Generic Structures:

Exponential Smoothing

Prepared for the
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1. ABSTRACT

Smoothing describes a decision maker's tendency to gradually react to changes in information. Smoothing appears frequently in human decision making. This paper shows that when decisions are based on smoothed information, action is delayed. Delay results when perceptions (upon which a decision is based) require time to adjust to changes in incoming information. Additionally, information smoothing filters out fluctuations.

Models representing perceived lemonade demand and perceived automobile quality are used to introduce the smoothing concept. This paper then presents a generic model structure often used to represent smoothing in system dynamics models. Additionally, the paper examines the behaviors produced by smoothing in response to step, pulse, and multiple-step information changes.

As part of the Generic Structures series, this paper discusses basic structures that appear frequently when studying systems. Familiarity with generic structures allows a modeler to transfer knowledge of one discipline to a seemingly unrelated subject by understanding how similar behavior is produced by two systems' common structure. Additionally, this is the first paper to discuss information smoothing, a topic that recurs often in Road Maps and in system dynamics.

2. INTRODUCTION

Most people do not take major action in response to every small fluctuation in their environment. For example, one does not cancel a camping trip after feeling just a single drop of rain. Nor does a bakery hire more employees if its workers are overexerted for just one day. After all, the skies might clear up, or muffins may have been unusually popular that one particular day. Major actions such as canceling plans or increasing a work-force are taken only after one is convinced that observed indicators are reflective of real, long-lasting environmental changes. Usually one takes significant action only after that first drop turns into a downpour or bakers are overworked for weeks.

The process of gradually perceiving environmental changes is called information smoothing. "Smoothing" means that a decision-maker does not instantly believe that a fluctuation in incoming information is indicative of a permanent change, and thus attempts to "smooth out" insignificant fluctuations. As a result, a person reacts gradually to a persistent change in information, so as not to overreact to what may turn out to be short-term changes.

Information smoothing can be accomplished through formal numerical averaging (for example, estimating current demand based on last month's average sales), or through a person's informal tendency to "wait-and-see" before taking significant action. Both of these processes involve averaging past information to form perceptions of current conditions. However, informal smoothing usually places more weight on recent events, while numeric averaging often places equal weight on all information within a discrete time period.[1] This paper examines the generic structure of exponential smoothing, which is used to model informal "psychological smoothing" present in many decision-making processes. Formal numerical averaging processes will not be discussed in this paper. People in real systems often "psychologically smooth" information before making decisions, even if that information has already gone through a formal numeric averaging process.

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[1] There are, however, methods of numerically weighting data which are uncommon in System Dynamics models. For a more thorough mathematical discussion of the process of information averaging, see Jay Forrester, 1961. *Industrial Dynamics*. Waltham, MA: Pegasus Communications, 464 pp.

3. EXPONENTIAL SMOOTHING

People base decisions for action on their perception of current conditions. This perception is often derived from informally smoothed information. This section discusses two models in which smoothed information is used to make decisions.

3.1 Example: The Lemonade Stand

Recall the lemonade stand model from *An Introduction to Sensitivity Analysis.*[2] The exponential smoothing component of the model is shown below in Figure 1. Howard, an owner of a lemonade stand, must estimate lemonade demand in order to determine how much lemonade to make. Howard forms his estimate based on his experience with past lemonade demand. If demand for lemonade changes suddenly from what it has historically been, Howard takes about an hour to distinguish a permanent shift in sales from random fluctuations, and then update his estimate accordingly. The rate at which Howard changes his estimate is determined by the difference between "BUYING LEMONADE" and "Expected Lemonade Buying" (the difference is embedded in the equation for "change in buying expectations"), and by a time constant that dictates how quickly Howard closes the gap. This time constant, called "TIME TO AVERAGE LEMONADE BUYING" and equal to one hour, is how long Howard takes to change his estimate of lemonade demand.

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[2] See Lucia Breierova and Mark Choudhari, 1996. *An Introduction to Sensitivity Analysis* (D-4526), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, September 6, 38 pp.
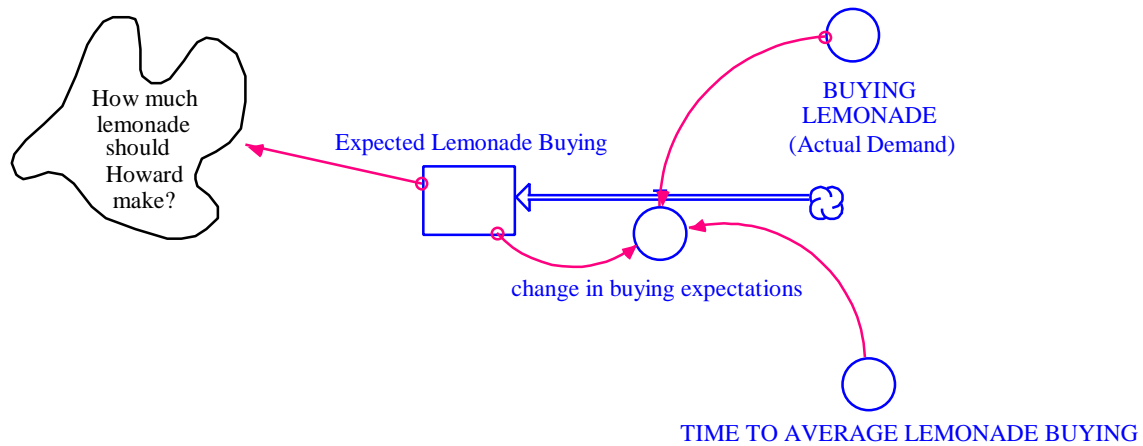
Figure 1: Exponential Smoothing of Expected Lemonade Buying

