Guided Study Program in System Dynamics System Dynamics in Education Project System Dynamics Group MIT Sloan School of Management¹

Solutions to Assignment #6 November 3, 1998

Reading Assignment:

Please refer to Road Maps 3: A Guide to Learning System Dynamics (D-4503-4) and read the following:

- Introduction to Computer Simulation,² by Nancy Roberts, et al.: Chapter 15
- Study Notes in System Dynamics,³ by Michael Goodman: Sections 3.10 and 3.11, and Exercises 6 and 7

Then refer to Road Maps 7: A Guide to Learning System Dynamics (D-4507-1) and read the following paper from Road Maps 7:

• Mistakes and Misunderstandings: Examining Dimensional Inconsistency, by Michael Shayne Gary (D-4452-1)

Exercises:

1. Introduction to Computer Simulation

Please read Chapter 15 of Introduction to Computer Simulation by Nancy Roberts. This reading will reinforce your understanding of positive and negative feedback. There are no exercises assigned on the reading, but you should be sure you are comfortable with all of the concepts covered before proceeding.

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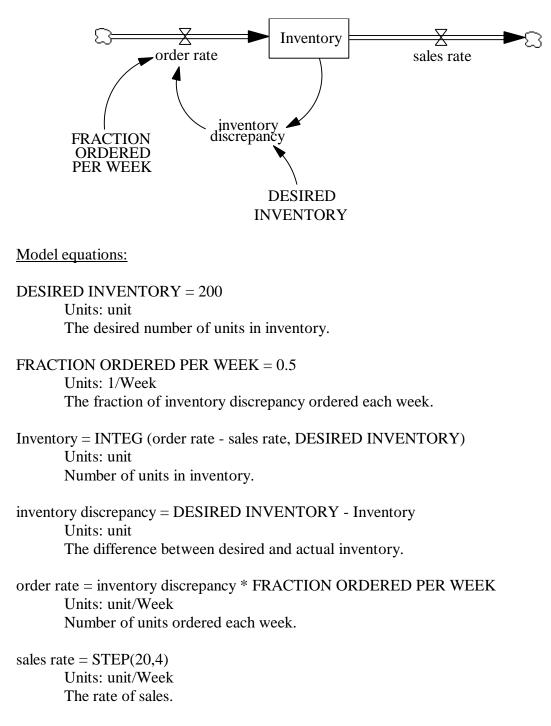
² Roberts, Nancy, David Andersen, Ralph Deal, Michael Garet, and William Shaffer, 1983. *Introduction to Computer Simulation: A System Dynamics Approach*. Portland, OR: Productivity Press. 562 pp.

³ Goodman, Michael R., 1974. *Study Notes in System Dynamics*, Portland, OR: Productivity Press. 388 pp.

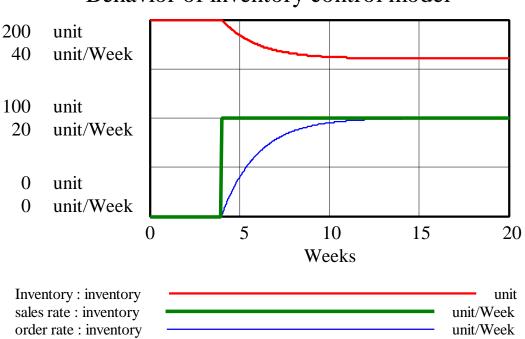
2. Study Notes in System Dynamics (Sections 3.10 and 3.11, Exercises 6 and 7)

A. Read section 3.10. Build and simulate the inventory control model on page 52. In your assignment solutions document, include the model diagram, documented equations, and a graph of model behavior.

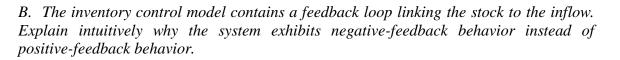
Model diagram:



Model behavior:



Behavior of inventory control model



The inventory-control system is an example of a goal-gap negative-feedback system. After the step increase in "sales rate," the "Inventory" starts decreasing, creating a positive "inventory discrepancy" between the "DESIRED INVENTORY" and "Inventory." Hence, the "order rate" inflow starts increasing, but "Inventory" keeps falling, although at a slower rate. The "inventory discrepancy" therefore keeps increasing, causing the "order rate" to increase further. The system keeps "correcting" itself as long as the "order rate" inflow is smaller than the "sales rate" outflow. Notice that when the "Inventory" stabilizes at equilibrium, the "Inventory" level is not equal to its desired value.

C. What is the half-life of the inventory? Give a mathematical answer and compare it with results from graphs of model behavior.

The half-life of the inventory is:

half-life = 0.7 / FRACTION ORDERED PER WEEK = 0.7 / 0.5 = 1.4 weeks

Notice that this half-life is the half-life of the "inventory discrepancy" between the "Inventory" and the "DESIRED INVENTORY," and not of the stock itself. The half-life now represents the amount of time it takes for the "inventory discrepancy" to close one half of the difference between its initial and equilibrium value, or for the stock to close one half of the difference between its initial value and its equilibrium goal value. These results can be confirmed by looking at graphs of model behavior.

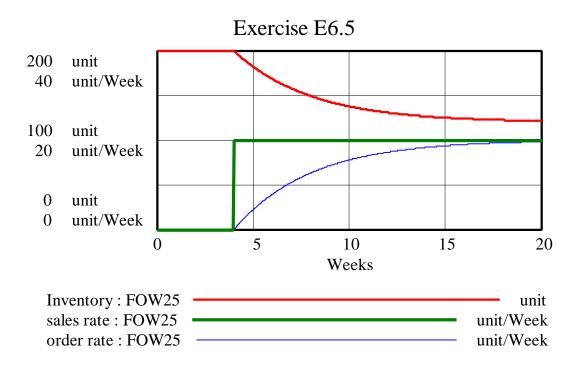
D. Do Exercise 6 on page 183 by hand, then verify your results by simulation. In your assignment solutions document, include graphs of model behavior for questions E6.4 to E6.7.

E6.4

The model behavior is the same as in part A.

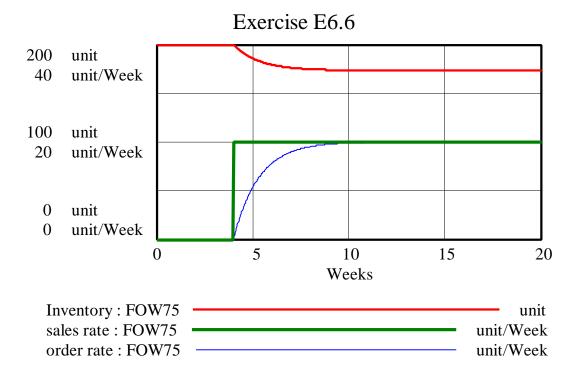
E6.5

When the "FRACTION ORDERED PER WEEK" drops to 0.25/week, the half-life increases to 0.7 / 0.25 = 2.8 weeks. That is, the system delay for responding to changes in the "sales rate" is doubled. The stock then reaches equilibrium after a longer time, and its equilibrium value is lower than in part A. The "order rate" inflow is at equilibrium when it equals the "sales rate" of 20 units per week, but because the half-life is doubled, it takes twice as long to close a half of the gap between the "order rate" and the "sales rate." Hence, the inflow takes longer to reach equilibrium as well.



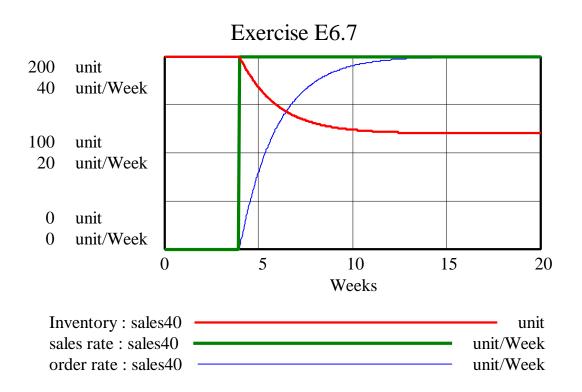
E6.6

If the "FRACTION ORDERED PER WEEK" increases to 0.75/week, the negative-feedback loop closes the gap faster, with half-life equal to 0.7 / 0.75 = 0.933 weeks. The "order rate" as well as "Inventory" approach their equilibrium values quickly, and the equilibrium value of "Inventory" is higher than in part A. Notice how steep the "order rate" is compared to that in exercise E6.5 with a much smaller "FRACTION ORDERED PER WEEK."



E6.7

If the "FRACTION ORDERED PER WEEK" is 0.5/week and the "sales rate" steps up to 40 units/week, the half-life is again equal to 0.7 / 0.5 = 1.4 weeks. The "discrepancy" between "Inventory" and "DESIRED INVENTORY," however, is twice that of part A. Therefore, it takes the same amount of time to reach equilibrium as in part A, but the equilibrium value of "Inventory," 120 units, is twice as far from 200 units as in part A (160 units).



