesis's strength and see whether it changes the behavior? So any suggestions for how would you change the strength of this hypothesis?

STUDENT: [INAUDIBLE]

HAZHIR Anne, go ahead.

RAHMANDAD:

STUDENT: OK, cool. I think I missed the explanation for why death rate-- or rather, why birth rate and not death rate rates cannot affect prey population.

HAZHIR Sure, let's look at the prey population again. So the prey population, we looked at the causes. It can be either

RAHMANDAD: birth or death. What is it we are trying to explain? The fact that prey population starts to go down. So the fact that it starts to go down means that its birth should have been going down or its death should have been going up.

The death is not going up. So that means the only plausible explanation is that the birth was going down, which is the case. You could actually use the valley thing as well here. That is, if you go to the valley of the prey population, you see it happens after the prey birth rate, the valley value of the prey birth rate. So that is the driver of what is happening to the population. Whereas, for the death, It is happening at the same time, which is coming from the stock directly. OK. Is that clear, Anne?

STUDENT: Yes, thank you.

HAZHIR Sure. OK, so now, we need a test for our hypothesis. Our hypothesis was that the prey birthrate, prey population,

RAHMANDAD: degeneration, vegetation is the cause of the oscillations. So if that is the case, if I disrupt this feedback loop at some point in that period we are looking at, it should disrupt the oscillations.

Now, I need to disrupt that feedback loop without disrupting anything else. So that part is not trivial. One way might be to stop the prey birth rate from oscillating. If I can rewrite the prey birth rate to not oscillate, to switch to a fixed number, then I have disrupted the oscillatory mode. That's one way to do it.

Now, we could write equations to get that going. Or we could see what was the initial value for this variable. So it was around 15. So I'm going to say to do this. So I right click on it. I override the default behavior. And I would say at time-- well, if I make it a constant, and the value should be 15-- let's see if it is starting at 15. Yeah, m is at 15. So what are you seeing? All of the oscillations went away. And the vegetation is now going all the way to 0 because we brought it below its equilibrium level. And then it cannot restore itself.

That's partly a function of how these are formulated here. I haven't even looked at the equations. But now, we have a good hypothesis for what feedback loop is driving that oscillatory mode. This was the practice I wanted you to go through because this is how you identify potentially important structures that are driving the behavior you are seeing. Alexander?

STUDENT: Instead of changing the value of the birth rate itself, it seems like there's that parameter that's next to it that controls the relationship between vegetation and birth rate. Could you have set that parameter to 0 or whatever it is?

HAZHIR The problem with that is-- yes. But that parameter perhaps is not doing what we wanted it to do because that
RAHMANDAD: parameter-- my most likely guess is that it is multiplied by vegetation. So if I bring this one to 0, then my prey birth rate is going to vanish. And then I'm going to change the prey beyond its equilibrium change.

So this way, at least, I'm looking at its value being at its equilibrium value, which was 15, and seeing how that intervention impacts it. If I basically kill all the prey or don't let them be born, there are more-- it is a more disruptive intervention into the dynamics. It will achieve the same thing, in the sense of disrupting that feedback loop, but it disrupts a lot of other things. Even the disruption I introduced is too much.

So the best way to intervene is to keep this as it was. We could even do that. But basically, put an if/then/else equation here that before time, let's say, 12 it is whatever is the equation. At time 12, it switches to 15, to the value that was the equilibrium. And then you see whether the oscillations stop at that time 15-- at time 12. We could do that just so that we-- I mean, now a lot of things will change. So I'm not sure if we'll get what we need.

But this would be if/then/else time is smaller than 12, then use this formulation. Otherwise, be 15. Now, this is all bad. I'm putting parameters into equations and so on. Don't follow this practice. I'm just showing a simple piece here. And then we come here. We go there. We just give a shock. And we make the-- yeah, we don't need all of this long and laborious shock. OK, so everything is happening as we discussed, except that this might be-- prey birth rate never goes down. So why is that happening? Anybody knows?

STUDENT: When is your--

HAZHIR Oh, because at time 12, we have the shock. And I switched this at time 12. So exactly, this was not-- if I do it as
RAHMANDAD: 14, then we start to see the oscillation before this goes. So that's my guess. OK. Yes. Now, you can start to see the oscillations start to happen. But then as soon as we turn them off-- turn the switch off, at this point in time, there is no more oscillation going. So this tells you that this was a good hypothesis for explaining the oscillations.

STUDENT: Why do you need the step as opposed to just changing the parameter value as you're saying? Because I was thinking that you could just-- when you were doing it earlier, when you just changed the parameter value for birth rate to always be 15, you can just compare the two runs because that's the only thing you're changing. Why was that too much of a disruption?

HAZHIR So I like to see that the oscillations are starting, and then disrupt it and see that the disruption happens
RAHMANDAD: instantaneously. As soon as I introduce this, oscillations go away. That's a very strong signal for me. The other way, I would never see this start. So there could be other things that are happening.

Now, in this case, it's easy enough that I'm comfortable going with that, but it may not always work as well. OK. I think this is good. And basically, you folks had the core structures in place. The most common problems-- so the two most common problems-- first was these outflows from the prey or from the vegetation not being constrained correctly. That is, you let things go out too much, even if you don't have predators, for example, with the multiplicative formulations that were very common.