In the lemonade stand model discussion, it was shown that a delay was introduced into Howard's reaction to demand because it took time for Howard to change his perception of lemonade buying (demand) if demand changed suddenly. This delay was partially responsible for Howard's inability to keep his lemonade inventory at its desired level following a permanent change in actual demand. Howard did not recognize a shift in demand immediately, and thus he did not change his lemonade production fast enough to respond effectively.

3.2 Automobile Quality

As another example, consider the quality of automobiles made by Lux Motor Company (LMC), as perceived by customers. When deciding whether or not to buy an LMC car, a consumer will consider how long she thinks a typical LMC car will last. She will base this estimate of average LMC car longevity on reports she hears from friends and media about the lifespan of LMC cars. Figure 2 shows a model structure representing perceived longevity of LMC cars.
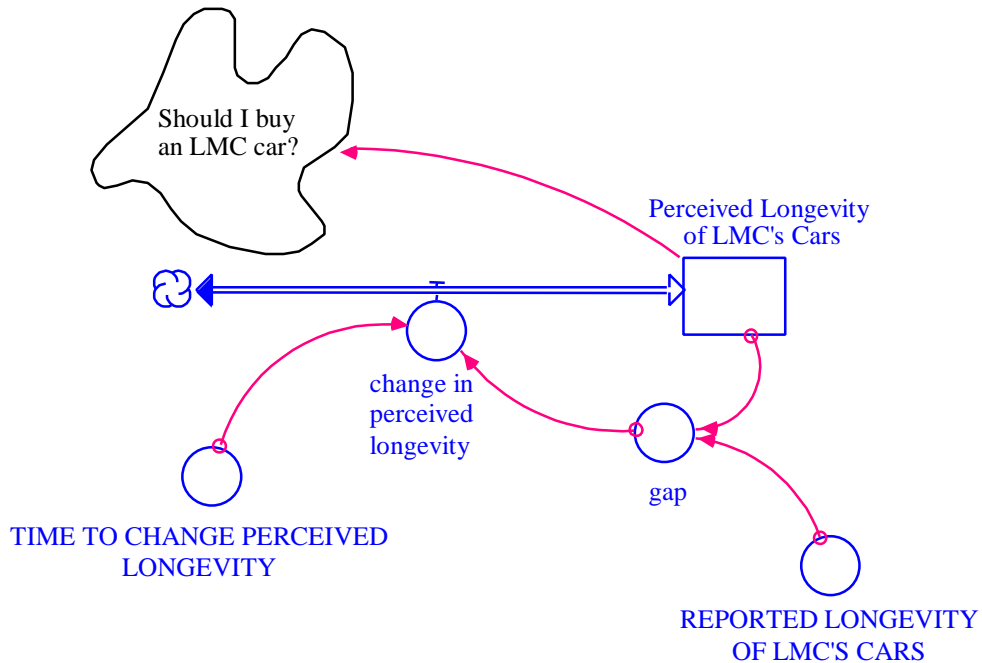
Figure 2: Exponential Smoothing of Perceived Car Longevity

"Perceived Longevity of LMC Cars," accumulates "change in perceived longevity." Changes in perception result whenever a discrepancy exists between reported and perceived longevity. "TIME TO CHANGE PERCEIVED LONGEVITY" determines how quickly "gap" is closed.

A consumer will not believe every report she hears. If LMC has consistently produced long-lasting autos and most consumers speak highly of LMC's quality, a potential buyer might dismiss one bad report she hears as a fluke. If bad reports persist for some time, however, the buyer will eventually be convinced of an overall decrease in LMC car quality and become hesitant to purchase an LMC car.

In this example, a buyer's habit of not immediately believing every report she hears prevents her from overreacting to changes in incoming information.

4. THE GENERIC STRUCTURE

4.1 Model Diagram

The previous two examples share the same basic first-order, negative feedback structure. The generic structure most often used to represent exponential smoothing of information is shown in Figure 3. Equations can be found in Section 4.2.
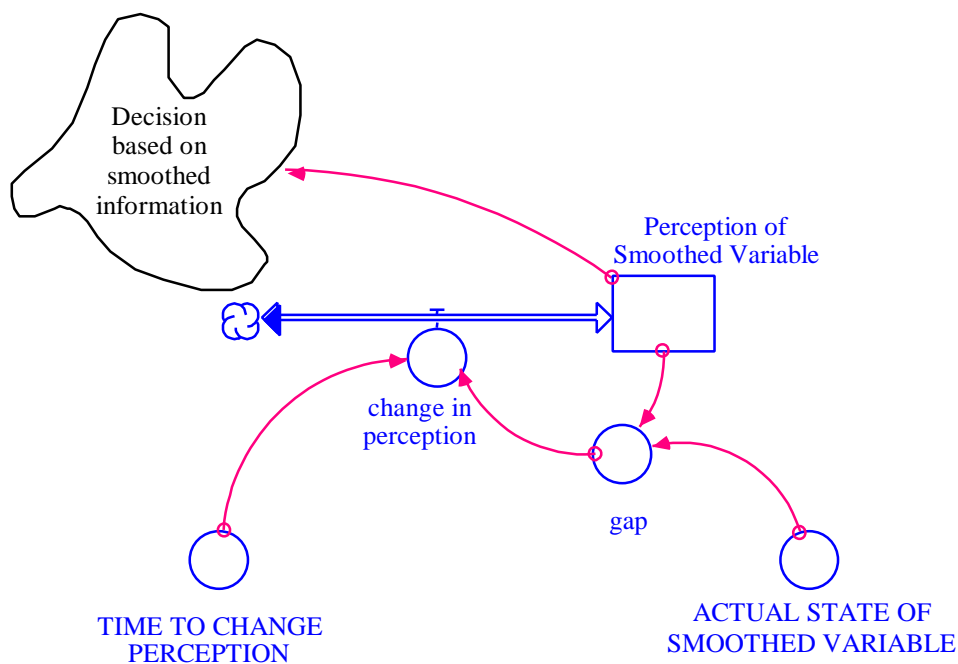

Figure 3: Exponential Smoothing Generic Structure

The structure consists of one negative feedback loop surrounding the stock, "Perception of Smoothed Variable." The stock accumulates "change in perception" as information is updated. Changes in perception are driven by a discrepancy between perception and incoming information. The rate at which "gap" is closed is determined by "TIME TO CHANGE PERCEPTION." "TIME TO CHANGE PERCEPTION" aggregates many influences that determine how quickly information is perceived and believed. These influences might include a belief of information source reliability,

consistency of past information, and how entrenched prior beliefs are. A large "TIME TO CHANGE PERCEPTION" indicates that one is slow to perceive and/or believe a change in "ACTUAL STATE OF SMOOTHED VARIABLE." Figure 4 shows the effect of changing "TIME TO CHANGE PERCEPTION" (TTCP) on the speed with which the discrepancy between perceived and actual state is closed. [3]
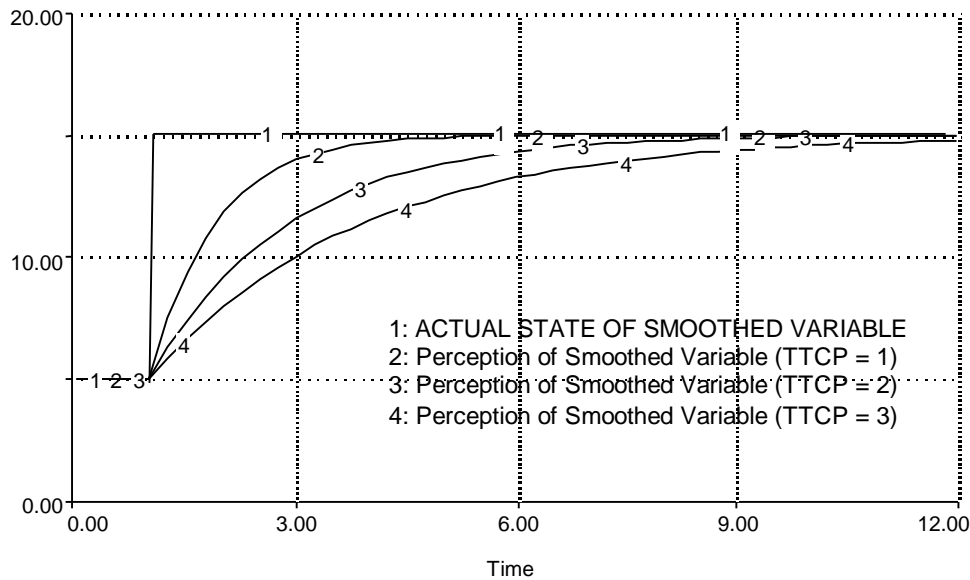

Figure 4: Changing "TIME TO CHANGE PERCEPTION"

Furthermore, an exponential smoothing structure is often one component of a larger system model. Smoothed information contained in a perception stock is often used as a basis for making decisions at other points in a system. For example, "Expected Lemonade Buying" in the lemonade stand model is used to determine how much lemonade Howard makes, while "Perceived Longevity of LMC's Cars" might be used to determine whether or not a customer will buy one.

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[3] For a discussion of the effects of changing "TIME TO CHANGE PERCEPTION" on model behavior, see Lucia Breierova and Mark Choudhari, 1996. *An Introduction to Sensitivity Analysis* (D-4526), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, September 6, 38 pp.

4.2 Model Equations

The following equations are used to formulate exponential smoothing as shown in Figure 3.

Perception_of_Smoothed_Variable(t) = Perception_of_Smoothed_Variable (t - dt) + (change_in_perception) * dt

       INIT Perception_of_Smoothed_Variable =

       ACTUAL_STATE_OF_SMOOTHED_VARIABLE

       DOCUMENT: A decision-maker's perception of the actual state of a smoothed variable.

       He makes decisions and takes actions based on this perception.

       Units: units of smoothed variable

change_in_perception = gap / TIME_TO_CHANGE_PERCEPTION

       DOCUMENT: The rate at which a decision maker's perception changes.   This is

       determined by the discrepancy between the perception and actual states ("gap"), and by the

       time it takes the decision-maker to react to changes in incoming information.

       Units: (units of smoothed variable) / (unit of time)

ACTUAL_STATE_OF_SMOOTHED_VARIABLE = *can be a constant, a changing input, an auxiliary variable, or a stock.*

       DOCUMENT: The actual value of a variable that a decision-maker is trying to estimate.

       The actual state often changes with time due to changing real conditions of the

       environment.

       Units: units of smoothed variable

gap = ACTUAL_STATE_OF_SMOOTHED_VARIABLE - Perception_of_Smoothed_Variable

       DOCUMENT: The gap is the difference between the actual and perceived value of the

       variable of interest.   The presence of a "gap" causes a decision-maker to update his

       perception.

       Units: units of smoothed variable

TIME_TO_CHANGE_PERCEPTION = *usually a constant*

DOCUMENT: The average time it takes a decision maker to recognize and believe that a change in the actual state of the smoothed variable reflects a permanent shift in the state of the system, as opposed to a random fluctuation.

Units: unit of time

5. BEHAVIOR PRODUCED BY EXPONENTIAL SMOOTHING

This section will discuss behaviors produced by an exponential smoothing structure when changes occur in the actual state of a system. It will become clear that using smoothed information as a basis for decision-making has two effects:

1. A delay is introduced into the decision-making process.

2. A decision-maker "filters out" (does not completely react to) fluctuations in the actual state of a system.

These two characteristics will be examined by observing the response of "Perception of Smoothed Variable" to step, pulse, and multiple step changes in "ACTUAL STATE OF SMOOTHED VARIABLE." "TIME TO CHANGE PERCEPTION" is equal to one time unit in all simulations.

5.1 Step Change in Actual State

Figure 5 shows the response of an exponential smoothing structure to a step increase in "ACTUAL STATE OF SMOOTHED VARIABLE."   After starting in equilibrium (where actual and perceived conditions are equal) actual state is increased at time = 1, and remains at a higher value. Perception slowly increases to adjust to the new condition. Eventually, the decision-maker comes to perceive actual conditions, returning the system to equilibrium. Note that perception is changing fastest immediately after actual state increases. The rate of change of perception is proportional to the discrepancy between actual and perceived states. This discrepancy is maximum at the instant following actual state's increase.
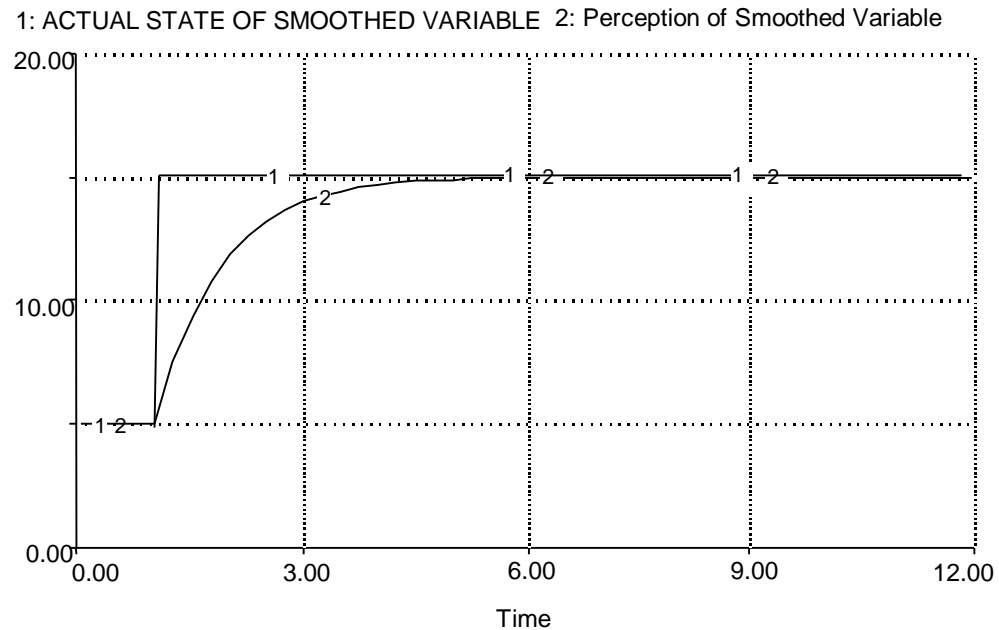
Figure 5: Response of Perception to Step Change

The response of an exponential smoothing structure to a step change most clearly demonstrates the delay introduced by information smoothing. Because perceptions change gradually, any action based on a perception will be delayed for some time after an actual change in a system. Delay length is determined by "TIME TO CHANGE PERCEPTION."

A gradual response is what one would expect from a real-life system. Recall Howard's lemonade stand. Suppose sales have been constant for some time so that Howard has a good idea of what demand actually is. If demand suddenly doubles, Howard will not respond by doubling his lemonade production immediately. Initially he will not believe that long-term demand has actually doubled because a jump could just be a short-term fluctuation, and he would not want to make more lemonade than he could sell. His estimate of demand might start increasing rapidly, however, because sales have dramatically increased. If demand stays high, Howard will eventually believe the increase is permanent and double his lemonade making accordingly.

5.2 Pulse Change in Actual State

Figure 6 shows the response of an exponential smoothing structure to a pulse change in "ACTUAL STATE OF SMOOTHED VARIABLE."   After starting in equilibrium, actual state jumps to a value five times its original, but only for an instant. Directly following the pulse, perception begins to rise quickly. As more information is received which indicates that the increase was short-lived, however, perception begins to decrease and smoothly approach its original value. As can be seen in Figure 6, the peak of "Perceived State of Smoothed Variable" is merely a fraction of the peak in "ACTUAL STATE OF SMOOTHED VARIABLE."

The response of the structure to a pulse change clearly demonstrates the second characteristic of exponential smoothing behavior: "filtering out" non-permanent fluctuations. Because the actual change was short-lived, slow-changing perception had very little time to increase before additional information caused perception to decrease to the original level. As a result, exponentially smoothed perception "filtered out" the short-lived fluctuation.
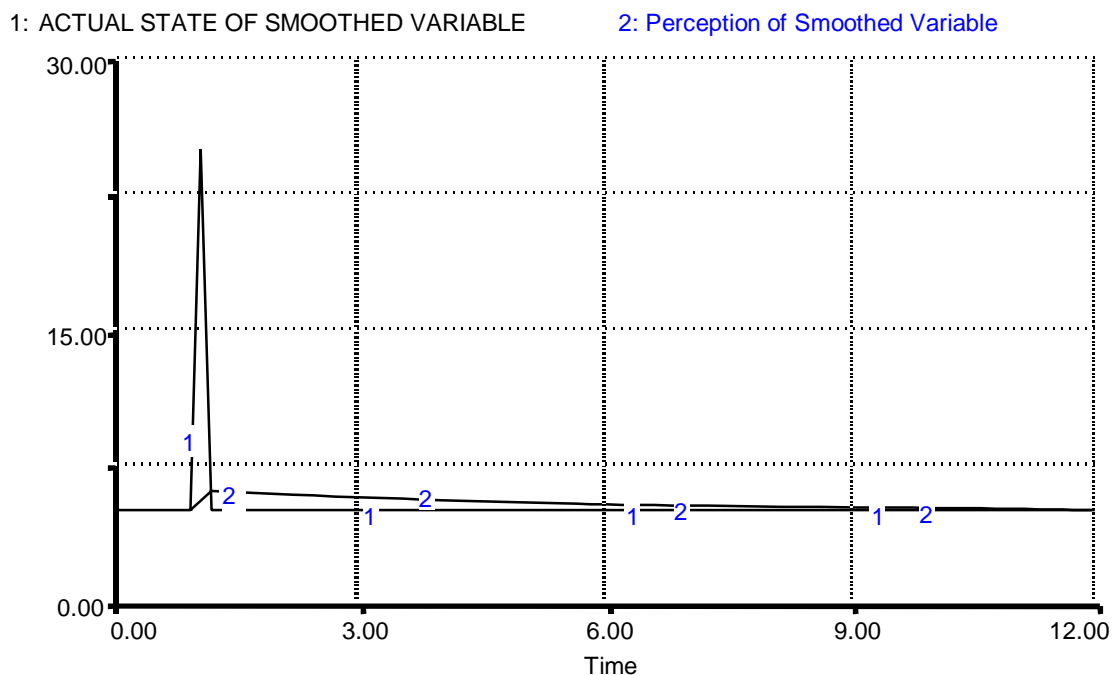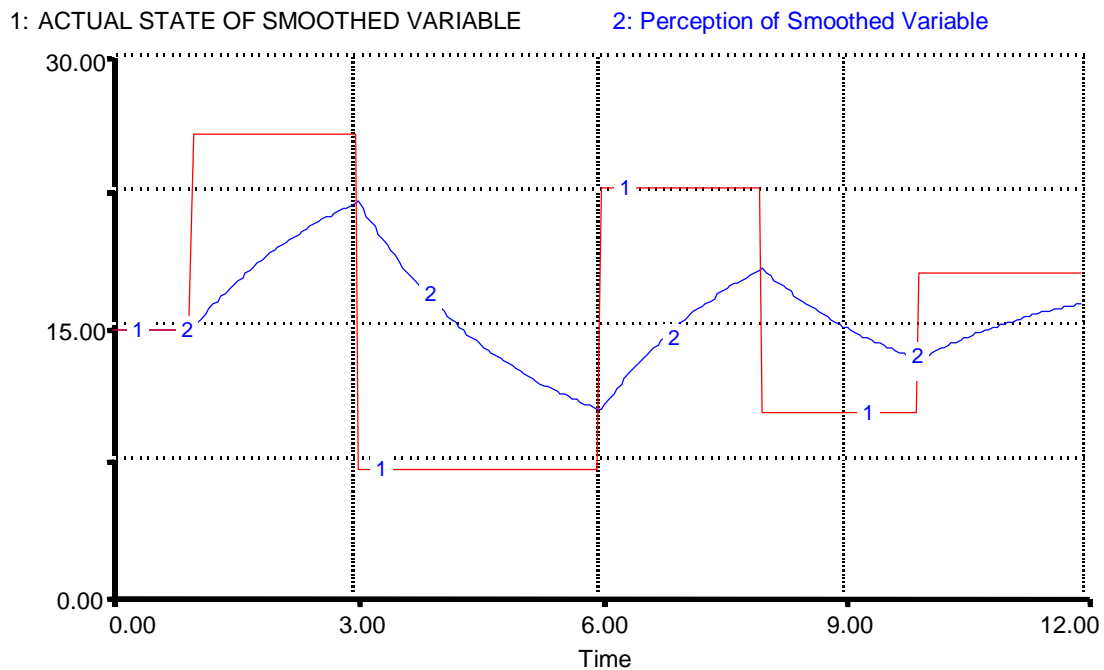
Figure 6: Response of Perception to Pulse Change

Recall the automobile longevity model. Imagine that Lux Motor Company has long been known for poor quality and a short car lifetime. If a potential buyer was aware of this reputation, but then was told by a friend that his LMC car had lasted a very long time, her perception of LMC car longevity would begin to rise. However, if she continued to hear reports claiming that LMC cars are short-lived, she would soon regain her original perception of poor LMC quality, thinking that her friend's tale was not representative of typical LMC cars.

5.3 Multiple Step Changes in Actual State

As a final consideration, this paper examines the response of an exponential smoothing structure to multiple step changes in "ACTUAL STATE OF SMOOTHED VARIABLE." In Figure 7, the actual state is a series of step increases and decreases of varying magnitudes and duration.[4] Following every change, perception begins to slowly approach the new goal. In each instance, however, the actual state changes before perception can adjust completely to the previous change.

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[4] Often in real systems, "ACTUAL STATE OF SMOOTHED VARIABLE" is highly fluctuating.

Figure 7: Response of Perception to Multiple Step Changes

The smoothing response to a fluctuating actual state clearly demonstrates both characteristics of typical exponential smoothing behavior. Changes in perception lag behind changes in the actual state (causing action to be delayed when decisions are based on this perception) and the magnitude of non-permanent changes is reduced (or "smoothed"). Both of these effects can be seen in Figure 7. The peaks of the graph of perception lag behind those of the actual state. Also, perception smoothing decreases the magnitudes of the fluctuations relative to the magnitudes of the actual fluctuations. The perception behavior in Figure 7 is a combination of the effects discussed in Sections 5.1 and 5.2. As with a single step change, response is gradual because of smoothing. Also, similar to a pulse change, amplitudes of fluctuations are reduced because perception is not given enough time to fully adjust to changes before another change occurs.

6. USING EXPONENTIAL SMOOTHING IN MODELS

6.1 Exercise: Road Conditions

This exercise discusses a driver's expectations of weather conditions. March usually marks the end of winter and the beginning of spring, though it is always unclear when this transition will come. Todd is trying to decide whether or not to remove the snow tires from his car. On one hand, he does not want to remove them if winter has not really passed and there is still a possibility of snowfall. On the other hand, his snow tires wear fairly rapidly on dry road, so he wants to remove them if spring has indeed arrived.

Using the generic structure from Section 4, formulate a model to represent Todd's perception of whether spring has come. This perception determines whether he should changeover his tires. Todd bases his perception on the road conditions (the actual iciness of the road) he encounters daily. Road iciness can be measured on a scale from either 0 to 1, or from 0 to 100, representing the fraction or percentage of the road covered by ice, respectively. Todd will change his tires if his expectation of road coverage ever drops below 10% (indicating that spring has arrived). Assume that it takes Todd a week to

become convinced that an improvement in road conditions is permanent. Also assume that Todd initially expects the road to be half covered by ice.

