Exploring

S-Shaped Growth

Prepared for the
MIT System Dynamics in Education Project
Under the Supervision of
Dr. Jay W. Forrester

by
Leslie A. Martin
October 3, 1996

Vensim Examples added October 2001

Table of Contents

1. Introduction

One spring a young couple and small child arrive in Provence, France, carrying bulging suitcases, ardent enthusiasm, and two abnormally large rabbits. They dream of settling down in the countryside and living off of the land. Jean, the husband, is unable to contain the excitement in his voice as he unveils his prized possession to his new neighbors: a carefully folded sheet of paper covered with ink-blotched calculations. Proud to have stumbled upon a get-rich-quick phenomenon known by the name of exponential growth, he explains how his two rabbits will become four then eight then sixteen and soon hundreds upon hundreds... and he is thought to be a madman.

So that summer the rabbits become intimate and produce more rabbits which in turn produce even more rabbits. By mid-summer Jean and his wife have more rabbits than they can control. Rabbits are everywhere. Jean and his wife are ecstatic. They dream eagerly of all the profits they will soon make. But one week Jean realizes that some of his rabbits are dying. They are dying of dehydration. The faster, stronger rabbits quickly lap up the small pools of rainwater in the meadow and leave nothing for the others. Jean prays for a rain storm, but the skies in Provence burn with a throbbing sun. Jean and his wife have to travel several miles to a well to get more water. The young couple finds itself unable to bring back enough water to care for all the rabbits. More die. Jean panics. Now the death rate is so high that for each young rabbit that is born a weak, older rabbit dies. Jean's population has stopped growing. A month later, a year later, he still has the same number of rabbits. The villagers mock the young couple for having bragged about such unrealistic dreams. A disillusioned Jean learns the hard way about limits to growth.

True, sustained exponential growth cannot exist in the real world. Eventually all exponential, amplifying processes will uncover underlying stabilizing processes that act as limits to growth. The shift from exponential to asymptotic growth, or from positive to negative feedback, is known as sigmoidal, or S-shaped, growth.
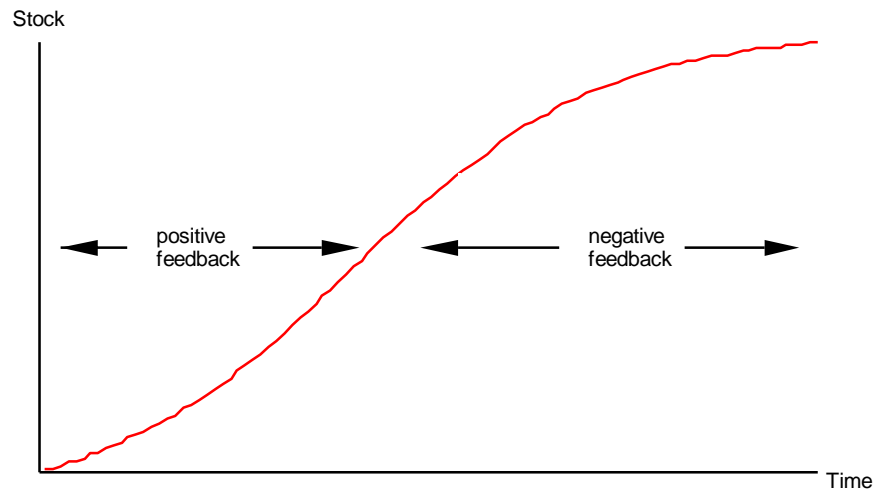
Stock

Time

Figure 1: S-shaped growth

Figure 1 displays a typical S-shaped growth curve. Positive feedback, which generates exponential growth, is tapered by negative feedback, which produces stabilizing growth. S-shaped growth can be observed in a wide variety of phenomena. The spread of fads, rumors, or even a religion is characterized by S-shaped growth. Attention span, concern, and interest also exhibit S-shaped growth. Market saturation and epidemics are classic examples of S-shaped behavior. The cellular growth of a plant and physical and intellectual development in small children, along with the body's immune response, are all subject to S-shaped growth. This paper will begin by exploring population dynamics, taking as an example Jean's population of rabbits.

Exploring S-Shaped Growth will first study the shifting loop dominance that produces S-shaped growth. It will then present a generic structure of stocks and flows associated with S-shaped behavior. Finally this paper will examine several different examples of systems which generate S-shaped growth, emphasizing how a whole range of very distinct phenomena can produce similar behavior.

2. The Generic Structure

2.1 Shifting Loop Dominance

Shifting loop dominance produces S-shaped growth. A system that will exhibit S-shaped growth starts out in a positive feedback loop. A large increase in the positive loop awakens a dormant negative loop. The negative loop does not just spontaneously appear. It is present the entire time, but its strength depends on the strength of a variable in the positive loop. When the positive loop begins to amplify all the variables involved in its cycle, the negative loop is also amplified until the dominance shifts and the negative loop takes over. Figure 2 represents shifting loop dominance.
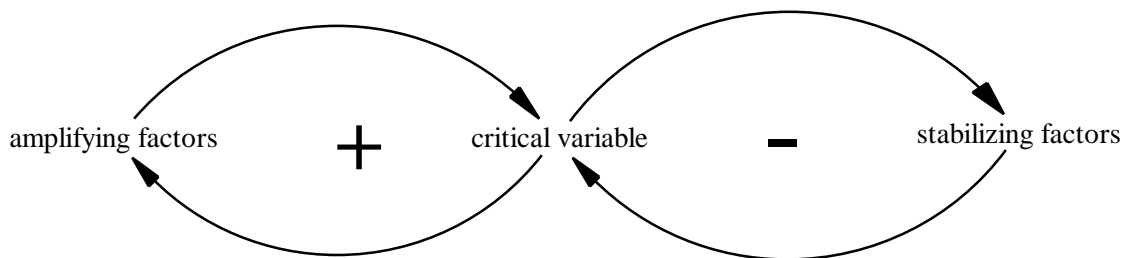

Figure 2: Shifting loop dominance

The critical variable in the behavior of Jean's rabbit population is the number of rabbits that roam around his farm. As the population of rabbits increases, so does the number of rabbit births. More rabbits, more happy couples, more baby rabbits... Births reinforce a positive feedback loop. A negative loop, however, lies dormant. As the population increases the total supply of water stays fixed, so the amount of water available for each rabbit decreases. When the amount of water per rabbit drops low enough, the rabbits will no longer have enough water to sustain themselves, and the weaker rabbits will start to die. The negative loop reduces the population growth rate until the amount of water per rabbit is just large enough to support the rabbit population.

Systems that exhibit S-shaped growth behavior are characterized by constraints, or limits to growth. In the case of Jean's rabbits, the system's constraint is a fixed supply of water. The constraint fixes the maximum number of rabbits that Jean's farm in Provence can support. Likewise, the constraint to the spread of a rumor is the number of people who could potentially be reached. That number could be the total student body on a college campus or a nation of TV viewers, depending on how juicy and pertinent the gossip. The constraint for the spread of an epidemic would be the total population exposed to the disease. Similarly, a company's product can saturate a consumer market. The constraint of the consumer market would be the size of the product's particular market. Exponential growth cannot last forever.

2.2 Generic Structure for S-Shaped Growth

Several different stock and flow structures produce S-shaped growth behavior.[1] Figure 3 represents an intuitive generic structure that explicitly shows both the feedback loops and the constraint on the system. See Appendix 5.1 for documented equations for the generic structure.