E. Read section 3.11. Examine the liquid cooling model on page 55. Using mental simulation, find the half-life of the coffee heat (use parameter values listed in the equations).

The half-life of the system is:

half-life = 0.7 / HEAT TRANSFER CONSTANT = 0.7 / 0.1 = 7 minutes.

F. Using mental simulation, determine how much time elapses before coffee temperature and room temperature are approximately equal. Explain your answer.

The coffee starts out at 200 degrees, and gradually decreases to a temperature of 78 degrees, the room temperature. The change in the coffee temperature is driven by the difference between the coffee temperature and the room temperature. When the coffee temperature equals the room temperature, the coffee temperature stops changing.

The initial discrepancy between the coffee temperature and the room temperature is 122 degrees. After 7 minutes (one half-life), the discrepancy is halved, bringing the coffee temperature down by (200-78) / 2 = 61 degrees to (200 - 61) = 139 degrees. After 14 minutes, the gap between coffee temperature and room temperature is halved again, by (139-78) / 2 = 30 degrees to 139 - 30 = 109 degrees. After 21 minutes, the temperature is 94 degrees and after 28 minutes, the gap halves again to 86 degrees. After 35 minutes, the temperature is at 82 degrees; after 42 minutes, the temperature is 80 degrees and after 49 minutes, the gap is 1 degree and the coffee temperature is 79

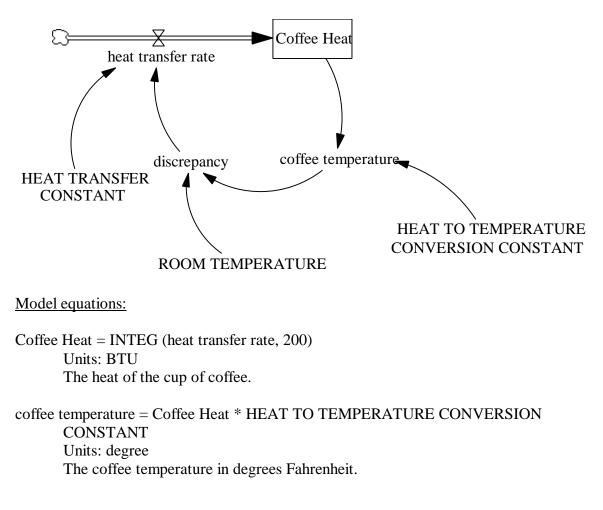
degrees. At 79 degrees, we can consider the coffee temperature and room temperature about equal. Notice that the time elapsed is 49 minutes, or about seven half-lives.

Some participants raised questions about how to decide when the stock has reached equilibrium (the goal) in the case of negative feedback. The stock in reality only approaches the equilibrium, or goal value, and therefore theoretically never reaches the goal value.

The period during which the stock value is changing significantly is called the transient period; the period after the transient period, when the stock is considered to have reached a steady state value, is called the steady state. Often, as an approximation, a time period equal to four times the time constant is considered to be the transient period duration. After three time constants, the value of the stock equals 95% of the equilibrium value, which is often close enough. In this case, the system is in equilibrium after 21 minutes. It is not incorrect, however, to be more precise and give any time greater than 21 minutes as the time when the coffee temperature "reaches" equilibrium. The answer depends on your criteria for what is "close enough" to the goal.

G. Build and simulate the liquid cooling model. In your assignment solutions document, include the model diagram, documented equations, and graphs of model behavior. Were your answers for parts E and F correct? Why or why not?

Model diagram:



discrepancy = ROOM TEMPERATURE - coffee temperature
 Units: degree
 The difference between the room temperature and the coffee temperature. If the
 coffee temperature is higher than room temperature, the discrepancy is negative.

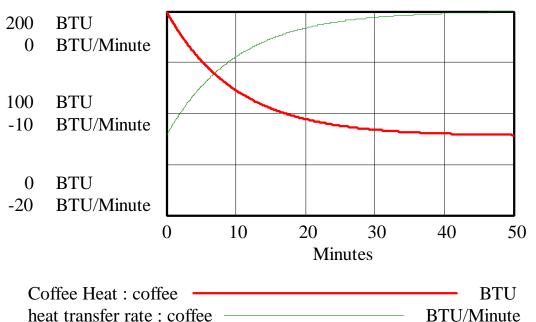
HEAT TO TEMPERATURE CONVERSION CONSTANT = 1 Units: degree/BTU Assume that each BTU converts to 1 degree Fahrenheit.

HEAT TRANSFER CONSTANT = 0.1 Units: BTU/degree/Minute The fraction of coffee heat that is transferred to or from the coffee, given the temperature difference.

heat transfer rate = discrepancy * HEAT TRANSFER CONSTANT Units: BTU/Minute The rate at which heat is transferred to or from coffee.

ROOM TEMPERATURE = 78 Units: degree The normal room temperature in degrees Fahrenheit.

Model behavior:



Coffee Heat and heat transfer rate

The answers in parts E and F were correct.

H. Do Exercise 7 on page 193 and check your answers with the solutions on page 203. You do not need to submit anything for this part. Please let us know, however, if you had problems with any of the exercises.

3. Mistakes and Misunderstandings: Examining Dimensional Inconsistency

Please read this paper carefully. You do not need to submit anything for this reading. The lesson you should learn from it is to always use the units checking feature in Vensim PLE!

4. Independent Modeling Exercise: Population Demographics

This exercise will develop a population dynamics model and use the model to test several different scenarios.

A. Imagine a small town in rural Kansas with 3000 inhabitants. The population is initially evenly distributed between children aged 0 to 20 years, child-bearing adults aged 20 to 45 years, and seniors aged 45 to 75 years (assume that no one lives past the age of 75). In its marital relations, the town is quite typical; when two people are married, they have, on average, 2 children together. You are interested in studying how the demographics of the small town will change over time. You can assume for the purposes of this model that all deaths are related to old age.

The model will contain the elements listed below. Identify each element as either stock, flow, or constant, and label its units. For each stock, determine its inflows and outflows.

- *maturing*
- years as a child-bearing adult
- children
- *number of child-bearing couples*
- deaths
- years as a child
- aging
- births
- seniors
- babies per couple
- *child-bearing adults*
- adults per couple
- years as a senior
- *total population (sum of the populations of all age groups)*

Name	Туре	Units
maturing	flow out of children into child-	people/year
	bearing adults	
years as a child-bearing adult	constant	years
children	stock	people
number of child-bearing couples	auxiliary variable	couples
deaths	flow out of seniors	people
years as a child	constant	years
aging	flow out of child-bearing adults	people/year
	into seniors	
births	flow into children	people/year
seniors	stock	people
babies per couple	constant	people/couple
child-bearing adults	stock	people
adults per couple	constant	people
years as a senior	constant	years
total population	auxiliary variable	people

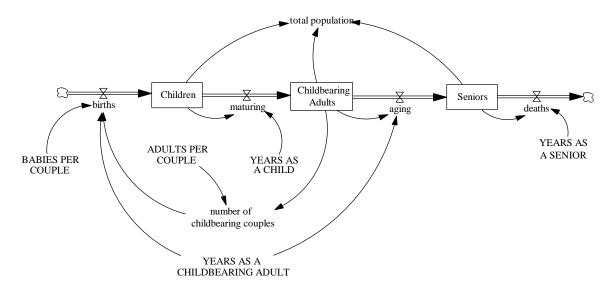
Please note that some elements in the model are auxiliary variables, which are used to disaggregate rate equations into more understandable components.

Notice that "BABIES PER COUPLE" has units of people/couple. One could also define "BABIES PER COUPLE" to have units of people/people, making the units effectively dimensionless. Using more specific units, however, often makes it easier to keep track of different variables while formulating equations. The rate of "births" is then the "NUMBER OF CHILD-BEARING COUPLES" multiplied by the number of "BABIES PER COUPLE," divided by the number of years over which the couple may have children.

Also, notice that the "total population" should not be modeled as a stock with "births" as inflow and "deaths" as outflow. Rather, the "total population" should be modeled as an auxiliary variable that is the sum of the other three stocks. Although in reality, the total population is a stock, modeling the "total population" as a stock would be duplicating other stocks in the model. In addition, if one modified the model to have outflows of deaths from each stock, then modeling the "total population" as a stock would be wrong, but modeling it as an auxiliary sum would be correct.

B. Using Vensim PLE, combine the elements to represent the structure of the system. In your assignment solutions document, please include the model diagram. Hint: One time constant will be used twice in the model to define two different rates.

Model diagram:



C. Using the description of the system provided above, define the equations for all model elements. In your assignment solutions document, please include the documented equations.

Model equations:

ADULTS PER COUPLE = 2

Units: person/couple Number of adults in a child-bearing couple.

- aging = Childbearing Adults / YEARS AS A CHILDBEARING ADULT
 - Units: person/year

If people are child-bearing adults of n years, then on average 1/n of all childbearing adults become seniors each year.

BABIES PER COUPLE = 2

Units: person/couple

The number of children born by each couple during the couple's child-bearing years.

births = number of childbearing couples * BABIES PER COUPLE / YEARS AS A CHILDBEARING ADULT Units: person/year The number of children born by all child-bearing couples per year.

D-4721-1

Childbearing Adults = INTEG (maturing-aging, 1000) Units: person Number of people between 20 and 45 years of age.

Children = INTEG (births-maturing, 1000) Units: person Number of people 20 years of age and younger.

deaths = Seniors / YEARS AS A SENIOR
 Units: person/year
 If people live n years as a senior, then on average 1/n of all seniors die each year.

maturing = Children / YEARS AS A CHILD Units: person/year If people live n years as a child, then on average 1/n of all children will mature to adulthood each year.

- number of childbearing couples = Childbearing Adults / ADULTS PER COUPLE Units: couple Number of couples who are able to have children.
- Seniors = INTEG (aging deaths, 1000) Units: person Number of people between the ages of 45 and 75.
- total population = Children + Childbearing Adults + Seniors Units: person The total population is the sum of all three population groups.

YEARS AS A CHILD = 20

Units: year

The number of years a person is a child. Assuming that a person is a child from birth until the age of 20, a person lives 20 years as a child.

YEARS AS A CHILDBEARING ADULT = 25

Units: year

The number of years during which people can bear children. Assume that people can bear children from when they are 20 years old until when they are 45 years old, over a period of 25 years.

YEARS AS A SENIOR = 30

Units: year

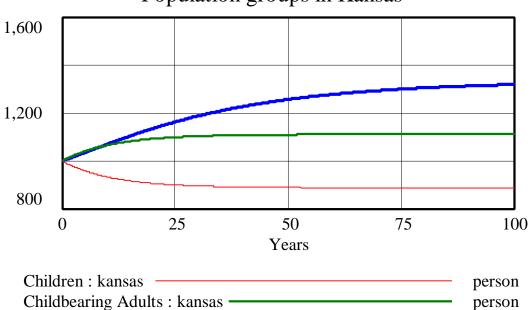
The number of years during which a person lives as a senior. Assume that on average, a person lives as a senior from age 45 until age 75, over a period of 30 years.

D. Before you simulate the model, draw reference modes for all stocks in the model. You do not need to include these in your assignment solutions document, but you do need to draw them in order to be able to answer the next question.

E. Simulate the model over 100 years. Make sure to use a DT that is less than oneeighth of the shortest time constant in the model. In your assignment solutions document, include graphs of the behavior of all stocks and of total population. Did the model generate the behavior you predicted? Why or why not?

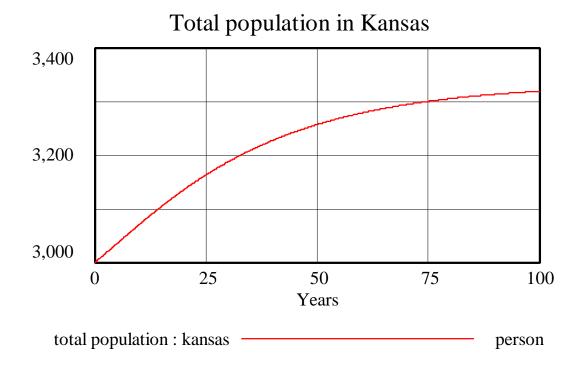
Model behavior:

Seniors : kansas



person

Population groups in Kansas



The behavior of the model makes sense. Each stock starts with 1000 people, and according to the rate equation formulations, during the first time step (DT=0.0625) the stocks behave as follows:

Children: Each person lives 20 years as a child, so 1/20 of the children (50 people initially) leave the stock per year. The stock is, however, being replenished by "births." Initially, there are 1000 child-bearing adults, thus 500 child-bearing couples. If each couple has 2 children over the course of 25 years, then each year a couple has 2/25 children. The total number of children born per year is 500 * 2 / 25 = 40 children. Because the inflow to the "Children" stock is 40 people while the outflow is 50 people, the net flow into the stock is -10 people per year, so the number of "Children" decreases by 0.0625 * 10 = 0.625 people during the first time step. The stock will keep decreasing until the inflow "births" equals the outflow "maturing."

Childbearing Adults: Each person lives 25 years as a child-bearing adult, so 1/25 of the adults (40 people initially) leave the stock each year through the aging process. The stock is, however, replenished by 50 maturing children. The net flow into the stock is 10 people/year, so the number of "Childbearing Adults" increases by 0.0625 * 10 = 0.625 people during the first time step. The stock will continue to grow until the inflow "maturing" equals the outflow "aging."