     After building the model, sketch how you estimate Todd's perception will change when confronted with the changing weather conditions shown in Figure 8. (You should sketch your estimate on the graph.) Will Todd change his snow tires? If yes, when?
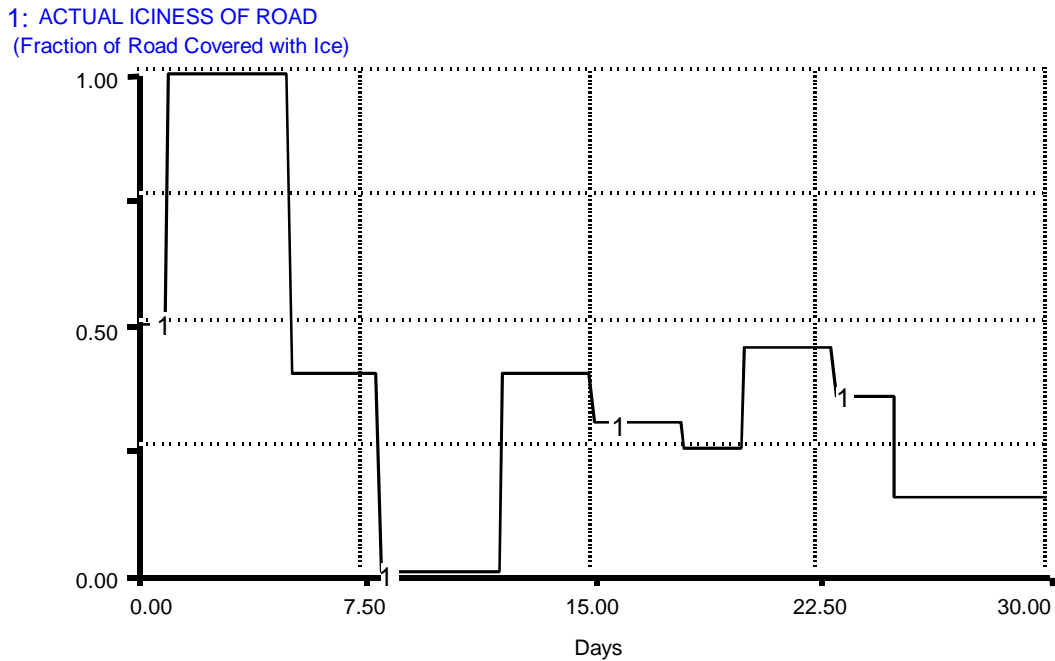
1: ACTUAL ICINESS OF ROAD
(Fraction of Road Covered with Ice)

Figure 8: Behavior of "ACTUAL ICINESS OF ROAD"

6.2 Solution to Exercise

Figure 9 shows a possible road condition perception model.   Documented equations for the model can be found in Section 7.3 in the Appendix.
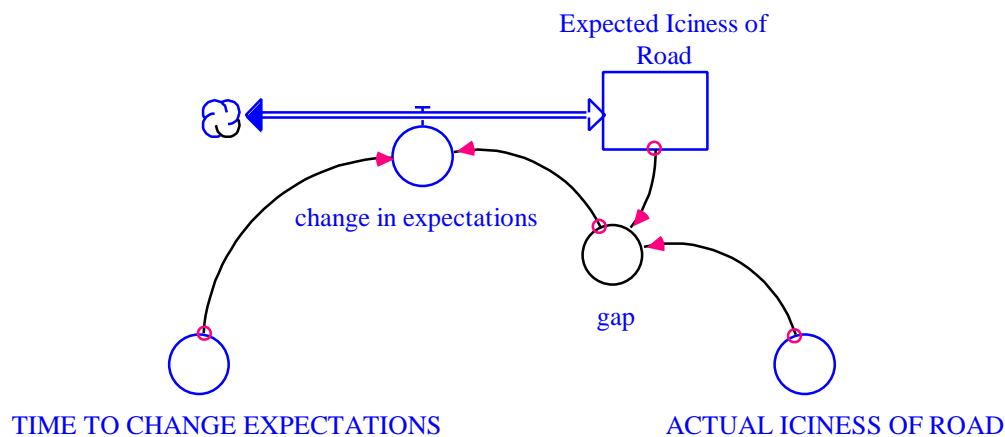
Figure 9 : Model of Expectation of Road Conditions

Todd's decision of whether to remove his snow tires can be modeled as an exponential smoothing of "ACTUAL ICINESS OF ROAD." If Todd sees a sudden drop in the amount of ice on the road, he is unlikely to believe that spring has arrived and that the roads will remain clear, unless the warm weather holds for an extended period of time. Similarly, if the roads are suddenly covered with ice one day, Todd will probably not immediately give up hope for an early spring. Consequently, if the weather is highly fluctuating, Todd is unlikely to change his expectations much either way. A simulation of this model is shown in Figure 10. Despite the fact that road iciness does drop below 10% at one point during the month, Todd's road condition expectation does not. Thus, Todd will not change his tires.
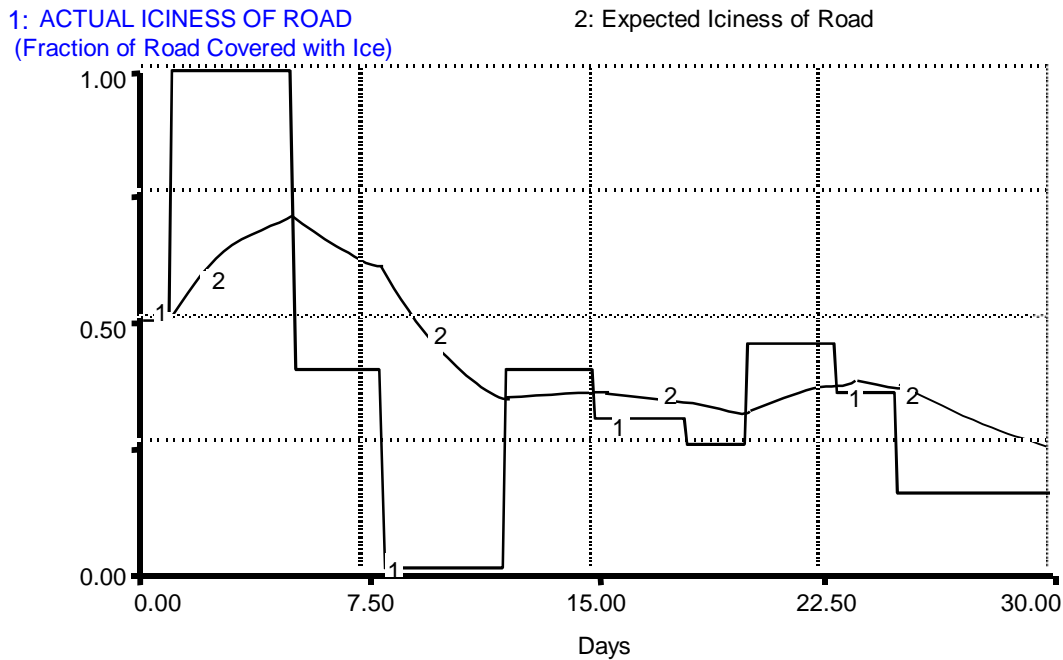
Figure 10: Response of Todd's Perceptions to Changes in
"ACTUAL ICINESS OF THE ROAD"