The second issue was the death rates that are due to starvation. We wanted robust formulations for those. And some of them were not very well formulated. And the third thing is to make sure they are not getting their signals from different parts of the model. That is, you don't create new parameters that are overlapping.

We also learned these constrained formulations for removing things out of stocks that does not overwhelm the stock. This is something you have seen before. It is in the molecules as well. If you look at the molecules, you will see that as well. OK. Yeah, so we talked about these.

Yeah, I think that's good for the predator prey. Let's spend the last few minutes on the next week's assignment. So next week, you are working on a model of drug development pipeline. So drug development has multiple stages. You have coming up with compounds. Then you have animal trials and laboratory tests. Then you get into phase I trials. And then you get to phase II trials and phase III trials. And then you launch it into the marketplace.

So if you want to make some amount of money, you have to manage this pipeline from a decade or more before to have the revenue today. This is also very stochastic. For our purposes, don't worry about the stochasticity. That's too much to capture in your model. Focus on a deterministic version that basically says each stage has a failure rate.

These failure rates are actually pretty high, typically. So across the three phases of trials, total failure rate is in the order of 90% or more. But it is what it is. So with that, your goal is to formulate the start rate for phase I for entering into phase I. So think of those previous steps as easy to manage. So the company can decide to have 100 new drugs per year this year started into phase I or five or whatever is the number, so that they can get the revenue that they want to have a few years down the line.

So basically, they need to have some revenue projection. And that revenue projection, think of it as a exponential curve. That is, we want our current revenue, but growing at a 5% rate per year. We want our revenue to be growing 5% per year. How do we start the initial entry initial start of the phase I so that we get the desired revenue trajectory in the long term?

And this is a stock management problem. That is, if you have too many items also in the pipeline, you don't want to get started too much. And it should be robust to any percentage. So if rather than 5% you go to 0%, it's a steady state. That should work fine. Even if you have a declining rate-- that is, I want to actually shrink my operation and close shop in 10 years or 20 years or whatever-- that should also work. So that's your marching order for this week.

STUDENT: Resource constraints, should we think about those?

HAZHIR So no, unless you're really pushing yourself. So right now, no resource constraints on the start of the projects. In
RAHMANDAD: reality, there is actually a very serious optimization problem because, as you increase the number of drugs that you start, the likelihood of success actually comes down because you are pushing in lower probability items. So a good model would actually capture that. And then you have to decide how much cost there is versus how much expected benefit given this reduction in the likelihood of success. Hassan? Go ahead, Anne.

STUDENT: So how does the company get the revenue at the end of the number of drugs that make it to the last stage?

HAZHIR Yes. Then they go into the market. They stay on the market for a while. And then they come out of the market
RAHMANDAD: because, basically-- they don't come out of the market. Their patents expire. Generics come in. You stop making money from them. Now, you don't fully stop making money out of them. But for first order approximation, let's assume, once the patents expire, you are not going to make any more money on that drug. Now, reality is a little bit more complex.

STUDENT: [INAUDIBLE] the clinical trials system has threshold dynamics embedded to the last stage of [INAUDIBLE]

HAZHIR No. So the threshold dynamics in the sense that the availability of other drugs on the market impacts the
RAHMANDAD: acceptance rate or what?

STUDENT: Yeah, so for it to get FDA approved, it has to outperform whatever's on the market in some sense.

HAZHIR Yeah, exactly, so the availability of other drugs. No, don't worry about that. That's too much to do in this model.
RAHMANDAD: So this is really the stock management, but with failure rate and with growing demand and how you incorporate those into your modeling. So as such, it is not very different from your stock management in assignment 6. But it has these additional pieces.

So in light of the time, we have, let's just think about-- basically, you should start from what you already know for stock management, but then incorporate failure rates. Make sure you are incorporating adjustments for different stocks. So there are desired levels for each of these stocks in the work in progress. That is, you have phase I, phase II, phase III. These are your work in progress equivalents for the inventory management model that you need to adjust.

And then when you get the full model in place, you want it to be able to match your desired revenue over the longer term. You want it to also get there relatively quickly. So if it is starting from a point that it is not matching the desired trend, it can quickly move towards that desired trend and not overshoot or oscillate in a weird way when you're exposing it to a step function, a step increase in desired revenue, for example. So those are the key consideration sets for a good model. Questions?

STUDENT: Failure rates can happen at any stage?

HAZHIR Yes, failure rates at any stage. Put a parameter for the fraction that get failed. Use first order delays. Don't worry
RAHMANDAD: about the fact that these clinical trials take time and are not really first order. It is OK for our purposes to go with first order delays.

STUDENT: [INAUDIBLE]

HAZHIR No, they also expire. So there is a life cycle, yeah.

RAHMANDAD:

STUDENT: But they don't have a failure rate [INAUDIBLE]

HAZHIR No, they don't have a separate failure. So for now, assume each drug on the market creates some amount of
RAHMANDAD: revenue per year. And that's all you care about. Jason?

STUDENT: Should we assume constant failure rates [INAUDIBLE]

HAZHIR Constant for now. So don't worry about that. Good? OK.

RAHMANDAD: