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[CLICKING]

HAZHIR So today, we are working on endogenous diffusion model. So this model came from basically Jack Homer's work.

RAHMANDAD: And I was interested in the diffusion space when I was doing my PhD. And towards the end of it, he shared ideas for potential thesis areas or research areas.

And then later on, we had a follow-up. And he shared some very small, nice model that captured his view of important dynamics. And I turned that model basically into a problem definition. So here, you are seeing a problem definition that captures what Jack's model had-- a very, very simple model. So I'll ask him if he's OK. I'll share that model with you as well.

So basically, you have now worked on it. So everybody knows the problem. It's an adoption of a product that itself is evolving over time with addition of new features, but also discovery of bugs and problems. And Jack's thesis was on pacemakers. It's a life-saving technology for people who have heart conditions. And it has evolved a lot from something that was for a very niche market of people who have very serious heart problems to now capturing a much wider range of conditions and providing incremental benefits for people who otherwise could live pretty fine without it as well, at least. So that's the expansion of the scope of use.

And then in his work, there he also identified impacts of identifying problems and assessing quality through research. And so these things come in that are promising. People may start adopting these. And this is not-- it's a little bit of different from drugs. It's not clinical trials that drive ability to market your product. But you need to show safety and show effectiveness in the longer term. And if there are problems, you need to get them discovered.

Anyway, so research becomes an important source of information in the pacemaker example. More broadly, basically, you discover problems. And those discoveries are typically done by the customers, many of them. In many other markets, they just find the product to be useless or not, as effective as it was sold for. It becomes a source of-- negative word of mouth potentially may bring down the scope of uses that people would have.

So that tug of war between expanding the scope and use cases and problems that are discovered that may need to be fixed, and then once the problems are fixed, those problems may go away. That is, the features become more useful. But some damage is already done. And this is the big picture. So let's see what challenges you have faced as you have worked on this problem. So what are some of the common challenges that you have experienced? Anybody want to share?

AUDIENCE: I think I wasn't sure how to model the bugs having a problem, whether it should be a flow of people with buggy pacemakers or some kind of enduring story from the adopters who did ever have bugs. I went with the latter. But I wasn't very confident in it.

AUDIENCE: For me, it was a little bit hard to think about the relation between the scope of use and the technology advancement, so these two sides. First of all, [INAUDIBLE] technology advancement [INAUDIBLE].

HAZHIR Very good. That's also an important question. Others?

RAHMANDAD:

AUDIENCE: I started this one too late, but, on previous ones, I would formulate each relationship incrementally. And for this one, that strategy didn't work. I had to lay out the big picture, and then go back and do the formulations in detail, which I didn't complete. But I think it draws attention to this question of incremental relationship formulation versus laying out a picture so that you have an idea of how the whole thing should work, and then going back and putting the relationship [INAUDIBLE].

HAZHIR That's a very good question. So we should have a chat on that. Other observations, challenges that you faced
RAHMANDAD: with this model?

AUDIENCE: [INAUDIBLE] units were a little bit more complex than this one, but [INAUDIBLE] didn't work as [INAUDIBLE].

HAZHIR Interesting. Good. So I think these are good items to look at. And as you see, this one was even more loosely
RAHMANDAD: defined. And that's by design. As we move forward, I'm trying to make the problems a little bit harder by making them less well-defined, so that you can't just say, OK, I know this is this chunk, this is this chunk, this is this chunk. And the next one would also be somewhat similar in that you need to decide for yourself what chunks of formulation are going in.

And that goes also partly to Jason's question. What's the best approach to modeling, in terms of having a big picture in place before you start with chunks or you go more incremental? And there are two parts in answering that. One is that the general iterative process for modeling means you do want to iterate. And you don't need to think about all the things that will be, in future, coming into your model's boundary. You start with what you can manage.

But within any of those iterations-- and usually, the first iteration is the big part, the bigger challenge. In that first iteration, I would say you definitely need to have a big picture view of the model that, OK, this is the sector and features and scope. And the way I would think about it is x, y, z.

So I would have features as a stock that is being developed. And I may have a coflow for bugs or-- so have it in mind before touching your Vensim and have that, OK, I think I know how to manage that. Then there is the question of impact of features on the scope of use and the number of customers. Do I want to just have an analytical function there? Yeah, maybe that's fine.

And then the demand side-- do I use a Bass diffusion model? Or do I set a feasible scope we're interested in, population, and just adjust towards that and make the time constant? So I have those in the back of my mind when I'm starting with the structure, I may take quite a bit of time deciding on that.

For the city dynamics model, I remember, for a few days, I was just walking and thinking, OK, what are the chunks that would work together and try to make reasonable decisions on those before I start working? Because as soon as you start building the model, it becomes too concrete to deviate. And therefore, you don't easily change bad decisions that you have made. And you start to get attached to bad decisions. And then they are inconsistent with other parts. And it just becomes messy. And you get locked in.

Yeah, so that's the approach. Now, in the previous assignments, to a large extent, I had simplified the problem enough that you could read it and you would have the image of all of those chunks in place. So you didn't consciously need to go through this process that you had to do this time. And you would probably have to do some version of it next time. So this is exactly the process.

Now, at that stage, you may also say, OK, I'm going to ignore this dynamic x or dynamic y. That is, OK, this is my first iteration. I'm not going to worry about measuring profitability and how it impacts feature introduction. This is beyond the scope. I'm OK with that. But I'm conscious of it.

It's important so that, if somebody asks, you say, yes, you're right. That's an important feature. That's not part of this version of the model-- next iteration or five iterations from now. In a real project, I would have a list of features to be added. I have one right now for my city dynamics model. It has-- I don't know-- about 100 rows with different features. And about 70 of them are incorporated already. But there are 30 more to come, even within the current sectors.

And this is beyond adding new sectors. So it doesn't have the transportation sector being added and so on. It still has quite a bit. So that's the big picture of it. So let's get started with your models. So while we are opening those, these are things that I was thinking about, what I'm looking for. So for the demand sector, and then the feature development sector, well, you may take a look at this. I'll just bring up the models that we have.

OK. So let's take a quick look at some of the structures we have. This is Ross's. And let me increase the size. So just quick review of the stock inflow structures-- so Ross has total population, potential users, current users. There is an adoption process.

Good. And he's thinking about potential users, those who may find this relevant. And that comes from features. And then those may adopt and become actual users. So this does make sense. I like this breaking down as one way of where the features operate and where utility might be operating.

There are issues about one-way flow of adopters, users. But we can talk about those. Then we have feature development creating side effects. These side effects are unknown, then become known, and then they are fixed. That's also a nice way of doing it. Good.

OK. So let's look at a few more. So this is Alban's. We have adopters here coming from basically some total population of sorts. And then we have feature expansion that sets the scope for indicated adopters. Then we have the bulk generation with feature generation, undiscovered, and discovered, very similar to Ross's model. Utility is coming from-- there is some utility function that is interesting.

It is kind of a coflow of current scope and-- so it's something I have to look into more details to understand what exactly is happening. And then perceived utility informs what fraction of customers might be interested. So this is not breaking down the adopters into three sectors, but into two sectors. That also works.

OK. This one is from Sercin. So we have adopters and their perceived utility, similar idea as Alban had, though not exactly as a coflow. And the adopters are coming from total population and what fraction may find the product to be useful. And this time to adopt is adjusted based on utility.

Scope is basically based on investment in R&D. And usefulness of product is an interesting concept. So you are trying to capture something about the marginal utility of the product. That is, as we increase the scope, each increment becomes less important. So that's an important and interesting feature that we didn't see in other parts. And then we have unrecognized problems, recognized problems not fixed, and then those that are solved. Great. So I think those are fairly consistent with the previous ones we saw.

This is Hesam's. So similar to Ross, we have potential adopters, adopters in scope, and current adopters, unknown bugs and known bugs, features. OK. So this is also fairly similar to Ross's big picture of components. Here, we have Gerardo's model with adopters and potential adopters. And this is based on the scope, I guess. This is a little bit more Bass diffusion-y type. And then available scope, this is basically the features. This is adding to the utility for customers.

Then we have unknown issues, discovered issues, and fixed issues. This is an interesting thing to have a stock here to calculate some things. I'm sometimes skeptical when I see this because this one will be ultimately accumulating everything. So whether people care about that in calculating anything is not clear to me. But we should look at those equations later.

Is this the one that I was just looking at? Yes. This is Jason's. So we have adopters and non-adopters, so two underlying stocks there-- discovered bugs, undiscovered bugs, similar to the previous ones. This is product capabilities, features that come. And then we have utility coming. And this is perception of utility, the delaying the perception of utility that impacts scope. OK.

And then is this what we just went through. So this is Cathy's. So we have features. We have bugs. There is adopters and addressable market, which could expand based on population here. So this is more population dynamics not the scope impacting that.

Then we have adoption, dynamics impacting what fraction of bugs are discovered or something about discovery of bugs. This is actually about customers that have problems and negative perceptions that come with it. OK, interesting. So this is a little bit of a different formulation we can check out here in Cathy's model.

Next one, Ali's. So we have issues, known and unknown, fixing time, if the issues that may depend on some additional structure here for desired time to fix things. But all of this looks like just a number at the end to me. So there is no feedback into this structure.

AUDIENCE: [INAUDIBLE]

HAZHIR Yeah. Maximum scope of use is coming from current scope of use. So there is some adjustment. There is a scope
RAHMANDAD: of use. And this maximum is also exogenous.

AUDIENCE: [INAUDIBLE]

HAZHIR OK, that's fine.
RAHMANDAD:

AUDIENCE: [INAUDIBLE] trade-off between the capability to [INAUDIBLE] scope [INAUDIBLE].

HAZHIR OK. Capabilities of timing, what do you mean?
RAHMANDAD:

AUDIENCE: The capability of investment in fixing the bugs [INAUDIBLE].

HAZHIR I see, I see. Interesting, OK.

RAHMANDAD:

AUDIENCE: [INAUDIBLE]

HAZHIR Yeah, that's fine. I think this is good. You had the big picture. You had these components in your mind. But then

RAHMANDAD: time-wise, you didn't get to fix all the bugs, so that's OK. Scope of use impacts the adoption. And we have here the two-stock version of the adoption. And yeah, so this is also fairly similar in the bug fixing.

Let's see. This is Jake's. So we have three-stock version of the adoption process. We have the two-stock version of the bug discovery. We have utility that is coming from capabilities or capability development rate. So this might be closer to-- so this is closer to the coflow idea. And then perceived utility is impacting something down here I'm sure.

OK, good. So I think it's actually interesting. It's fairly consistent across the models that we have. So we have two or three stock versions of the adoption chain. And then we have typically two-stock version of bugs and bugs that need to be fixed and one-stock version for features. And then we have variants for other things. So some folks have utility explicitly. Some just capture it within the other pieces of the model.