Seniors: Each person lives 30 years as a senior, so 1/30 of the seniors (1000/30=33) people initially) leave the stock through "deaths". The stock of "Seniors," however, also gains 40 new people through the "aging" inflow. Therefore, the net flow into the stock is 6.67 people per year, so the number of "Seniors" increases by 0.0625 * 6.67 = 0.42

people during the first time step. The stock will continue to grow until the inflow "aging" equals the outflow "deaths."

total population: Add up the net increase or decrease into each stock to see that the total population increases by 0.42 people during the first time step.

To find out whether the system will reach equilibrium or will exhibit exponential behavior, one must analyze the dynamics of the birth loop.

Notice that each couple, in its lifetime, gives birth to two children who essentially replace the couple in the population. The exact substitution gives the impression that the total population will remain unchanged. The population will only be unchanged, however, if the time delays for all stocks are the same. Because it takes 25 years for each couple's two children to enter the "Children" stock but only 20 years for children to mature, the net effect is that the number of maturing children will always be larger than the number of children born. The faster children become "Childbearing Adults," however, the more "Childbearing Adults" in the population, hence the greater the birth rate. Equilibrium is reached when the sum of inflows into each stock equals the sum of outflows from the stock:

For the stock of "Children" to be in equilibrium:

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births = maturing
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number of childbearing couples * BABIES PER COUPLE / YEARS AS A
CHILDBEARING ADULT = Children / YEARS AS A CHILD
(Childbearing Adults / 2)*2 / 25 = Children / 20
Childbearing Adults / 25 = Children / 20
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For the stock of "Childbearing Adults" to be in equilibrium:
maturing = aging
Children / YEARS AS A CHILD = Childbearing Adults / YEARS AS A
CHILDBEARING ADULT
Children / 20 = Childbearing Adults / 25
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For the stock of "Seniors" to be in equilibrium:

aging = deaths

Childbearing Adults / YEARS AS A CHILDBEARING ADULT = Seniors / YEARS

AS A SENIOR

Childbearing Adults / 25 = Seniors / 30
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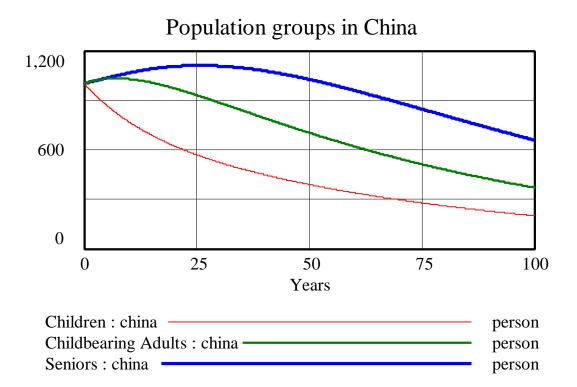
The first two equilibrium equations give the same equality. The third equation is different but does not conflict with the first two. Equilibrium is reached when the system reaches the following proportions:

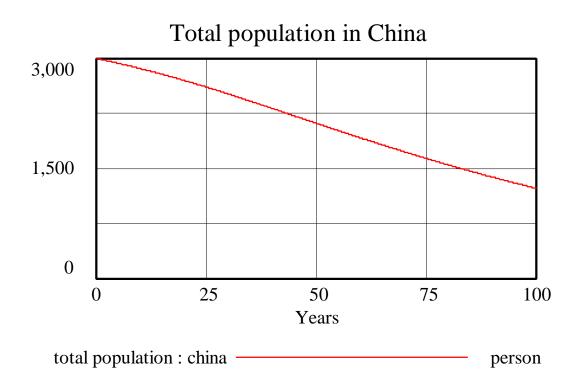
Children / 20 = Childbearing Adults / 25 = Seniors / 30

F. Imagine a small agricultural village in central China. Again, a population of 3000 people is initially evenly distributed between children aged 0 to 20 years, child-bearing adults aged 20 to 45 years, and seniors aged 45 to 75 years. In this village, however, each couple only has one child.

Change the original population model to represent the new scenario. What happens to the population distribution now? Draw reference modes for each stock. Then simulate the model over 100 years. In your assignment solutions document, include graphs of the behavior of all stocks and of total population. Did the model generate the behavior you predicted? Why or why not?

