# Guided Study Program in System Dynamics <br> System Dynamics in Education Project <br> System Dynamics Group <br> MIT Sloan School of Management 

## Assignment \#4

## Reading Assignment:

Please refer to Road Maps 2: A Guide to Learning System Dynamics (D-4502-4) and read the following papers from Road Maps 2:

- An Introduction to Feedback, by Leslie A. Martin (D-4691)
- Graphical Integration Part One: Exogenous Rates, by Alice Oh (D-4547)

Also read the following:

- Study Notes in System Dynamics, ${ }^{2}$ by Michael Goodman: Chapter 2 and Chapter 3, Sections 3.1 to 3.9

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## Exercises:

1. An Introduction to Feedback

The purpose of this exercise is to reinforce your understanding of positive and negative feedback, and to prepare you for modeling systems containing both types of feedback.
A. The exercises attached to "An Introduction to Feedback" include a simple model of a bank account on page 19. The model exhibits exponential behavior through positive feedback. Using Vensim PLE, build this model and enter initial conditions and constants that you think are realistic. Simulate the model for 50 years. From a graph of model behavior, estimate the doubling time for this system. That is, how many years pass before the amount of money in your bank account doubles? In your assignment solutions document, please include the model diagram, documented equations, and a graph of model behavior.
B. In real life, you may notice that many systems do not grow unchecked. Unfortunately, our bank accounts are no exception. Let us study, through building another model, how negative feedback prevents us from becoming millionaires (darn!):

Often, a person's spending habits depend on the amount of money he currently has in his bank account. Let's say that you are comfortable with spending about $50 \%$ of your current bank balance per year. Build a model that you can use to simulate the behavior in this scenario. If your bank account starts out at $\$ 2000$, how long does it take for the balance to halve? If your bank account starts out with $\$ 10,000$, how long is it before the balance halves? Again, estimate the half-lives from a graph of the model behavior. How do you intuitively explain the relationship between these two half-lives? In your assignment solutions document, please include the model diagram, documented equations, and a graph of model behavior.

Hint: In order to answer this question correctly, you should make sure to change the time step of the simulation (also known as solution interval or DT). In Vensim PLE, the default setting for the time step is 1 time unit. The time interval denotes how often Vensim PLE solves the equations for all variables. If the time step is set to 1 year, the first time Vensim PLE performs the calculations for the system is at time $=1$ year. The model says that each year, $50 \%$ of the balance is spent; hence, Vensim PLE removes half the balance at the end of the first year. This would be correct if we assumed that you withdraw the $50 \%$ of the balance at one time, at the end of the year. Instead, we assume that the withdrawing is a continuous process.
To make the simulation a more accurate representation of the continuous process, you need to decrease the time step of the simulation. To do this, go to the "Model" menu in Vensim PLE and click on "Time Bounds." To reduce the value of "TIME STEP," pull down the menu next to the "TIME STEP" window and choose a value smaller than 1 .
As a general rule of thumb, the time step in any simulation should be less than one half, but larger than one-fifth of the smallest time constant in a model; for small models, we recommend using a time step of one-eighth of the smallest time constant. The time
constant is the reciprocal of the growth or decay fraction. In this model, the only (and thus smallest) time constant is the reciprocal of the "SPENDING FRACTION," equal to 1/0.5, or 2 years. You should therefore choose a time step less than 1 year, say 0.5 years or smaller. When the time step equals 0.5 years, Vensim PLE performs the calculations $1 / 0.5$ or 2 times a year. Setting the time step to a number greater than one-half of the smallest time constant may generate incorrect system behavior.
C. Now, take the two models and put them together into one. Start with a stock of $\$ 10,000$ and assume that your account earns interest of $5 \%$ annually. In addition, each year you have a steady income of $\$ 5000$ independent of your account balance. You feel comfortable spending $50 \%$ of your account balance per year. Formulate a model of this system and simulate the model. In your assignment solutions document, include the model diagram, documented equations, and a graph of model behavior. How do the positive and negative feedback loops interact? Do they cancel each other, and if so, when? Does one loop dominate the other?
D. Now, simulate the model developed in part C again with a steady income of $\$ 4000$. What happens now and why? Include a graph of model behavior in your assignment solutions document.
E. Finally, simulate the model developed in part $C$ with a steady income of $\$ 4500$. What happens now and why? Include a graph of model behavior in your assignment solutions document.

## 2. Graphical Integration Exercise Part One: Exogenous Rates

Please graphically integrate the following flow:


The flow starts out at 0 , steps up to 10 at time $=2$, then up to 40 at time $=4$. At time $=5$, the flow drops to 30 , then remains steady until dipping to -20 at time $=8$. At time $=10$, the flow steps back up to 0 and remains constant thereafter.

In your assignment solutions document, you should either submit a graph ${ }^{3}$ showing the integration of the above flow, or describe the integration verbally. If you choose to describe the stock behavior verbally, you should make sure you clarify what changes take place at a certain time, and what the slope of the stock is at all times.
3. Study Notes in System Dynamics, Chapter 2, and Chapter 3, Sections 3.1 to 3.9
A. Read Chapter 2.
B. Chapter 2 (page 16) shows a causal-loop diagram illustrating the feedback loops in arms proliferation of two warring nations. Build a model in Vensim PLE based on the diagram, with initial weapons of each nation equal to 10 missileheads. Make the "threat perceived by nation A" and the "threat perceived by nation B" equal at $0.2 /$ year. Simulate the model and examine the accumulation of weapons for each nation over time. What are the doubling times of the stocks? In your assignment solutions document, include the model diagram, documented equations, and graphs of model behavior.
C. What if nation $A$ is more sensitive to the threat of armament than nation $B$ ? Change the "threat perceived by nation A" to 0.4/year. Simulate the model and discuss the results. How do the numbers of weapons of each nation differ over time? How are the doubling times of the two stocks related? Include graphs of model behavior in your assignment solutions document.
D. Read Chapter 3 up to section 3.9. The examples from chapter 3 will be covered in a later assignment.

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[^0]:    ${ }^{2}$ Goodman, Michael R., 1974. Study Notes in System Dynamics, Portland, OR: Productivity Press. 388 pp.

[^1]:    ${ }^{3}$ You can submit graphs by creating them in a graphics application and then pasting them into your assignment solutions document. If neither of these options is convenient, draw the stock by hand, and then describe its behavior carefully in words in your assignment solutions document.