So let's dive into some of them. We could start with Jake's here. So first, on the demand side-- so I think the more standard formulation is the demand side. So population out of scope of use, they become addressable based on total utility. We should get back to see what this is. This total utility, how does it turn into the--

AUDIENCE: Utility, it's going to be like 0 to 1.

HAZHIR OK. So utility is bounded between 0 to 1. 1 means everybody who potentially could ever use pacemaker is now

RAHMANDAD: within the scope. That is, there is a pacemaker that could be useful for them. And then as you get this ratio closer to-- and then yeah, you want to bring down this ratio to 1 minus it. Yes, that makes sense.

And then this is, over time, these people move to the right-hand side. And if we have a utility that goes down-- yes, it would get negative. It should get negative. Now, this one has a max 0. So this is an interesting question here. Should it get negative or not?

So if the utility goes down-- now, maybe in your formulation, it never goes down. So you wouldn't have the use case where this is negative. But conceptually, I could see a situation where we have expanded the scope because we thought this is useful. And then later on, we realize, actually, there is so much side effects and problems that this shouldn't be in scope. And therefore, the addressable population should go down. So there would perhaps be a negative flow backwards.

And in that case, it's conceivable that just having this flow not maxed out by 0 be OK. It may work fine. There is a little bit of a tricky business here that usually, when you have side effects, people decide, oh, this thing is out of scope. The movement backwards is very fast. The whole community would know about it. This is a scandal that has been behind it that has led to that point.

So the time constant for movement to the right versus to the left would be potentially asymmetric. But having that in mind, if you're aware of that, then it's OK. I'm OK with this formulation here. And we have indeed looked into the utility. Then once you are a potential adopter, you have the word of mouth formulation here. And you have a different formulation based on adoption fraction and adoption gap.

AUDIENCE: So yeah, I mean what you basically just said is being [INAUDIBLE]. There is a literal utility, but you're adding scope that is going to [INAUDIBLE] from population [INAUDIBLE] use cases to potential adopters. And then, they consider themselves part of scope with the perceived utility versus adoptive [INAUDIBLE], actual [? adopters. ?]

HAZHIR So what I'm saying here, there is the adoption gap, which is-- and the units here start to become tricky because
RAHMANDAD: adopters are typically in the units of people or something. Let's see. So this adoption gap, this is fine.

AUDIENCE: [INAUDIBLE] perceived adopters [INAUDIBLE].

HAZHIR So what this formulation does for you is that it says, what is the gap between fraction that have adopted, this
RAHMANDAD: guy, and fraction that should be here, but may not still be here.

AUDIENCE: [INAUDIBLE]

HAZHIR Or thinks that they should be here. And if that number is positive, it allows the flow to continue. When it goes to
RAHMANDAD: 0, it goes to 0. It shuts off the flow. That's the idea. Now, the word of mouth formulation has contact rate. Word of mouth adoption effectiveness, potential adopters, adopters total population. So this is the standard base model. But you have added the max zero thing. So where did these max zeros come from?

AUDIENCE: Well, I mean, maybe I was just sticking too close to the idea of [INAUDIBLE]. I didn't know if you can remove the pacemaker, but [INAUDIBLE].

HAZHIR That's fine. But this formulation should never go negative. So why do you need a max zero?
RAHMANDAD:

AUDIENCE: It's probably at some stage in development when I was like, why is this graph going negative?

HAZHIR OK. So this is actually a very good process point. So if, in your modeling, you see something going negative that
RAHMANDAD: it shouldn't, that is actually a very good sign to trace and figure out why it is happening. Putting a max zero there will basically wipe the problem, but wouldn't solve the problem. So the problem is now happening, but you don't see it.

AUDIENCE: But if you run it before and after, you see what fixed it.

HAZHIR Yes, but it really hasn't fixed it because you haven't understood what led to the negative flows here. So basically,
RAHMANDAD: negative flows here mean your potential adopters have gone negative. That's my guess. And my guess for that is this perceived adoption gap is doing something a little bit problematic.

AUDIENCE: I think I ultimately found it and fixed it.

HAZHIR It might already be fixed. So that's exactly the point that you wouldn't know from this signal. So don't delete the
RAHMANDAD: signal. Signals are good, even if it means there is a problem with your formulation. OK. So the second thing is that you're kind of double-counting an important concept.

So Bass diffusion model has this limit of, if I don't have potential adopters, I wouldn't have any adoption. So that's already there. Now, you have added this additional perceived adoption gap mechanism that says, basically, do I have enough potential adopters? If not, I'll shrink adoption rate further.

So this is kind of double-counting the same mechanism. And as such, I wouldn't have that. I would probably go with one of these two pathways. That is, either you would have a perceived adoption gap-- not in the dimensionless view of the world, but in the difference between the current adopters and what they should be, which is the potential adopters plus adopters.

So the gap is the potential adopters, basically. So it would be potential adopters divided by a time constant. That time constant says how fast they move. And that movement, you could say, depends on how many adopters you have. If you have a lot adopters, there is word of mouth, people talk about it. I can change the time constant. So that would be a simple alternative formulation here.

OK, great. So this is the adoption side. Now, let's go and understand what the supply side of features come in. So you have an R&D process that generates technological capabilities. This R&D process has utility. So it is kind of endogenous. That is, you have a max effort. And then you have an incentive factor that depends on--

AUDIENCE: How much [INAUDIBLE] you're actually going to get.

HAZHIR Yes, so the utility that you are getting per new development. So this is an important, interesting mechanism.

RAHMANDAD: That is, it's trying to endogenize the creation of new features. Now, the measurement issues, when I see this, is I don't know how it has been captured. So I'm interested to see how the utility per new development feature is incorporated. That's the core. Marginal utility of a new feature is key to the decision to whether to invest into R&D or not.

So as such, conceptually, it is a good concept here. Now, how it is formulated, we have to see. But before we go further to see that, it is R&D incentive multiplied by max R&D effort. And max R&D effort is the units of capabilities per unit of time. And this is, I assume, dimensionless. It's the ratio of utility per new development, max utility gain per development.

That's OK. I think this is introducing a couple of parameters that are left out. And as such, some people may be hesitant to have as many free parameters. But I'm OK with it for our purposes because we are not doing serious cost-benefit analysis. So it's a good first order approximation. The question is, is this utility per new development? You have a question?

AUDIENCE: What if you have [INAUDIBLE]?

HAZHIR For technological capabilities? If it is not used anywhere, we don't need it. But it should be used somewhere

RAHMANDAD: because this is what is really determining the current scope of use. Now, we'll see where it is being used. It is not being used.

AUDIENCE: Yeah, right now, it's not. I did try to make it depend on that rather than the flow [INAUDIBLE], I thought it easier to make the scope [INAUDIBLE].

HAZHIR OK. So this is important, I think. If you are capturing a stock that is capturing the key inertial feature here, which
RAHMANDAD: is the-- let's call it features or capabilities-- then it should be used somewhere. Now, probably the way you're doing it is that this guy is going to some other places as well, yes.

So it is not only going to flog, but also it's going to utility growth rate. So basically, your utility growth rate is capturing this inertia. But you also have this decreasing return built into your utility growth rate. So you can think of the total utility to be a coflow of the technological capabilities.

But that coflow makes the incremental utility smaller and smaller as you go to higher levels of current utility or current features. That's the idea that Jake is capturing. It's a good idea. That is, basically, each increment of a technology is not as valuable. And then you can calculate-- OK. So one part to consider is whether this feedback on-- so utility of new development is max utility e to the power of utility drop-off sensitivity by total utility.

So basically, you have a declining function. As you build more utility, fewer incremental utilities are added. So that creates an exponential curve that converges for utility as you add more features. That's, I think, the idea that Jake is capturing. It works. I think the basic idea is reasonable. And it should work fine. Then this utility per development is the same thing. So this is exactly-- no, sorry. Utility per-- yes, this is exactly the same thing. So this is actually the increment of utility that is added.

AUDIENCE: Jake, you mentioned that-- I think you mentioned [INAUDIBLE] utility is found between 0 and 1.

HAZHIR And because max-- there is-- so for that to be bounded between 0 and 1, how do you enforce that?
RAHMANDAD:

AUDIENCE: [INAUDIBLE] exactly, [INAUDIBLE] utility for [INAUDIBLE].

HAZHIR Ah, OK. If it goes above 1, you kill the inflow. So this is not a good way to normalize it, to get--
RAHMANDAD:

AUDIENCE: I did it another way at first. And it never actually converged to 1. And I was like, well, at some point, it's going to stop.

HAZHIR But this creates a discontinuity. Basically, you have positive marginal utility at one day. And the next day, it goes
RAHMANDAD: to 0, whereas the actual process is much more like the underlying slow convergence. So it's an interesting question.

So you might be able to find an analytical solution for what would be the maximum-- the total utility at the end of time using this formulation that you have, as this is basically a first order system, assuming that you get max utility-- you get, sorry, capability development rate to be at its maximum-- that is, always maximum-- even though there is a feedback here that slows it down.

Even if you assume it is at its maximum, what would be the total utility? What would it converge to? There should be some analytical solution for that. And if that is the case, then you can use that as your maximum. You divide this stock by that maximum. So that's one way to normalize it to 1.

AUDIENCE: That's close to [INAUDIBLE]. Yeah, I just, in my head, was like I wanted it to actually hit 1, where they have a responsibility to fix the bugs that are still in there.

HAZHIR And the bugs are not yet showing up here, yeah.

RAHMANDAD:

AUDIENCE: Make it the whole product.

HAZHIR Yeah, OK. So then you have perception of utility that starts with the total utility. And then it looks at bugs being fixed. So here, we have a little bit of a unit problem in the way I'm looking at it, based on the fact that bugs may be corresponding to these units of technological capabilities. But they do not directly correspond to units of utility.

And therefore, this comparison here becomes harder. So you have total utility minus sensitivity to bugs to bugs being fixed. So you have this added parameter of sensitivity to bugs to fix the units. But then you probably have observed that total utility could be much smaller than the bugs being fixed.

And therefore, if you just had total utility minus sensitivity to bugs multiplied by bugs being fixed, that could go negative. And then that messes up everything. So then you have added max zero and the other thing. And that approach is not right. That's one of those use cases of max zero that you don't want to have.

Basically, you should say, OK, what is the concept that is comparable to bugs? It is my technological capabilities. So with respect to the current technological capabilities, how many bugs do I have that are being fixed that are known? That becomes a ratio-- if you keep the units of bugs to be in features-- so buggy features, think of them-- so technological capability-- buggy features divided by technological capabilities gives you a good rate for what the effective features are. And those effective features could be driving what is the current perception of utility or perceived utility would be.

Now, the problem there is that you are capturing utility with a stock and, therefore, with an accumulation process. So now, if I shift the total features, I can't directly go and see what is the corresponding utility. And that makes the complication. So that is one place where I don't necessarily go with the stock version of total utility.

I would go with a simple function that says, as you increase technological capabilities or effective technological capabilities based on what fractions are bugs, you are going to have more utility, but with decreasing returns. So that would be a simple decreasing return formulation for-- and you could use logarithm.