Model behavior in the "China" scenario:





Let us examine the initial behavior (during the first time step of 0.0625) of each stock in this scenario to find the equilibrium values of the system:

Children: The initial outflow is, as before, 50 people per year. The population contains 1000 "Childbearing Adults" forming 500 child-bearing couples. Each couple has only one child in 25 years, or 1/25 children per year. The inflow of "births" is therefore 500 * 1 / 25 = 20 children/year. Thus the net flow in the stock is -30 people per year, so the number of "Children" decreases by 0.0625 * 30 = 1.875 people during the first time step.

The stocks "Childbearing Adults" and "Seniors" behave as before, changing by 0.625 and 0.42 people during the first time step, respectively. The change in the "total population" is -1.875 + 0.625 + 0.42 = -0.83 people during the first time step.

Intuitively, one would think that because each couple is replaced by only one person, the stocks should be decreasing throughout the simulation, until the population of each stock reaches zero. The initial increase in the stocks "Childbearing Adults" and "Seniors" is caused by the delays in the system. That is, although the number of "Children" is decreasing quickly, the decrease does not affect the other stocks right away because of the delays. Only after the number of "Childbearing Adults" starts decreasing because of the decrease in "Children," does the number of "Seniors" start decreasing. Also, as the delay for a stock becomes longer, the effect is felt later. Therefore, the "Seniors" stock peaks after the peak of the "Childbearing Adults" stock.

Using the stock equilibrium equations from before, one can find the relationships of the stocks at equilibrium and show that equilibrium can occur only when the all stocks equal zero:

For the stock of "Children" to be in equilibrium: births = maturing number of childbearing couples * BABIES PER COUPLE / YEARS AS A CHILDBEARING ADULT = Children / YEARS AS A CHILD (Childbearing Adults / 2)*1 / 25 = Children / 20 Childbearing Adults / 50 = Children / 20
For the stock of "Childbearing Adults" to be in equilibrium: maturing = aging Children / YEARS AS A CHILD = Childbearing Adults / YEARS AS A CHILDBEARING ADULT Children/20=Childbearing Adults / 25
For the stock of "Seniors" to be in equilibrium: aging = deaths

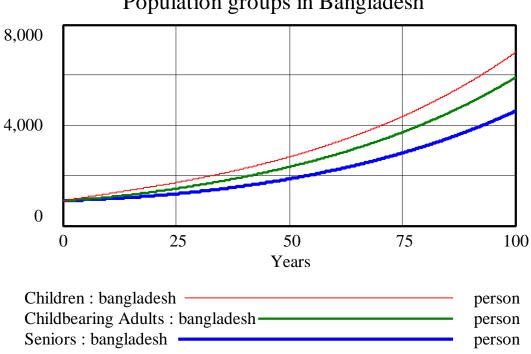
Childbearing Adults / YEARS AS A CHILDBEARING ADULT = Seniors / YEARS AS A SENIOR Childbearing Adults / 25 = Seniors / 30

One can see that the equilibrium equations for "Children" and for "Childbearing Adults" cannot hold true at the same time. The equilibrium equation for "Children" states that for every 20 "Childbearing Adults" there are 50 "Children," while the equilibrium equation for "Childbearing Adults" states that for every 20 "Childbearing Adults" there are 25 Children. Both equations only hold true when the value of both stocks is zero, which implies that the equilibrium value of the "Seniors" stock must also be zero.

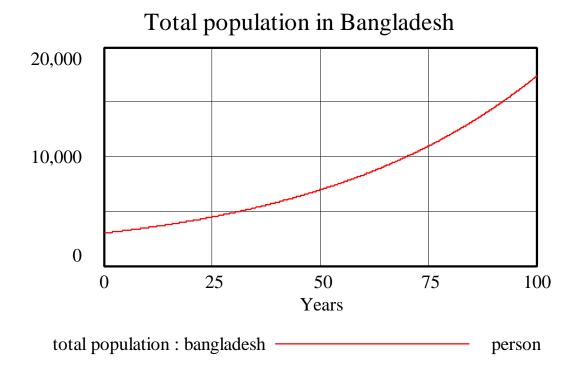
G. Imagine a small village in Bangladesh. A population of 3000 people is also initially evenly distributed between children aged 0 to 20 years, child-bearing adults aged 20 to 45 years, and seniors aged 45 to 75 years. In this village, however, each couple has four children.

Change the original population model to represent the new scenario. What happens to the population distribution now? Draw reference modes for each stock. Then simulate the model over 100 years. In your assignment solutions document, include graphs of the behavior of all stocks and of total population. Did the model generate the behavior you predicted? Why or why not? Can you foresee any problems in Bangladesh that would be caused by the behavior you observed?