---

[1] See also Marc Glick and Terri Duhon, 1994. *Generic Structures: S-Shaped Growth I* (D-4432), System Dynamics is Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, August 24, 30p. and William Glass-Husain, 1991. *Teaching System Dynamics: Looking at Epidemics* (D-4243-3), System Dynamics is Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, October 3, 58p.
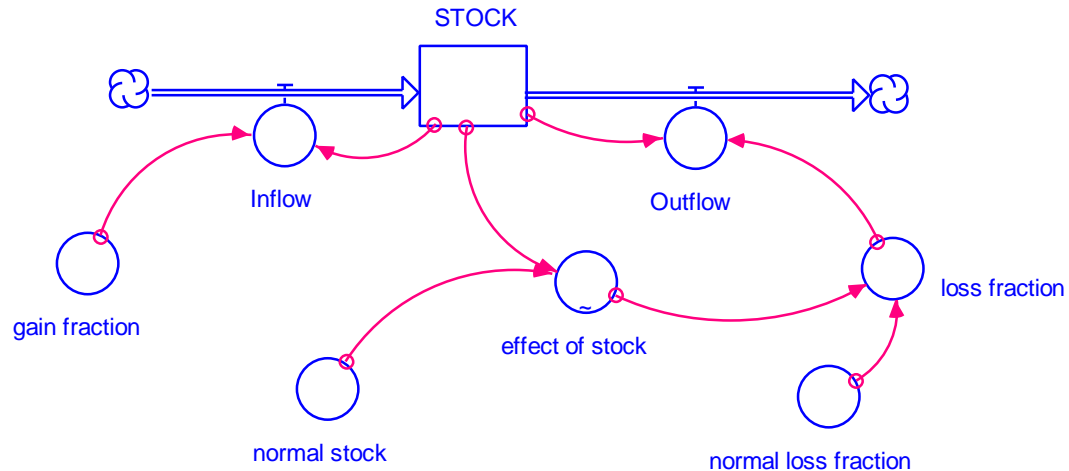
Figure 3: Generic structure that produces S-shaped growth

The model in Figure 3 is composed of three feedback loops. The positive feedback loop is associated with the Inflow to the STOCK. The Inflow, the product of the STOCK and the constant gain_fraction, drives the STOCK into exponential growth.

Two negative feedback loops regulate the Outflow from the STOCK. A connector ties the current value of the STOCK to the Outflow from the STOCK. The Outflow shapes a negative loop. The second negative loop, which passes through the loss_fraction, is responsible for shifting loop dominance.

The STOCK grows initially only if the gain_fraction starts off larger than the loss_fraction. When the gain_fraction is greater than the loss_fraction, the Inflow is greater than the Outflow, and the system experiences exponential growth. The STOCK, however, cannot grow forever. As the STOCK increases, so does the effect_of_stock multiplier. The effect_of_stock multiplier determines the effect of the STOCK on the loss_fraction. When the effect_of_stock multiplier assumes values larger than 1, the loss_fraction increases. The effect_of_stock multiplier will eventually become large enough for the loss_fraction to equal the gain_fraction. Then the Outflow will catch up to the Inflow and growth will cease. The system will reach equilibrium.

Even though the positive feedback loop that originally determines the system's growth is initially stronger than the negative loop, it is at a disadvantage when compared to the negative loop. The strength of the positive loop is constant. The negative loop, on the other hand, becomes stronger with the growth of the STOCK. The Outflow from the STOCK is the product of the STOCK and a *variable* loss_fraction. Here lies the key to the shifting loop dominance: the loss_fraction increases as the STOCK increases. When the STOCK is small the negative feedback loop goes almost unnoticed. But as the STOCK gets larger, the negative loop becomes stronger and stronger. Finally the negative feedback loop drives the system to equilibrium.