You could use something to the power of a small number smaller than 1. Something like that would do the trick. So that's the trade-off that would-- but once you have put the stock of total utility, I see how everything else goes in this direction. That creates a little bit of a dead end at this point inside the model.

OK. I think, conceptually, it's all in the right space. But this creates some challenges that you have probably faced. I like the idea of getting some measure of incremental value of utility for the decision to expand technological capabilities. And if you were going with this alternative that I was saying, then you actually needed to capture that marginal utility in the curve of technology to utility.

And that marginal utility is-- you probably needed it either analytically or some simulation-based way of calculating it. I would go with an analytical solution. That is, if I have features to the power of alpha, then marginal utility is just the derivative of this thing with respect to features. So that becomes our marginal utility.

OK. This is nice. Let's go to some other models. Which one is this? This is Jason's.

AUDIENCE: Just warning-- about 50% of the formulations are not [INAUDIBLE].

HAZHIR OK, so there are open things here.

RAHMANDAD:

AUDIENCE: A lot. But the hardest ones.

HAZHIR OK. Then why don't we focus on things that you want us to focus on, if there are places that you think are-- you

RAHMANDAD: were struggling. Let's look at those.

AUDIENCE: Maybe to continue with the last conversation, I guess, scope and utility.

HAZHIR Yeah.

RAHMANDAD:

AUDIENCE: I like the idea of having scope move people into the addressable market, and then utility determining what fraction of that addressable market is realized. That was the idea I wanted to capture. I didn't get there.

HAZHIR OK. So let me see. So there are really two different ideas that I think are happening in different models. One is

RAHMANDAD: that scope and features determine the addressable market. And then bugs determine quality within that space and the movement of people between the two different-- I don't think that's what you're doing here because, if you were doing it that way, then the non-adopter and adopter, you needed another stock.

You needed the stock of-- so I think what Ross was doing was closer to that idea. We should get to other models to see exactly how it is done. But right now, here, you have indicated fractions from this idea of perceived utility, which is coming from indicated utility--

AUDIENCE: Going from [INAUDIBLE].

HAZHIR So yes. And then you have addressable market captured here. So you're doing two operations at the same time.

RAHMANDAD: This is OK. This is also similar to what Jake was doing. I'm not saying that is wrong. But there are two different ways to do it. One is exactly this way, that you combine these effects and just have the indicated adoption from both of them. Or you separate the two. One is operating on the first gate of getting people into the addressable market. And then the second one works on the gate of moving people into adopting--

AUDIENCE: It's clearer to make the second option [INAUDIBLE].

HAZHIR Yeah, so these two are two different options. I don't have a strong preference for one of them. I think the second

RAHMANDAD: one is a little bit cleaner, conceptually easier to follow. And so maybe I would go with that if I was building the model. But I haven't thought through all of the trade-offs here. So we should see some of those in practice. Anything else?

AUDIENCE: No, that was the main, I think, conceptual hangup. It's like, yeah, I'm trying to think. Oh, for the bug accumulation in discovery, so I have adoption as a coflow into undiscovered bug accumulation. You can see the [INAUDIBLE].

HAZHIR Yes. So this is tricky because what you're saying here is that the units of bugs conceptually is dependent on how

RAHMANDAD: many people are there. So if you have 100 people, you may discover one bug per month. But if you have 1,000, you may discover \$10 per month.

And conceptually, however, these bugs are unique bugs, that you want them to be unique bugs. Whereas, the version that you are highlighting this way, they are double-counting and triple-counting. If you and I both have the same bug, both of us add to this undiscovered bugs stock. Is that a correct understanding?

AUDIENCE: [INAUDIBLE] it was a correct understanding [? because-- ?]

HAZHIR Now, if you do it-- as soon as you do it that way, things get messed up because now there is no comparative
RAHMANDAD: stock of bug-free features in the world. Now, you could create that. You could basically count the number of features that you have already sold by number of features in the product multiplied by the number of people who are buying the product.

But that's way too convoluted. And you're not getting much out of it because then you're dividing it to get-- or maybe you would need to divide it, at some point, to get what is the fraction of buggy features. So just focus on features rather than-- but why did you have that?

AUDIENCE: Because the thinking was there's a difference between a fluke, a freak accident and a legitimate bug. And you need volume to amount to it being a real bug and not just a mistake.

HAZHIR OK, that's-- so to capture that, essentially, what you have is that, first of all, it is undiscovered bugs. So if it is
RAHMANDAD: undiscovered, it is not yet even known. So these are bugs that are there in the product, but we don't know about it. So that part, as soon as it is developed poorly and released into the market with those bugs, it goes into this stock. So this should be a coflow with the feature development. So that's very clear.

Now, once it is a bug that is undiscovered, there is a discovery process. And the time to discover could very much be dynamically related to your adoption rate or your recent adopters or something along those lines. I would probably just anchor it to adoption rate. But recent adoption rate might be even a better one because recent adopters are observing, experiencing those challenges. That reduces the time to discover bugs. So that would be the right way of doing it.

AUDIENCE: This is helpful.

HAZHIR Sure. OK. So any questions on Jason's? OK. Actually, let me just see something else here. So here, you have
RAHMANDAD: product capabilities increasing based on potential capability. And potential capability is coming from non-adopters. So this was just the idea not yet--

AUDIENCE: [INAUDIBLE]

HAZHIR So I think what Jake had, which was something about the marginal value of new features, is a pretty smart way of
RAHMANDAD: thinking about these that economists would like. Now, whether companies are exactly tracking marginal utility-- these days, yes, by and large, the whole product development process is organized around, OK, let's assess the value of each new feature for customers and decide, cost-benefit-wise, does it have the ROI? So I think it is a decent approximation of what even happens in the real world. So we should follow something along those lines for adding new features.

But it is also tricky because the utility on the eye of the company is not just this concept utility that I said, OK, take the features, raise it to power of some alpha, and that becomes utility. What do you do with that final utility? You now take it back into the customer sector. And you would say, OK, how many customers would I get with that marginal utility?

You need to calculate that before you can-- and then what is the profitability I would expect from getting those additional customers so that you get to something about the cost benefits? So there is more that goes into a very serious, robust formulation there. But the core is the idea of marginal utility. OK. Let's--

AUDIENCE: [INAUDIBLE] better to include the decision-making process [INAUDIBLE].

HAZHIR
RAHMANDAD: So what do you mean by including in the decision making process?

AUDIENCE: Of the [INAUDIBLE].

HAZHIR
RAHMANDAD: In the company. Yeah, but how do you want it to be represented?

AUDIENCE: Yeah, like I was thinking [INAUDIBLE], how should we assess if adding some feature was basically bringing us customers or profit or whatever. [INAUDIBLE]. I don't know. But another thing I was thinking [INAUDIBLE] having limited resources and having synthetic problems or developing features. Then you have to decide [INAUDIBLE] was helping in that decision-making. [INAUDIBLE] having this discussion separately [INAUDIBLE].

HAZHIR
RAHMANDAD: We should talk separately?

AUDIENCE: [INAUDIBLE]

HAZHIR
RAHMANDAD: But you have the discovered bugs that are accumulated here. And then once they are fixed, they are in the fixed bugs. Which one are you saying?

AUDIENCE: I was saying I was going to make that decision. [INAUDIBLE] this number as well, which is [INAUDIBLE].

HAZHIR
RAHMANDAD: Yeah, so the resource allocation problem is a real one. So that is, basically, do I invest in fixing bugs or do I invest in developing new features/ And that is not always obvious that I should always spend more on bug fixing or prioritize bug fixing because there are some bugs that are not that important.

So in a sense, it is-- but then bugs that are not so important, maybe we could ignore them, in a sense. Defining bugs as things that are high enough a priority that, once you have discovered them, it is better to fix them first. That would be my option for a simpler model without going through too much details there because the alternative is to think about the marginal utility of fixing a bug and to capture in that marginal utility of fixing the bug, or marginal profit from fixing a bug, you have to trace the impact of that bug on your sales, on losing customers who are now, because of the bug, disappointed and are moving into, OK, I'm not going-- even though I'm within the scope, I'm not going to do this. And that's too much work. And it becomes messy, messy.

AUDIENCE: Yeah, I was thinking [INAUDIBLE].

HAZHIR Yeah, so I think the first order solution is to actually think of bugs as bad enough that you should fix them. It is,

RAHMANDAD: cost-benefit-wise, more important than the new features to be added. First, fix those bugs. And once you have defined it with that boundary condition, then the resource allocation becomes pretty straightforward.

OK, you have a bunch of bugs. You have a desired bug fixing rate. That captures a subset of your R&D resources. The remainder can be allocated to building your new features, but provided that those new features are worth it. And that becomes the--

AUDIENCE: Also, [INAUDIBLE].

HAZHIR So adding of a new product feature, we went through it. It's more driven by the marginal benefit of the new

RAHMANDAD: feature. I mean, that would be one formulation-- that is, the marginal benefit of a new feature, what is it. And calculating that could be also tricky. We talked about an analytical function from feature to utility. Then you have an analytical function from utility to number of customers who are adopting. Then these two marginals multiplied by each other give you the total marginal that you would get.

OK. So let's look at this one. It is Hesam's. So we have the three-piece customer sector with potential adopters. And Hesam doesn't have unit errors, even though he didn't like the units here.

AUDIENCE: [INAUDIBLE]. I started with the two-stock thing. Then I moved to three because the expansion of scope was a goal-seeking behavior. And then that created a jump at the beginning in the adoption rate, if we weren't starting at equilibrium. And then I didn't like that. So I moved to three.

HAZHIR Sounds good. So that's one reason to do it. Now, that one you could manage in other ways. So I think the better

RAHMANDAD: reason-- not the better reason, but structurally more problematic part, more challenging part exactly, is related to these marginals. And if you combine the two effects of bugs and features, it becomes too complex to trace the marginals. So separating them might be a little bit easier conceptually.

AUDIENCE: [INAUDIBLE]

HAZHIR Yeah, no, but that's fine. OK. So the scope is based on indicated potential adopters. That comes from total

RAHMANDAD: population and the scope of use. Scope of use comes from features.

AUDIENCE: S-shape of the features [INAUDIBLE]. So additional features don't get you [INAUDIBLE].

HAZHIR That's very interesting. So you're using an S-shaped, which is fine. In fact, there is an argument for going with an

RAHMANDAD: S-shaped because, at the beginning, if you have no features, it's a pretty useless thing. Even the first feature is still not very useful. So there is a minimum viable feature.

So if you are interested in a startup-- that is, at the very beginning of the growth of the market as well-- then having the S-shape makes sense. Now, for once you have a minimum viable product, then you really are looking at the decreasing return part of the scope. So that part is not as much. So I'm fine with it.

Now, when I'm thinking about the marginal parts, as soon as you have an S-shape, I'm saying, OK, do we have analytical solutions for the S-shape you have chosen? Probably, yes. If this is a logistic curve, we have an analytical solution. It's easy to use the marginal thinking in that phase as well.