7. APPENDIX: MODEL EQUATIONS

7.1 Lemonade Stand Model

Expected_Lemonade_Buying(t) = Expected_Lemonade_Buying(t - dt) +
(change_in_buying_expectations) * dt
    INIT Expected_Lemonade_Buying = BUYING_LEMONADE
    DOCUMENT: The hourly demand for lemonade that Howard expects.
    Units: cups/hour

change_in_buying_expectations =
(BUYING_LEMONADE - Expected_Lemonade_Buying) /
TIME_TO_AVERAGE_LEMONADE_BUYING
    DOCUMENT: The rate at which Howard's expectations about demand for
    lemonade change.
    Units: (cups/hour)/hour

TIME_TO_AVERAGE_LEMONADE_BUYING = 1
    DOCUMENT: The time it takes Howard to recognize a permanent change in
    demand for lemonade from random fluctuations.

Units: hours

BUYING_LEMONADE = 20 + STEP(5,1)
  DOCUMENT: The hourly demand for lemonade.
  Units: cups/hour

7.2 Automobile Quality Model

Perceived_Longevity_of_LMC's_Cars(t) = Perceived_Longevity_of_LMC's_Cars (t - dt) + (change_in_perceived_longevity) * dt

  INIT   Perceived_Longevity_of_LMC's_Cars   =
  REPORTED_LONGEVITY_OF_LMC'S_CARS

  DOCUMENT: The perceived longevity of the cars produced by Lux motor company is changed by the reported longevity. The perceived quality will influence a buyer's decision to buy an LMC car.

  Units: years

change_in_perceived_longevity =
 gap / TIME_TO_CHANGE_PERCEIVED_LONGEVITY

  DOCUMENT: The rate at which a buyer changes her perception of the quality (longevity) of a typical Lux Motor Company car.

  Units: years / month

gap = REPORTED_LONGEVITY_OF_LMC'S_CARS -
Perceived_Longevity_of_LMC's_Cars

  DOCUMENT: The discrepancy between LMC car longevity reported to a potential buyer and the longevity perceived prior to receiving the new information.

  Units: years

TIME_TO_CHANGE_PERCEIVED_LONGEVITY = 3

  DOCUMENT: This reflects the time it takes to the buyer to change her perception of LMC in response to changes in the quality reported to her.

Units: month

REPORTED_LONGEVITY_OF_LMC'S_CARS = 5 + PULSE(5,1)

DOCUMENT: The   longevity of Lux Motor Company Cars reported to the potential buyer by the media and by friends.

Units: years

7.3 Road Conditions Model

Expected_Iciness_of_Road(t) = Expected_Iciness_of_Road(t - dt) +

(change_in_expectations) * dt

INIT Expected_Iciness_of_Road = ACTUAL_ICINESS_OF_ROAD

DOCUMENT: The expected iciness of the road is simply the fraction of the road that the driver expects will be covered by a layer of ice the following day.

Units: dimensionless (expressed as a percentage)

change_in_expectations = gap / TIME_TO_CHANGE_EXPECTATIONS

DOCUMENT: the rate at which the driver's expectation about road conditions change.

Units: dimensionless/days

ACTUAL_ICINESS_OF_ROAD =

.5 + STEP(.5,1) - STEP(.6,5) - STEP(.4,8) + STEP(.4,12) - STEP(.1,15) - STEP(.05,18)

+ STEP(.2,20) - STEP(.1,23) - STEP(.2,25)

DOCUMENT: The actual iciness of road is the actual fraction of the road that is covered by ice. This depends on the unpredictable weather.

Units: dimensionless

gap = ACTUAL_ICINESS_OF_ROAD - Expected_Iciness_of_Road

DOCUMENT: The gap is the difference between the actual iciness of the road and the road conditions the driver expected to find.

Units: dimensionless

TIME_TO_CHANGE_EXPECTATIONS = 7

DOCUMENT: the time it takes the driver to believe a change in road conditions is permanent.

Units: days