What type of curve is in the effect_of_stock table function? The curve in the effect_of_stock table function is determined by the characteristics of the constraint acting upon the specific system in question. In Figure 3 the constraint is modeled as a normal_stock converter which determines how much the STOCK can grow. As the value of the STOCK becomes large with respect to the normal_stock, the loss_fraction increases steadily due to the increased presure of the constraint. So the curve in the effect_of_stock table function increases as the ratio of the STOCK to the normal_stock increases. A sample curve is included in Figure 4.
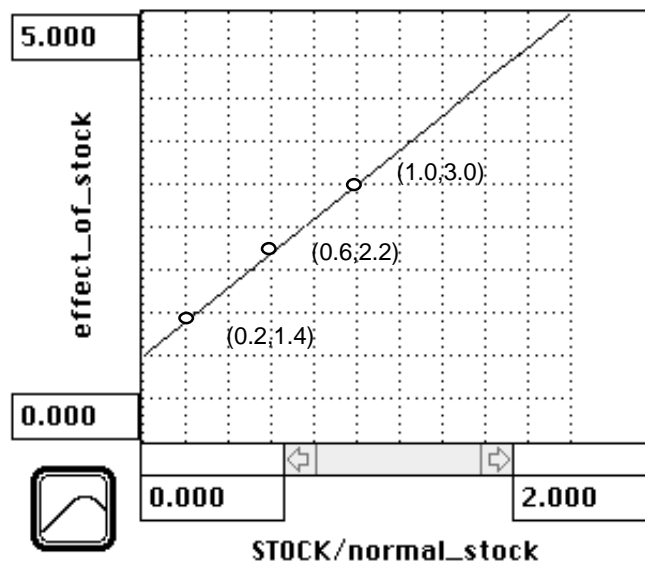
Figure 4: Sample effect_of_stock curve

The effect_of_stock curve outputs a fraction steadily increasing from 1. When the STOCK is very small, the multiplier is close to 1. The loss_fraction is almost identical to the normal_loss_fraction. The positive loop easily dominates. As the STOCK increases, however, so does the effect_ of_stock multiplier. When the STOCK is 20% of the normal_stock, the ratio of STOCK to normal_stock is 0.2. The effect_of_stock multiplier outputs a value of 1.4. So the loss_fraction equals 1.4 times the normal_loss_fraction. When the STOCK is 60% of the normal_stock, the effect_of_stock multiplier outputs a value of 2.2. So the loss_fraction is 2.2 times the normal_ loss_fraction. As the STOCK grows the loss_fraction grows, catching up to the gain_fraction.

Equilibrium occurs when the loss_fraction reaches the gain_fraction. In the generic structure model in Figure 3, the gain_fraction is set to equal 3 times the normal_loss_fraction. So the STOCK reaches equilbrium when the loss_fraction is equal to 3 times the normal_loss_fraction. When does the effect_of_stock multiplier reach the value of 3? The table function outputs a 3 when the ratio of STOCK to normal_stock is 1. So the system finally reaches equilibrium when the STOCK grows to the size of the normal_stock.

Note that the equilibrium value of the STOCK is determined by the relative values of the gain_fraction and normal_loss_fraction and the steepness of the curve in the effect_of_stock multiplier. If the curve in the table function is very steep, then a small growth in STOCK has a strong effect on the loss_fraction. The loss_fraction grows quickly and the Outflow from the STOCK catches up with the Inflow before the STOCK has the chance to become very large. The STOCK ceases to grow before it reaches the size of the normal_stock. On the other hand, if the ratio of the gain_fraction to the normal_loss_fraction is very large, then the loss_fraction has much catching up to do. If the gain_fraction was 5 times the normal_loss_fraction, then equilibrium will not be reached until the effect_of_stock multiplier outputs a value of 5. The effect_of_stock

table function in Figure 4 does not output a value of 5 until the ratio of STOCK to normal_stock has a value of 2. In other words, the STOCK would need to be twice as large as the normal_stock for the system to reach equilibrium.