The problem, though, is that the marginal thinking doesn't work for the introduction of features at the very beginning because, at the beginning, every startup, in expectation, is going to be wiped out. And therefore, nobody would invest. And therefore, nothing happens. So that is a little bit additional consideration in the back of my mind when I look at this S-shaped alternative. OK, then I need to define a honeymoon period, an irrational investment period for the startup phase before we get into the point that it is marginally viable to invest anything. OK, all good.

But I think the structure here looks solid. You get a scope of use, which is a number between 0 and a max scope, which is what fraction of total population would ever be interested. So this is an important concept to keep in mind. For pacemakers, I don't think ever we would have 80% of the population be in scope for having a pacemaker. I mean, I don't know. With the current world where it is going, maybe we have this. There is a good term for it. We are biologically enhanced people or something.

Yeah, anyway, but that's the meaning of that parameter. It's important to keep that meaning in mind. This is the total fraction of population that could ever be interested in this product. Now, if your product is not pacemaker and it is something else, then it may very well--

AUDIENCE: I was thinking of potential adopters being all people who have heart problems, but [INAUDIBLE].

HAZHIR But I'm highlighting that because then your definition of total population is total population with heart problems.

RAHMANDAD: So then you need to make that more clear when you are talking about this, the main total population. Anyway, we are all on the same page.

AUDIENCE: I remembered the text of the question suggested that there would be always a minimum scope, as in that there are some people that, no matter how flawed these devices are, have no other choice but pacemakers. And that is the main focus.

HAZHIR Yeah. So in that sense, it is a little bit tricky because, once you have that minimum scope, then your definition of

RAHMANDAD: feature-- what is the zero feature, the first feature? Is it already built? Zero means there is already a bunch of features there. And then at that point, do we really have an S-shape in utility? So you see, these are kind of small. But they actually could matter.

So we want to be clear what's our theory of feature to utility or to potential adopters. OK, good. So then you get the potential adopters that go into in scope adopters. And then scope adopters turn into adopters with a time constant. And that time constant depends on perceived utility and depends on--

AUDIENCE: The perceived utility was coming from a combination of bugs and--

HAZHIR Yeah, so that formulation has a little bit of a problem. So think about the extreme condition where you had very fast adoption of these in-scope adopters. Now, pretty much all of them are adopters because this sounded like a great idea. And now, a year later, there is research that shows this thing has lots of bugs. And now, these people actually want to stop using the product. They want to move back. And they don't have any option here.

So that adoption rate here, using just the first order correction, can create this challenge. So I would probably go with some version of, OK, this is based on the current utility, the fraction of these two groups that needs to be in the adopters box so that the effect of bugs is much more clearly captured here. And then the time to adopt, the way you're formulating it is a linear--

AUDIENCE: And perceived utility between 0 and 1.

HAZHIR Yeah, so you're doing a linear increase from minimum time to adopt to-- so if the perceived utility goes to 0, you
RAHMANDAD: have minimum time to adopt--

AUDIENCE: Plus [INAUDIBLE].

HAZHIR Plus this nominal time to adopt. So this is a very stylistic thing. I would probably have minimum time to adopt
RAHMANDAD: multiplied by perceived utility plus 1 minus perceived utility multiplied by maximum time to adopt so that the two concepts are easy, independent, clear, mathematically identical. And linear, fine, fine, because you have control over those time constants that you're using.

But in fact, I wouldn't have used perceived utility in the first place. So the thing I would have probably used more clearly is the fraction of adopters or the recent adoption rate or something that tells if there is more customers, there is more word of mouth, that increases the speed of adoption. So that would be the effect on the time. And then there is this desired level that you are shifting. And that desired level is not all of the in-scope adopters, but may depend on how many people have already adopted versus potential. Then you have bug discovery that comes from fraction of adopters--

AUDIENCE: I listened to what you were saying to Jason, which makes a lot more sense than how I--

HAZHIR Yeah, so this is not-- it shouldn't come from the total stock. It should come from the recent adoption rate, even
RAHMANDAD: from adoption rate itself directly for the bug discovery time.

AUDIENCE: Introducing, right [INAUDIBLE]?

HAZHIR What?

RAHMANDAD:

AUDIENCE: You guys are introducing [INAUDIBLE].

HAZHIR Introducing bugs? Right. Yes, that's coming directly from bugs per feature. Now, yes, this is fine. I would even
RAHMANDAD: think of bugs as buggy features so that this bugs per feature becomes something like error rate or error fraction, so that it is simplified. Then I can compare bugs and features directly. That just keeps the formulation a little bit cleaner. Then you have the debugging R&D budget is going to the--

AUDIENCE: My budgeting style was the other way around. What you were talking about makes sense to go with simple budgeting needs to go prioritize debugging. Mine was based on the opportunity left.

HAZHIR Yes. So basically, you prioritize R&D first and bug fixing second. OK.

RAHMANDAD:

AUDIENCE: And then if the scope gets high enough, then the opportunity goes down. And then the majority of budget--

HAZHIR Yeah, so we haven't yet looked at that formulation for the budget. So you have profit margin.

RAHMANDAD:

AUDIENCE: And that's a switch because I wanted to test how it works without this.

HAZHIR Yes. So you are capturing a total opportunity thingy here, which is the remainder population. But the way you are

RAHMANDAD: calculating it is potential adopters plus in-scope adopters divided by total population.

AUDIENCE: Well, I was thinking that this is like the firm, the company, is seeing that this many people who are potentially my customers haven't bought the product yet. So I can lure them in with more features.

HAZHIR But in your formulations, you don't lure them in with more features. Is that a correct?

RAHMANDAD:

AUDIENCE: No, I think the features then feed into scope, right? So I want to--

HAZHIR Yes, you can bring in more in-scope adopters. But you can't take in-scope adopters and turn them into adopters

RAHMANDAD: with more features. Whereas, this formulation, it's saying it is a good idea to invest more if we have a lot of in-scope adopters. So it's internally inconsistent. Is it clear what's happening? OK.

And then this opportunity thing is not really tied anyway to-- OK. So if you have a lot of potential adopters simplified-- and by the way, also this cannot go above 0.8. And that 0.8 is a parameter you have made an assumption about somewhere else. So it makes this formulation of opportunity dependent on a parameter that is not part of-- the scale of this construct is bounded by that other parameter.

So if I change that parameter, the scale of this will change. Therefore, wherever this is going to go is going to have a change in scale. That you don't want. So this is, again, one of these general rules. If you have a parameter that is changing some construct downstream and it's going to change this construct, then I have to keep in my mind all the traces where this is going to impact.

Whereas, if I could normalize it right here, maybe divide it by 0.8, then this is always between 0 and 1. Then I can use it without worrying about tracing that impact and making sure it remains consistent. And that will always fail. It's just no doubt about that. You will forget some pathway. So as soon as you can, normalize so that you don't need to keep things in mind.

OK. So again, the opportunity idea-- the idea is right. That is, yes, it becomes smaller as you don't have anybody left. And that should reduce R&D budget. But it is not as precise and clean as a marginal utility type of a calculation. But it is good to have that idea.

Now, you could use the same idea and say this is the desired R&D budget. And then I have the desired bug-fixing budget. And if you wanted to prioritize that, get that bug fixing budget it taken care of. And then whatever remains versus what is the desired gives you the R&D budget. If desired, is more than what it remains, you give what remains. If the desired is less, then you just give what is desired. Good. I think we have covered the key formulations here. So this all makes sense.

Let's see. Let's go to Ross's model. So we have users. So here, we have a very similar d fraction of potential users. This is potential users who are in the scope, so to speak. This definition of total population starts with the total people who have a heart disease. So this is not the total population altogether. That comes from feature, which has a return on feature thingy.

Let's take a look at-- so the return on features is-- OK. so this is an interesting formulation. It's normalized by initial features. So typically, initial features are smaller than features. So this number can go easily way above 1. So we need to keep that in mind, that the scope of use is now increasing.

And then you have minimum of baseline fraction of potential users multiplied by scope of use. And then you have minimum of this and 0, 1. So Ross, I think I like the first part of your formulation to think about this scope of use to have decreasing return with features. But the way you have formulated it here, you get a stock in the next step because this scope of use starts above 1 and goes above 1.

Then you need a parameter that, when multiplied by it, you want to keep that still below 1. But then you cannot enforce it because the first part, which is features to the power of return on features, is not bounded. It's not bounded by anything. And therefore, you get a stock. You don't know what to do with that.

So what do you do in these situations? So one way-- there is no easy way because-- I mean, because the features are growing unlimited, potentially. And as long as they grow unlimited potentially, no matter how you normalize it, it can grow above 1.

But I could-- if I was making the feature investment endogenous to marginal return from features, then I could, here, rather than initial features, use the maximum features or something like that as my normalizing factor. So I look at the features divided by maximum features as the factor raised to the power of return on features.

Then as long as I could keep features under the maximum features, things will be fine. So that's an easier starting point. And then I don't also need to worry about this baseline fraction of potential users. So I don't introduce additional parameters up there.

Now, keeping the features under the maximum is still tricky. How do you do that? If you do the marginal utility calculations, you probably will get there. Or there is-- you could get there. It's not guaranteed either. You could do something that is more ad hoc, like putting a bound on feature productivity that goes to 0 once you get close to the maximum features. So you could make that, no matter how much you're investing, if the gap between maximum features is 0, you basically shut off the productivity of feature development. That's an ad hoc solution that I don't like. But if I don't have any better solutions, I'm OK to use something like that.

OK. Now, that creates a fraction that leads to indicated potential users. There is a time to adopt or to move. This all makes sense. Then you have potential users with a fraction of these potential users based on perceived utility. You get the adoption rate. And let's take a look at what is happening here. This is indicated users minus users divided by time to adopt. So this is a two-way flow.

Now, in this case, theoretically, if the indicated users go down, you could have negative numbers and, therefore, push people from users go back into potential-- to indicated users. That's the right formulation here. This is actually what you want to have. That allows reductions in indicated users if perceived utility goes down. And perceived utility is coming from known side effects.

Now, the way Ross has formulated known side effects is based on some sensitivity parameter that you just multiply-- oh, OK, so this is a more complex formulation-- 2 divided by 1 plus known side effects divided by initial side effects to the power of this thingy. So if known, side effects are the same as initial, you have perceived utility of 1.

AUDIENCE: [INAUDIBLE]

HAZHIR Yes.

RAHMANDAD:

AUDIENCE: [INAUDIBLE]

HAZHIR Yes. And if you have a hell of side effects, then it goes to 0. So this is kind of an S-shaped curve that goes from 0,

RAHMANDAD: is changing slope second derivative at 1, and then saturates at 2 for the utility. It's a little bit ad hoc-y for me. It's not bad because it is capturing the trend of the utility. But it leaves too many things on the initial side effects. It's an interesting one. OK, so--

AUDIENCE: [INAUDIBLE]

HAZHIR Can you speak up?