Model behavior in the "Bangladesh" scenario:



Population groups in Bangladesh



Because the number of "BABIES PER COUPLE" is 4, it is easy to see that the system exhibits exponential growth. It is interesting to note, however, that the "Children" stock has the sharpest growth, followed by the "Childbearing Adults" stock, and then the "Seniors" stock. Again, this effect is produced by the delays in the system.

Many problems could be caused by such explosive population growth. Depletion of resources, overcrowding, and pollution are just some of them. These factors will slow down the exponential growth and cause the population to stabilize at some equilibrium point by increasing the death rate. Social pressure may cause the birth rate to decrease before these natural factors increase the death rate as a population control measure.

H. Humanitarian envoys attempt to convince the couples in the small village in Bangladesh to reduce the number of children they have. Assume that for some reason the villagers decide to comply with the envoys' request. Starting in year 50, couples go from having 4 children to having 2 children each.

Change the original population model to represent the new scenario. What happens to the population distribution now? Draw reference modes for each stock. Then simulate the model over 100 years. In your assignment solutions document, include graphs of the behavior of all stocks and of total population. Did the model generate the behavior you predicted? Why or why not? How could a system dynamics perspective help in proposing recommendations for developing countries with high birth rates?

To model the change in the number of "BABIES PER COUPLE," the equation should be modified to:

BABIES PER COUPLE = INITIAL BABIES PER COUPLE + STEP (CHANGE IN BABIES PER COUPLE, TIME OF CHANGE)

where:

INITIAL BABIES PER COUPLE = 4

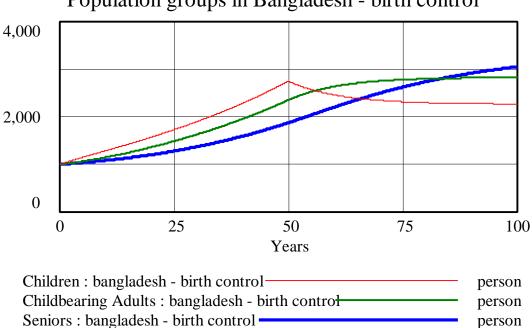
Units: person/couple The initial number of babies per couple in Bangladesh.

CHANGE IN BABIES PER COUPLE = -2

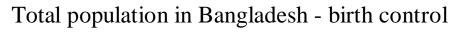
Units: person/couple The change in the number of babies per couple resulting from the efforts of humanitarian envoys.

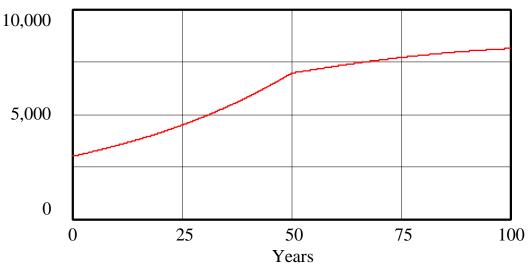
TIME OF CHANGE = 50 Units: year Time at which the change in the number of babies per couple occurs.

Model behavior for the "Bangladesh – birth control" scenario:



Population groups in Bangladesh - birth control





total population : bangladesh - birth control person

The behavior of all population groups and of "total population" shifts from exponential to asymptotic in year 50. Up to year 50, the behavior is the same as in part G (where the number of "BABIES PER COUPLE" was 4). After year 50, the behavior has the same shape as in part A (where the number of "BABIES PER COUPLE" was 2). The envoys' recommendation was effective in controlling population growth.

It is, however, not obvious that the population would continue to increase even after the population-control measures were set in place. Even though the birth rate was reduced to two people (one couple) having two children after the 50th year, the number of "Childbearing Adults" and "Seniors," as well as the "total population" continued to grow. The growth continues because of the delays in the system. Babies born before the 50th year will still grow up, increasing the stocks of adults and seniors. Therefore, for the time of an average lifespan, about 75 years, the population continues to grow.

This problem is similar to the "aging population" situation that many developed countries are starting to face now. Although their birth rates have been low for the past few decades, the fraction of their populations that are comprised of seniors has been increasing. This situation is worsened by the fact that the number of babies per couple in most countries is still a little above 2. This shift in population demographics tends to pose huge problems for Medicare and social security programs.

System dynamics tools can help determine important factors such as how many years it will take before the population halves, or what needs to be the number of "BABIES PER COUPLE" allowed in order to control the population by a certain date. Even the awareness of many of these counter-intuitive behaviors of relatively simple systems helps us to make better decisions because we know that such unexpected things can happen and we are more careful and consider non-obvious results. Implementing these policies, however, is another story.