RAHMANDAD:

AUDIENCE: [INAUDIBLE]

HAZHIR So 2 is basically what Ross decided to normalize around. And then when it goes into the fraction of users, it

RAHMANDAD: should not go to 2 because, otherwise, the fraction of users becomes twice the current. OK. So this is actually an actual problem here, Ross. First of all, fraction of users ultimately should be between 0 and 1. So if you use perceived utility and perceived utility can go to 2, then fraction of users becomes 2. Then it doesn't work, if it is fraction.

AUDIENCE: I think I was trying to do the baseline fraction thing again, but probably [INAUDIBLE].

HAZHIR You were trying to do what?

RAHMANDAD:

AUDIENCE: Like the one that I did in the [INAUDIBLE], multiplying by the [INAUDIBLE].

HAZHIR Yeah. You wanted to multiply it by something to scale it. You could scale it by that maximum, which, in this case,

RAHMANDAD: was 2 if I-- yes, maximum is always 2, which comes from known. And that could work. But the more structurally problematic issue is this indicated users. It shouldn't be just multiplying fraction of users by potential users. It should be multiplying it by potential users plus users. Am I right?

AUDIENCE: Yeah, yeah. What changes [INAUDIBLE]?

HAZHIR Hm?

RAHMANDAD:

AUDIENCE: What changes [INAUDIBLE]?

HAZHIR Yeah, that would work.

RAHMANDAD:

AUDIENCE: And I am trying to figure out. So when somebody adopts into a user, because there's also an indicated potential user, that adoption would drive someone from total population to come into the potential users stock, but not through--

HAZHIR Wait, wait. From total population, nobody should come here just because somebody left. Let me see. What's the
RAHMANDAD: becoming relevant formulation here?

AUDIENCE: I guess indicated potential users made me think that.

HAZHIR Oh, this is also problematic, yes. So you don't want to look at the indicated potential users minus potential users
RAHMANDAD: for becoming relevant. You want to look at indicated potential users minus the sum of potential users and users. So that's the total that you're comparing with, and then closing that gap. Is that clear, Ross?

AUDIENCE: Yeah, yeah, yeah.

HAZHIR OK. And similar here, basically here, your indicated users should be the sum of the potential users and users
RAHMANDAD: multiplied by this fraction. That becomes the total that you would expect to be in the users category. Now, you compare that with the current users. And you can close that gap.

So it would be indicated users minus users divided by time to adopt. But indicated users is the sum of users and potential users multiplied by this fraction. And then the time to adopt, you're looking at endogenizing it. But you're endogenizing it using the current users. And that's OK. It's people who have it have it word of mouth. So this is the idea.

Now, this formulation here is baseline time to adopt divided by fraction of users in population plus 1. So as that fraction goes down, you get to the baseline. As that fraction goes to 0, you go to baseline. If that fraction of users in population is 1-- that's the maximum-- then time to adopt goes down to half that value.

So this is not a very effective way of scaling your time constant because, basically, it limits it between baseline and baseline divided by 2. And even 2 is very unlikely because this fraction of users in population is a number that usually doesn't go that high. It's bounded by how much potential users can increase the scope and so on.

So I would go with something more flexible that has another parameter for maybe some kind of a minimum time and maximum time, and then scale it using this fraction. And the fraction could be fraction of users in the population. That's fine, but yeah. So these are the key questions here.

Then feature development here, you're using a fixed rate. And why do you have this baseline at feature rate divided by time to add features?

AUDIENCE: I did not go as far [INAUDIBLE]?

HAZHIR Yeah, I think you can just have one parameter, if you want to fix it. But then if you fix it, you will not be capturing
RAHMANDAD: the marginal utility and the decision to add features endogenous to the return from features and so on. And then the bug discovery and fixing right now is a fixed one. Again, this is skipping R&D structure and formulations there. OK, good.

Let's take a break. I think it's a good time. But before that, any questions on this last model that we went through?

AUDIENCE: [INAUDIBLE]

HAZHIR Yeah, short question.

RAHMANDAD:

HAZHIR Mm-hmm.

RAHMANDAD:

AUDIENCE: If you want to [INAUDIBLE] stop-- so for the features we had a maximum feature level that we preordained and then just close the gap to that level. Is there a problem? What problems do we have [INAUDIBLE] if the marginal [INAUDIBLE]?

HAZHIR You don't get anything-- any big problem. The only problem we would have is that your marginal becomes

RAHMANDAD: incorrect. Basically, because you have one pathway that is using marginals to decide on adding features, and then you have this other kind of a very strong limiting factor that ignores the marginals and says, OK, I'm going to shut off feature development, even if it is why it is OK marginal-wise.

AUDIENCE: But if you don't build out the decision making structure, then you can still capture that marginal effect. If you don't have the profitability of the [INAUDIBLE], then you still get that goodness of marginal return on investment [INAUDIBLE].

HAZHIR I mean, that cap alone is not a great formula for marginal because it is disconnected from what is happening

RAHMANDAD: with your fraction of potential users. This return on feature parameter is the key. This is really what determines your marginal or, by the first order determinant of the marginal, is your return on features parameter.

So the derivative of fraction of potential users with respect to features is dependent on this parameter. So in this formulation for add features, I ignore that parameter. I am completely ignoring the actual process by which marginal is generated. So you don't want that. And that's why I was a little bit hesitant of adding that very sharp thingy.

AUDIENCE: That's helpful.

HAZHIR Yeah, OK, cool. Let's take 10 minutes of a break.

RAHMANDAD:

OK. We went through Ross's model. Let's see which models did we miss. So this is Ali's. Let's take a look. Did we go through your model, Ali?

AUDIENCE: Uh, no.

HAZHIR No, OK. Oh no, we did go through the stocks. Yeah, but we didn't go through the formulation.

RAHMANDAD:

AUDIENCE: So I haven't collected [INAUDIBLE].

HAZHIR I see, I see.

RAHMANDAD:

AUDIENCE: [INAUDIBLE]. The reason that I used this formula [INAUDIBLE] S-shaped logistic.

HAZHIR So you want to have S-shaped logistic for what?

RAHMANDAD:

AUDIENCE: For the scope.

HAZHIR OK. So the scope right now, you have a maximum scope of use. And then you are moving towards that maximum

RAHMANDAD: scope of use. And it's basically looking at the gap--

AUDIENCE: [INAUDIBLE] carrying capacity.

HAZHIR Yeah. And so you slow down. So it has a little bit of that reduced investment in the scope over time built into it.

RAHMANDAD: And then--

AUDIENCE: [INAUDIBLE] adopters.

HAZHIR Yeah. So one thing that I just remembered we haven't discussed is we talked about this marginal utility or

RAHMANDAD: marginal benefit of adding more features. So what that really does, in practice, is, if it is above a threshold-- that is, if it is profitable, it gives you enough return on investment-- you want to invest in feature development. And if it is not, then you don't want to invest in feature development.

So if you have a maximum capacity for investment, that maximum capacity-- if you don't make that endogenous, that maximum capacity is operational, as long as you are above this threshold, and then it shuts off or becomes much smaller when it's not worthwhile building new features.

And that formulation actually doesn't use the marginal utility to decide on the flow, on the level of product development. It just is a switch. It's 0 or 1. If it is above threshold, you're investing. If it is below threshold, you are not investing. And that might be an OK first order approximation. I'm just highlighting the way I was talking about it and because I haven't built that part of the model-- I haven't built this model recently. I did a version of it many years ago. I talk as if it is continuously changing the flow, but it is not, in that interpretation.

Now, there is a more subtle, complex interpretation that says the faster you develop features, the more inefficient is development process. It's just called time compression. Diseconomies, this is kind of-- academics want to generate terms that are fancy-sounding.

So time compression diseconomy is basically telling you that, if you pack in too many features together into next product development project, you have to spend a lot more in getting those features in place. If you account for that, then this marginal becomes important because you would pack enough features into your next development project so that the time compression diseconomy's cost of developing those features balances against the benefits that come from having more features. And that is coming from your marginal value of new features.

So there is a more complex version of the world in which the flow, at any point in time, is dependent on that marginal revenue or profit per feature. But in that world, to get to that world, you need another concept. You need something about these time compression diseconomies.

Absent that, the decision on how many new features I want to develop-- the decision is infinite until the point-- infinite flow-- until the point that I get to marginal goes to negative, below the threshold. And then I shut off. That is, go with the maximum features you can pack, and then go to 0.

That, of course doesn't happen in the real world because of those time compression diseconomies and the underlying mechanisms for those. You who don't know about the customer taste until you do some other things. And you don't have the technology until you do some other things. So a lot of features are not in the feasible frontier that is accessible to you. You have to develop other things before you get to the technologies that allow you to develop new features and so on.

Basically, those technology frontiers are typically-- knowledge and technology frontiers are what sets the pace of product development. That is, you're trying to develop whatever is feasible in that space that you think makes sense. And then that increases, that expands the frontier. Now, new frontiers show up. And you develop that.

Approximating that with a fixed product development rate or a max product development rate is OK. So for this project, that's perfectly fine. You just need a process to shift towards a point that, OK, it's not worth it anymore to develop more features or expand the scope. So that's just something to keep in mind.

OK. And this one is a fractional investment. So it reduces investment as we go forward. And then you get the maximum adopters from here. You look at the gap between the current adopters and maximum. That creates an adoption process with the time to adopt and some perceived utility. So how do the three come together?

AUDIENCE: [INAUDIBLE]

HAZHIR So in this formulation, you say adopters gap, which is those who are in a scope minus those who are already
RAHMANDAD: adopted, you multiply that by a perceived utility. So what fraction of those are interested is coming from your perceived utility. So in this formulation, if the perceived utility goes to 0, you would just shut off the flow. But you would not have a way for people to come back because they are seeing a lot of problems with the product. And this is also partly the challenge that we talked about with combining those two effects together.

AUDIENCE: It seems fair for a pacemaker [INAUDIBLE].

HAZHIR So yes, that's right. So once you have it in this other-- and there is a question of who is an adopter in this case.
RAHMANDAD: These are people who have a pacemaker or are kind of-- because pacemaker don't last forever. So you would go every few years to replace, upgrade. And that process, we don't model it explicitly. So if you think about adopters as people who are sold on the product and either have it right now or whenever this goes off they will be willing to replace it, then that replacement rate could go to 0 and can reduce the number of adopters, shifting people to the other side. So it becomes-- there are ways to consider flows back.

OK. Then we have perceived utility. That is coming from the bugs that are known issues. And by the way, depending on how we are defining issues, for a pacemaker, I think this is a good formulation, people who don't know about these problems until they have adopted it. And so unknown issues wouldn't count.

But for some other products and services, there is at the point-- so if you go to a website of a company and their website is poorly designed, you may decide not to adopt it at that point in time. And those bugs are the types of bugs that are-- even when they are unknown issues-- that is, the company is not monitoring those yet or has not identified those yet to start fixing them-- they could stop customers from adopting.

So for a lot of online businesses, that matters. You may go to Netflix or Amazon or wherever. And you may be disappointed with something and decide not to adopt. And that may not be in the known issues for the . Company, they would be investing. Their product development team would be looking at data from their online. So in those cases, you could have a fraction of these unknown issues also to play a role in whatever calculation for utility you're doing. I'm just highlighting that it is plausible to have that idea. So here, known issues--

AUDIENCE: [INAUDIBLE] collecting adoption rate to the [INAUDIBLE]. So right now, I'm not doing this, but instead, I'm commuting the [INAUDIBLE].

HAZHIR Using the perceived utility to drive that.

RAHMANDAD:

AUDIENCE: Yeah, I'm just talking about the bugs [INAUDIBLE] here for the recovery time or reporting--

HAZHIR Yes, reporting time.

RAHMANDAD:

AUDIENCE: So I'm just using that feedback from the stock to the time to report.

HAZHIR Yes. So you're using adopters rather than adoption rate or recent adoption rate to drive that.

RAHMANDAD:

AUDIENCE: [INAUDIBLE]

HAZHIR That's conceptually a little bit problematic because these adopters, many of them are very old timers who don't

RAHMANDAD: have anything to say about the current bugs. And therefore, the unknown issues, these are people who have very recently adopted.

AUDIENCE: So that's what I thought, maybe even-- probably even in the case of pacemakers, maybe it takes like five years or so.

HAZHIR Sure.

RAHMANDAD:

AUDIENCE: So that's why I thought maybe even very old adopters.

HAZHIR That's fair. And in those cases, I mean, you could break down the adopters into cohorts. So this is recent adopters

RAHMANDAD: and 2 to 3 years adopters and 4 to five years, et cetera. And then you would have different contributions to bug discovery from these different cohorts. You could do that. It's an overkill for this project, but that would be the way to tackle it.

So I know you have all these parameters to create these S-shaped functions. Again, kind of OK, but perhaps an overkill for some of these. And also, right now, because of doing the known issues in its own units-- it is its dimensionless. I would probably not make it dimensionless. Bugs have a unit. It's either a bug or it is in units of tasks that are buggy or features that are buggy. But then you have this bug scale thing that needs to come in and impact.

AUDIENCE: [INAUDIBLE]

HAZHIR Yeah. Right now, the unit problems that, I think, Hesam was also highlighting are showing up here-- that is,
RAHMANDAD: known issues by making it dimensionless. But this thing can go up and down. It's determined partly based on the scope. And the units for scope is also dimensionless. So this makes it harder for you to really use the units as a way of setting things right.

So this is another generally good insight. You can use dimensionless for everything, ignore units. But you're depriving yourself of a very helpful avenue for thinking clearly about the problem. So don't use dimensionless unless something is really dimensionless so that dimensionless is meaningful.

There are situations that, of the way Vensim reports unit errors, if you have x to the power of y , both x and y need to be dimensionless. Otherwise, Vensim gives you errors. Then you may need some kind of normalizing to get things dimensionless for those rare cases to just deal with Vensim. But conceptually, always have units that are consistent.

AUDIENCE: It's in scope something.

HAZHIR So this should be, in my view, in features. Scope should be in the unit of features. But Ali has put it as
RAHMANDAD: dimensionless. And when you put that as dimensionless, I can't really track what exactly is measured here, how it is counted. So is it this scale of it between 0 and 1? Or is it between 0 and 100? And I wouldn't know when you have dimensionless how to think about that.

But then also, it goes into bugs per change in scope of use. And it is not really changing scope of use. It's bugs per scope unit because this thing is feature per month. And this is also bug per month, if you want to have that. So you already see the lack of clarity seeps into the variable name selection as soon as you remove the unit. So it's pretty helpful to do it.

OK, unknown issues-- anyway, they are in the scale of-- I assume this is also small. Yeah, so this is 2. So these are the scale of what the scope of use is. So then up here, you come and use this scale thing to really try to grapple with how many bugs do I have versus how many adopters do I have.

And so first of all, you shouldn't be comparing bugs with adopters. Why are you normalizing against adopters? It is really bugs compared to features. So that's the real driver of the normalization, which is missing from this formulation here.

AUDIENCE: [INAUDIBLE]

HAZHIR Sure.
RAHMANDAD:

AUDIENCE: [INAUDIBLE]. I was thinking that if I put myself in the shoes of a person who's [INAUDIBLE].

HAZHIR Yes.

RAHMANDAD:

AUDIENCE: And I see how many people have adopted it and I also see how many bugs that we have so that is [INAUDIBLE].

HAZHIR So this is very important. What is the definition of a bug in your formulation, the way you are thinking about it?

RAHMANDAD: You kind of switched between two ways of thinking, maybe even again because of the units. One is the version that we have seen in most of the models, which is each bug corresponds to a feature that is buggy or bugs within a feature. So no matter you have 100 customers or a million customers, there is a buggy feature, it has a negative utility.

Then there was another feature-- way of defining that we saw in Jason's model, where he was tracking features multiplied by customers or buggy features multiplied by customers. This is how many bugs total are in the population. That conceptualization then requires you to normalize also against the total number of customers. And that's what might be driving your thinking in the second part. So in one part, you're using one definition. In another, you're using another definition.

AUDIENCE: Does it matter for bugs to be normalized against the number of features versus just the number of bugs? Because I mean--

HAZHIR What do you mean by normalize against the number of bugs?

RAHMANDAD:

AUDIENCE: Not against the number of bugs, but--

HAZHIR Non-normalized.

RAHMANDAD:

AUDIENCE: I [INAUDIBLE] to care that there's two bugs if it has a million features or two bugs if there's 10 features [INAUDIBLE] the bugs are a problem.

HAZHIR It's a good question. And the way I think about it is how serious those bugs are. And to get a feel for how serious those bugs are, you should compare it to the opportunity space for having bugs. If you have a million features, you could have a million bugs. And the fact that you are seeing only two means this is actually a very good product. If you had two features and you had two bugs, that would be a terrible product. So by not normalizing, we miss the picture on that opportunity space for having bugs and, therefore, what is considered a bug, how serious it is. That's the idea.

Now, when you're doing it for a single product, that normalization factor may not be as big a deal. You can deal with it OK by just think of it as a number. But to make a robust model that is more generalizable-- but also, in all the formulations, when you want to have dimensionless effects of these bugs on utility, on other things, you do need a normalization. So I think, for both of those reasons, it's a good idea to have the normalization. OK. I think we covered most of the things here. Anything else that you want us to look at, Ali?

AUDIENCE: [INAUDIBLE] have a quick question-- a comment that scope [INAUDIBLE]. Is that specific to this model or to Ali's model or just in general, how you would think about it?

HAZHIR I think, for this exercise, I would think of scope as something along the lines of features or--

RAHMANDAD:

AUDIENCE: And then that determines the maximum population.

HAZHIR Yes, so that transformation is what we saw in-- we were just looking at Ross's model. We talked about the

RAHMANDAD: possibility of normalizing that against the maximum viable feature so that you do have a dimensionless ratio that, then you can look at it's incremental value for having more customers, something along those lines. OK, good. Let's see what models we haven't looked at. This one is Kathy's. So let's look at this now.

AUDIENCE: [INAUDIBLE]

HAZHIR Could you speak up?

RAHMANDAD:

AUDIENCE: Yeah, I did discover a unit error at 10:00 AM. It's in risk perception.

HAZHIR OK.

RAHMANDAD:

AUDIENCE: It won't be clean [INAUDIBLE].

HAZHIR No worries. It has fewer unit errors than Jason's. OK. So let's take a look at features. So increasing features is
RAHMANDAD: based on a maximum feature idea, similar to Ali's and remaining features. You add them with the time constant. So initially, you add more features. And over time, you add fewer features.

And this part is pretty much built into it. That is, the whole function of this part of the model becomes creating a goal-seeking behavior that doesn't get any feedback from other parts of the model. And that's OK. Again, as we discussed, there are different ways to formulate it here.

AUDIENCE: It does get feedback from the R&D pressure.

HAZHIR Ah, because the time to add features is getting feedback from R&D pressure. OK, good. And then these features
RAHMANDAD: lead to effective features and marginal utility. So how is that formulated? It is features divided by initial raised to the power of marginal utility reduction curve strength. So this is this marginal utility curve strength. It's a mouthful.

AUDIENCE: It's from the VCR model formulation.

HAZHIR Yeah, that's OK. You basically have one parameter here. Now, you're transforming this parameter-- calculating
RAHMANDAD: this parameter using another parameter. It's an overkill. You then you need to explain exactly why. And I'm OK with it. But it's not adding much value to have two parameters there.

And then that feature rate leads to marginal feature utility. This is very similar to what Ross was working on as well. So this feature utility is now not a number that is bounded by anything. It can be above 1. And it can be below 1. It goes into the utility.

It's actually interesting. So right now, the formulation you have here is a little bit confusing for me because the marginal utility of features is the derivative of utility with respect to features. If you're calling that marginal utility, that's the definition of marginal utility. Then this marginal feature utility would not be going into the utility as an integral-- to get integrated into the utility.

What you would have is-- I have this number of features. That creates a utility for me, which can come from the effect of features on utility rather than marginal utility. So you would have initial utility multiplied by effect of features on utility. And that gives you the utility that you want to have.

AUDIENCE: OK. I might have not really been conceptualizing utility correctly. I was thinking of it as like a basic-- it's meant just to be a number of features based on that marginal add of each feature. So for instance, I think most people did utility as a combination of features and bugs. But for me, utility is just--

HAZHIR No, that's fine. I think yours is not wrong. I'm not saying it is wrong. I just see it as a more convoluted way of
RAHMANDAD: modeling it. It is not wrong. And the way you do it later-- that is, you then multiplied increase in features by marginal feature utility-- is correct-- that is, if you want to calculate total utility using this version.

So what you're doing is that you are calculating the marginal utility from the current feature level. And then you accumulate utility that way. And then this actually becomes the correct marginal utility. So this is actually mathematically correct. It's just a little bit more complexity.

Now, sometimes, it is a good idea to have that complexity because maybe I know how to calculate the marginal utility from the current feature level, but I can't calculate the total utility from current feature level because that's an integral that I can't take analytically. Then this formulation allows you to take that integral here using a different stock and have the utility and the marginal utility calculated at the same time. That's great.

So in a way, it is a good trick if you need a more complex marginal formulation that you know how it depends on the thing that is driving it. OK. Then you have the utility effect on indicated fraction of adopters. So you have this ad hoc maximum utility here. I don't know how you would make sure this is maximum utility.

AUDIENCE: There's a maximum number of features somewhere. And I just said the maximum number of features times 1 util per feature is maximum utility.

HAZHIR Yeah, but that is not your maximum of this integral. That is, if you increase the features over time using
RAHMANDAD: whatever inflow you want to put into it, you will get to a utility level that saturates at some point, perhaps. I mean, I don't know for sure, given this formulation. Maybe it never saturates. Maybe it continues forever. But once you get to your maximum feature, it will saturate at some level. That is the value that should go into the maximum utility. But you don't know that value beforehand because it requires solving this integral problem.

Now, you might be able to analytically solve that integral problem. So doing that is a little bit of work, back of the envelope or under some real paper and pen work because it might be more complex. But you could potentially do that calculation, and then use that maximum utility. That is correct.

The back of envelope one that you have done, which is features multiplied by 1 and 1 being the maximum marginal utility that you could conceive, is not quite satisfactory because then it doesn't scale between 0 and 1. It scales between 0 and some number. But that number you don't know what it is.

AUDIENCE: Could you say a little more about how I would go about doing that?

HAZHIR Sure. So the solution, whether it is feasible or not, depends on whether you can solve analytically the maximum
RAHMANDAD: level of utility once you get the integral of feature from initial feature to maximum feature. So it is integral over feature multiplied by marginal utility. That becomes the total utility-- the feature going from feature being equal to minimum or initial until it is maximum.

So it is feature multiplied by effective feature on marginal utility, which is feature itself to the power of marginal utility sensitivity. So it becomes feature f to the power of 1 plus this alpha parameter that you have. So integral of that from minimum f to maximum f is analytically calculatable because it is just a power function. So you can relatively easily calculate that, use that calculation to put it into your maximum utility. Is that clear? It's a little bit-- yeah I just hand-wavingly did an integral. So it may not be obvious what's going in it.

Now, the problem with that is-- it's OK. But it is a little bit tricky because then the formulation here is complex. You have to actually explain quite much with detail. And it has to come from these parameters. So the maximum feature, initial feature, and this marginal utility, all of them should go into that equation so that it is all internally robust and consistent.

If you do that, then it is fine. And then you make this a maximum-- sorry, initial function. It is initial value calculated based on those three parameters. Anyway, once you have done that, it all works. Then you have a utility effect on fraction because this number is between 0 and 1 . This indicated fraction will also remain between 0 and 1 .

Now, at this point, I'm inclined not to have a separate strength of utility. I'm inclined to make utility act as my fraction of customers who are interested in this market because, otherwise, I have this unobservable qualitative thing in between. And I have two nonlinear transformations going into it and coming out of it. It's very hard to then track what is happening. And it doesn't add value. Mathematically, it doesn't add value. So I would fold this additional parameter into whatever I think is the effect of this marginal utility reduction curve strength. And I'll just take care of it there.

AUDIENCE: And you said something a second ago. You said the maximum utility would be some initial value. What do you mean by that?

HAZHIR Maximum utility-- oh, in Vensim, I would make it an initial equation. So typically, auxiliary equations or initial
RAHMANDAD: equations-- initial was calculated once and is never updated throughout the simulation. So it acts like a parameter for you. That's why I would think of it that way.

Now, yeah, that's not a big deal. You could also just have a normal auxiliary. And then this indicated fraction here is being affected by flaws. So you have flawed features that go into the features introduction. And then the outflow is by fixing of these through third order delay or something, I assume.

That's all fine-- again, with the caveat of, OK, then this is kind of sidestepping the issues related to allocation of resources between bug-fixing and new features and so on. But that's OK, as long as you make that choice consciously. Flaws per feature, it's also well-defined. But the flaws per feature part is not normalized. Oh, OK, it is normalized because you are using the same units for features and flaws. Good, good. So that's of keeping it into the same units. And that makes sense.

And then you are looking at adopters with flaws. That's interesting. So you're saying folks who have problematic items are creating some kind of a risk perception. This has a little bit of a unit issue, I think. So flaws per feature--

AUDIENCE: [INAUDIBLE]

HAZHIR Yeah, it is dimensionless, but it's a scope. Let's say you have 1,000 features and you have 10 flaws in those

RAHMANDAD: 1,000 features.

AUDIENCE: [INAUDIBLE]

HAZHIR At that point in time, we are-- so that's 1%. And we are adopting. And these adopters have 1% flawed features.

RAHMANDAD: So adopters with flaws are bugginess-- average bugginess multiplied by how many adopters you have. You divide that by adopters, you get your average bugginess. Yeah, that works. That works, except--

AUDIENCE: [INAUDIBLE]

HAZHIR Yeah, because it doesn't drain this one. I mean, death rate-- let's for now put it aside. Because it doesn't drain,

RAHMANDAD: this is very accumulative. It is not the recent risk perception. This is the longer, over the whole course of this product, risk perception.

So I think, conceptually, it looks OK, but with that caveat. So I would probably have what is the time horizon of performance that impacts new customers' decision or market reputation. And that time horizon, after that, I'll just drain people or these adopters with flaws from this stock. And I'll send the adopters into a different stock.

These are people who we are tracking because their word of mouth matters because they are the recent adopters. And then they go into these old adopters that you don't need to track anymore. And then that would create your indicated adopters.

And indicated adopters, therefore, is having these two effects together. And that gap between indicated adopters and total population drives adoption rate with the time to adopt. That is affected by fraction of adopters in total market. So again, this is kind of OK. But because it is coming from the total adopters, the old adopters have the same effect as the new adopters. But that's OK. It's not a big problem there. And then you have a couple of parameters going into this. So this is--

AUDIENCE: It's just a reverse sigmoid. And then that max reduction says that we can't take time to 0. So I think I have it set at the max reduction is 0.75, so the best we can do is a quarter of the baseline time.

HAZHIR OK, that's all right. I probably would have-- these nonlinear curves, then they go into a time constant. And the

RAHMANDAD: time constant is in the denominator. So it adds another nonlinearity. The accumulation of these nonlinearities make it very hard to really mentally track what is happening and what is the impact of k_a or max reduction in time to adapt or something on the rest of the dynamics.

So my preference is for something simpler, like a linear function for the time constant that goes between a minimum and a maximum. And to get that, you would need a normalized fraction of adopters in the market. And it can easily do that for you. This is not wrong. It's just a little bit harder to track.

OK, I think-- yeah, pretty much everything is making sense with a few ideas here and there that could be improved. OK, which models haven't we looked at? So this is Jason's we looked at. This is Gerardo's. Let's look at Gerardo's model.

So we have quality issues, unknown quality issues, unfixed quality issues, fixed. Commercially available scope of use, that is coming from feasible scope of use discovered-- what is feasible scope of use discovered?

AUDIENCE: [INAUDIBLE]

HAZHIR OK, so that's the maximum. Then you have available R&D expenditure for technological advancement. And then
RAHMANDAD: you have the actual R&D expenditure. So, Gerardo, could you tell us what the available R&D expenditure versus R&D expenditure do for you here? What is the story?

AUDIENCE: [INAUDIBLE] how much improved [INAUDIBLE].

HAZHIR Yeah, I'm just not following what's the average R&D expenditure for technological advancement. What is it really
RAHMANDAD: doing? So it's maximum of 0 and feasible scope of use discovered minus commercially available scope of use divided-- so you don't need this max 0 because it should turn off the scope development anyway-- divided by average R&D expenditure divided by R&D expenditure.

AUDIENCE: So the [INAUDIBLE] balance between the feasible [INAUDIBLE]. But the other one would be [INAUDIBLE]. And basically, between these two, [INAUDIBLE], but probably simulation is wrong.

HAZHIR Yeah, so I think the formulation is at least too complex. So it's not intuitive. You have, in the denominator,
RAHMANDAD: something again divided by something. So this part makes me-- it just makes it hard for me to track what is happening. I want just the time constant for these types of adjustments.

Now, these two parameters going there, one of them is changing. So R&D expenditure is changing. So I don't even-- anyway, so this part is a little bit sketchy, in terms of getting everything easily traceable. It may work just fine. I think all that it is doing is creating a curve for technological advancement rate that slows down over time and perhaps maybe a little bit of S-shape, maybe not. It doesn't have S-shape, I think. OK.

AUDIENCE: [INAUDIBLE]

HAZHIR What's that?
RAHMANDAD:

AUDIENCE: [INAUDIBLE] shape [INAUDIBLE], but nothing really [INAUDIBLE].

HAZHIR So if you run it, it does create an S-shape or nothing happens. And so right now, it is just doing a linear thing. I
RAHMANDAD: don't know what is that.

AUDIENCE: [INAUDIBLE]

HAZHIR Oh no, yeah. OK. Anyway, I think this can benefit from something more standard formulation. I think the concept
RAHMANDAD: of average R&D expenditure for technological advancement, in the unit of dollar, that doesn't make much sense because it doesn't sound like expenditure. Expenditure is dollars per unit of time. This looks a pot of money. But what that pot of money is doing is not clear to me.

AUDIENCE: In my mind, the money [INAUDIBLE].

HAZHIR OK, this is per unit of technology. So this would be per task or per unit of scope. Or this is for total scope, how

RAHMANDAD: much you would be needing to get to the full scope or something like that. Then you have the scope that, combined with perceived quality, impacts scope of use-- qualitatively valid. So that's similar to what Kathy also had. With some delay, it is perceived that utility then impacts your Bass diffusion model. And it is just a multiplier for that Bass diffusion model.

But this commercially available scope is between 0 and 1. Perceived quality, I assume, is between 0 and 1. So in all cases, the perceived utility is above 0. And if perceived utility is above 0, Bass diffusion model will ultimately get everybody to adopt-- well, assuming you have some initial adopters. So you wouldn't have a case where any customers return-- that is, they decide not to adopt it after initial adoption.

Now, again, with the pacemaker example, it's understandable to not have people go backwards. But either way, the Bass diffusion is a very strong exponential. It has a strong exponential component. So for this perceived utility not to turn off the spigot of adoption, it needs to really go to 0. And for it to really go to 0, you need the perception of quality to go to 0 because the scope is always above 0.

So then the quality perception needs to be well-formulated. Here, the quality perception is coming from-- well, not only well-formulated, but also be prone to putting numbers close to 0. This is, right now, undiscovered quality issues that are discovered quality issues that are unfixed divided by fixed quality issues.

This is not a very good formulation here because, as we discussed, fixed quality issues in the past are not a good normalizing factor for the current discovered issues. So think about a project that is in its first year. So there is no fixed quality issues in the denominator. This unfixed ratio becomes a huge number.

Now, if you divide it by discovered plus fixed, then it is bounded. So it would go to 1. But either way, it is not very attractive. But then if you have a product that has been around for 40 years, the fixed quality issues are in hundreds. And you may have just a couple here. That ratio doesn't really scale correctly. So this needs to be normalized against something else.

Now, the something else here is perhaps the scope. And fractional technological issue that is being added with each technological advancement rate could take care of that. So the numbers around here are these very small numbers. So it's 1,000th as the flow variable.

So the perceived quality, after that very early phase, will become so small-- so high because the unfixed ratio is close to 0 that, yeah, the effect of quality on all of the dynamics goes away. It just wouldn't have much of an effect. That would be my guess, after the very beginning. So oops. Well, in this case, actually, it does have an effect. So the sensitivity parameter is strong enough.

AUDIENCE: [INAUDIBLE]

HAZHIR Yeah. I mean, maybe it is because the unfixed ratio is changing significantly. Yes, the unfixed ratio is changing.

RAHMANDAD: Oh, six months of simulation for something that should go 40 years is also not enough, yeah. OK, good I think all of these have-- so we have seen the key issues around it.

Let's see. Which model did we not discuss? Hesam's we discussed. Sercin's OK, let's take a look at yours. So scope is changing with R&D investment, effectiveness of R&D investment. So this average time to develop new apps is less clear to me why do we need that because R&D investment should be in dollars per month or whatever. And then effectiveness would be features per dollar.

So this dividing by average time to develop suggests to me that your R&D investment is not in the right unit. Yes, it is in dollars. It should be in dollars per unit of time. So this is important. Conceptually, investment is floating. And then you're getting the investment based on something that you call research gap, which come from the technological capabilities of the product. This is a fixed number. This is kind of a maximum. Is it a maximum of sorts? Yes.

AUDIENCE: [INAUDIBLE]

HAZHIR Yes, maximum number of applications. So then you are looking at the gap between the maximum and the
RAHMANDAD: current scope. And then this R&D investment is that gap multiplied by per unit. So if you have a bigger gap, you're more willing to invest. That's the idea. And then the bigger the gap as a fraction of the total-- so this is a fraction. You just multiply to get an effectiveness.

I don't know. Just multiplying for both of these are a little bit simple. If the fraction of applications that are still not realized, which is this number, is higher, you're saying effectiveness of investment is higher and linearly higher with that. Given that this number is actually between 0 and 1, I would probably use maybe a power function rather than just a multiplication of a linear.

OK. So anyway, all of this, what it does is that it speeds up initial change in scope because now you're investing more. And there is more return per unit of investment. And as scope grows, that starts to become smaller and smaller. The extra development gets smaller. So it just creates a goal-seeking behavior for the scope, I guess.

And then scope goes to fraction of population to be served. So here, you have this sensitivity parameter for effect of scope on fraction of population served. And it is normalized by that technological-- sorry, technological capability. So this is good. This will take care of the normalization. And it will always be remaining between 0 and 1.

And then you have this decreasing return on the scope or if you want to have that in the sensitivity parameter. This gives the fraction of population. And that fraction is partly dependent on technological capabilities, so max fraction that can get it. And then it gives you the max user base. And you move towards that max user base based on a time to adopt, which depends on the utility.

So the formulation here is reference utility divided by perceived utility raised to some power. So if perceived utility is smaller than reference utility, then you adopt more slowly because your time constant increases. OK. That makes sense. And then perceived utility is coming from this indicated value that itself comes from problems and usefulness.

Problems are recognized and not fixed problems. It is normalized against some reference number, which perhaps should be normalized against either total features or some measure of recent features multiplied by this number of problems per unit of application.

So right now, this recognized-- reference number of recognized problems is just an ad hoc number that you have put. It is not rooted in any of those other parameters that determine the scope of it. And that creates a problem downstream because then you have to remember, whenever I change any of these numbers, come and change this number accordingly. You don't want that.

But otherwise, the core ideas are good. Effective usefulness on utility-- so you already calculated that from the scope. You got the average usefulness. Wait, how do you do this total usefulness? OK. So you create a marginal usefulness. You get to total usefulness of product. And then you say per feature what is unit of usefulness per unit of a scope.

So this marginal calculation is very similar to Kathy's calculation of marginals to get to a utility. This is basically looking at change in scope, multiplying that by marginal utility. So that makes sense, in terms of marginals. But then you get this total usefulness of product, which is the utility of the scope that we are seeing. And then you calculate it as average usefulness per application. So as we increase the scope, the average usefulness per application goes down. And then that makes effective usefulness on utility to go down.

That's OK. But again, you have the max usefulness of product that is hanging in there without having-- it's a very similar issue as what we saw in Kathy's model, that you do need some max that is rooted in how far this usefulness can go up. But if you do that, then it should work. Cool?

OK. I think maybe we have only one more model that we haven't looked at. It says Alban's model. So let's do a quick look at Alban's model, and then go for the next week's assignment. So we have scope. Expansion of scope is coming from R&D investment and investment productivity. As you build more scope, your opportunities to build more goes down, I assume.

So this is balancing the return on investment from the R&D. That makes sense. Then this scope is having a coflow with the utility. And that is affected by this very similar idea of marginal utility. But you are getting it from these remaining opportunities to expand the scope. So they bring down the average utility of a new piece of feature or a new-- yeah, these are all dimensionless, which makes it a little bit harder, as we discussed. But it would be OK, potentially.

So here, this is the decreasing return on utility of new pieces that go into the total scope. Then we get the average utility of the product. This is per feature what is the average utility. So the indicated utility is actually going down over time?

AUDIENCE: Right.

HAZHIR And then perceived utility determines the fraction of population willing to adopt. So this part is problematic, in
RAHMANDAD: the sense that, over time, your average utility of product should go down in this formulation. Is that correct? Or we could simulate it and see. Let's not just assume.

So yeah, perceived utility initially goes down because the average utility of product is going down. That part is-- I think it's baked into the formulations you have. And the way it is baked into it is because you're dividing the total utility by scope. And because the marginal utility is going down, then the average utility will also go down.

And average utility is relevant for some use cases or some parts of the model. But it is not the factor that is determining the total scope of adoption or total fraction of population willing to adopt. So that's important to keep in mind. So this creates a weird behavior that, initially, the largest fraction are interested. And over time, fewer people become interested. Now, you have this additional feedback coming from the effect of bugs on utility that is very high at the beginning. We should see how that is happening.

AUDIENCE: [INAUDIBLE]

HAZHIR Uh-huh?

RAHMANDAD:

AUDIENCE: The scope determined the [INAUDIBLE].

HAZHIR Ah, interesting, interesting. Yes, yes. So this is not this fraction of potential adopters. Got it, got it. Sorry, thank

RAHMANDAD: you. That clarifies. So max potential adopters is coming from the scope. So this is actually closer to the formulations we did three stocks. But you basically have created-- you have assumed that first step is very fast, which is good, which is not a bad assumption. So you get the max potential adopters from a scope. And then the perceived utility could go down because this is, given that scope, how far the utility is. OK, that's good.

And this gives the indicated adopters. And then you have adopter adoption versus abandonment. And you have an asymmetric thing here. So if the indicated adopters is bigger than the adopters, then you have time to adopt, otherwise time to abandon. So that's nice because abandonment could be faster, perhaps. And then effect of adopters on adoption time is what we have discussed. So that's good. Great, I think.

And then the bug part with this scope comes bugs, discovered bugs, some time constant, create the effect of bugs. Now, discovered bugs, because you're using dimensionless for the current scope--

AUDIENCE: [INAUDIBLE]

HAZHIR Yeah, this needs some normalization because, right now, you have 200 bugs per unit of scope going into here.

RAHMANDAD: And then you don't normalize it at all. So this definitely needs a normalization. Cool. Otherwise, it's pretty nice. OK. Any questions? All right, let's then review our model for next week. Actually, it's easier to do this way.

OK. So next week's assignment is about modeling the growth of a professional field of practice. In some ways, it has relationships with this week's work. So this week, you were focusing on a product. Next week, it is a little bit perhaps more complex, perhaps not. It is more about a situation where supply is a service.

So think about system dynamics. Or think about any kind of field within consulting space or within law. There are practitioners who do the job. They are going through a learning process to become professional, become experts in what they do. And as they build these skills, they become better at serving the customers for that space.

They could take on projects perhaps, even when they are novices. And they could mess up those projects and create quality perception issues. So there is the quality dynamics and training dynamics. So rookie/expert type of a structure would be key there.

And then you would have the demand side. Demand side requires people, outsiders, to know about this service. There is a potential maximum market size ultimately. Not every decision in the world can benefit from that specific type of consulting, for example.

But from that total market fraction, people need to know about it and need to have a good sense of, OK, this is a good thing. And this is a reputable practice. And if they know about it and have a good, positive sense about it, then they bring in work.

And that work motivates more of these professionals to develop their skills, more of the novices to spend time and build their skills. And if there is not much demand, then they wouldn't stick around and they would leave. So that becomes the core of the model you will be developing.

Again, a little bit less well-defined. I've left my purpose to be you decide on what's the right constructs exactly. If you found time and you found it easy to add more pieces, you can think about endogenous learning time. So basically, if you have a lot of novices, they can't learn on their own that much. So working with professionals is where they learn on the job to do the job better and learn the tricks of the game.

On the other hand, they also take some time from the experts for training. So whether that taking of time from experts is important, you can think about it. If you have a job that is well-modularized, so the novice-- so the clerk in the law firm helps with the research, and then the partner is doing some of the more complex tasks, then both of them are happy. And the novice is learning and going up the chain.

On the other hand, if you need a lot of hand-holding, then that effect might be more challenging for the professionals. And if there is not much work to be done, of course, there is not much opportunity for novices to learn from this learning on the job side. So those are extra ideas.

And then if you had time doing some policy analysis, so thinking about policies like certification-- that is, who is eligible to do the work? So in the law, there is a very clear certification process. In consulting, there is not. Reputation becomes important. But certification is not part of it.

Training-- so in some settings, you have professional schools. We teach system dynamics in the university. But some types of consulting, there is not as much professional training. In law, there is law schools you have to go through and so on. So how does that play out?

And then matching markets-- how well you can match customers with expert professionals, how does that impact the dynamics? So those are interesting policy levers to think through and try to incorporate. Another possibility is thinking about, if you inject more novices, try to bring people into the field, advertise and bring lots of folks, how does that play out?

These are actual questions. A lot of professional societies are dealing with these questions. I talked with folks in the system dynamics society. These are exactly the questions of the day. So if you've got a chance, try to do some policy analysis as well. Questions for-- yeah?

AUDIENCE: [INAUDIBLE]

HAZHIR

So you need some feedback on the supply demand imbalances on both sides. So there is a quality feedback. If

RAHMANDAD:

you have too much demand and not enough supply, a lot of the work that gets done gets done by people who don't know what they are doing. And that quality reputational impact will be there.

If there's not enough work, novices will leave professionals. Experts may stick around, but be less engaged. They don't get enough incentives. So those you could capture. I don't think you need to go all the way into cost-benefit analysis, tracking profitability of a typical consultant, things like that for this purpose. For this exercise, it's not worth it.

OK, so that's the plan for next week. Next week, we will have only the first half focused on the discussion, and then the second half of this model. And then the second half, we will go through your problem definitions and the feedback from those who have looked at those problem definitions. So come prepared for those conversations. Any other questions?

OK. So have fun. And I'll see you next week. Of course, we have some journal club here right now. So you can stick around for that as well.