2.2.1 Applying the Generic Structure to the Rabbit Population Model

Figure 5 is a model of Jean's rabbit population. Assemble the model using STELLA, paying careful attention to the relationship between the variables in the rabbit population and the corresponding variables in the generic structure. See Appendix 5.2 for documented equations of the rabbit population model.
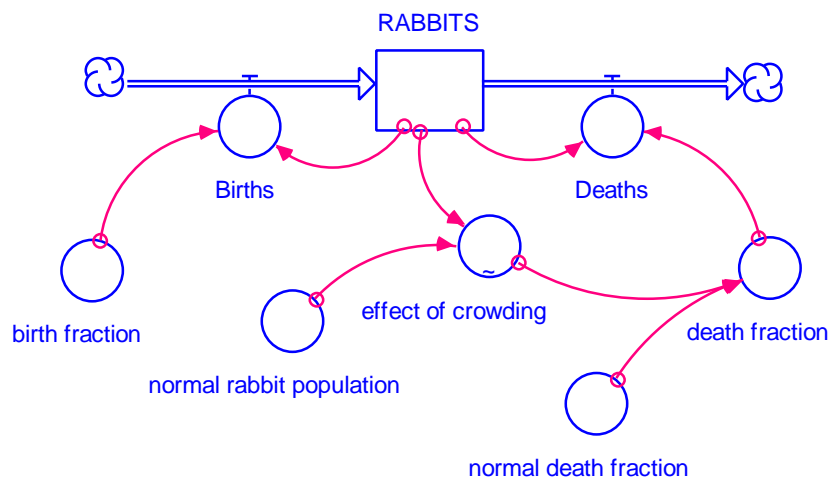
Figure 5: Stock and flow diagram of the rabbit population

Three distinct feedback loops are at work in the population model. The left hand side of the model contains the positive feedback loop. The right hand side of the model displays the two negative loops. The normal_death_fraction is equal to one third of the birth_fraction. Because the birth_fraction is larger than the death_fraction, Births will initially predominate. As more RABBITS are born there are more and more RABBITS ready to give birth to even more RABBITS.

The positive loop cannot go unchecked forever because of the constraint on the system: the environment can only support a maximum rabbit population. The total supply of water available to the rabbits is fixed. The ratio of RABBITS to the normal_rabbit_population determines the effect_of_crowding on the death_fraction. Figure 6 is a plot of the effect_of_crowding table function. The curve in the table function is a representation of the specific constraint of a constant water supply. When there are few rabbits, crowding has no effect on the death_fraction. Water is in abundance. The table function outputs a value of 1, so the death_fraction is equal to the normal_death_fraction. When the rabbit population becomes very large, however, the death_fraction starts to increase as the effect_of_crowding table function outputs a value larger than 1. As many rabbits as possible are drawing from the constant water supply, and the environment's maximum capacity for rabbits is finally reached. The effect_of_crowding table function outputs a value of 3. So the death_fraction equals 3 times the normal_death_fraction. The death_fraction has caught up to the birth_fraction. The number of Deaths reaches the number of Births and the rabbit population system stabilizes at equilibrium.
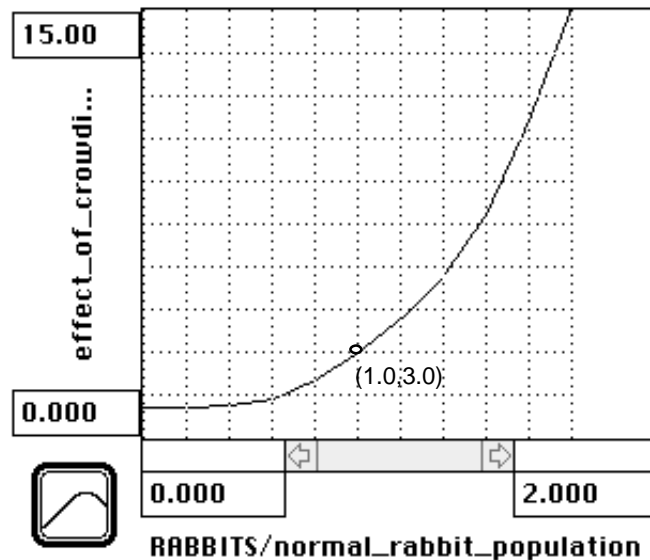

Figure 6: Effect_of_crowding table function

2.2.2 Running the Rabbit Population Model

Set up a graph for the population of RABBITS. Plot the birth_fraction, normal_death_fraction, and death_fraction on another graph. Create a converter that calculates the Net Flow into the stock of RABBITS. The Net Flow is Births (Inflow) minus Deaths (Outflow). Set up a graph for Births, Deaths, and the Net Flow over time. Then run the model over a 24-month time period.
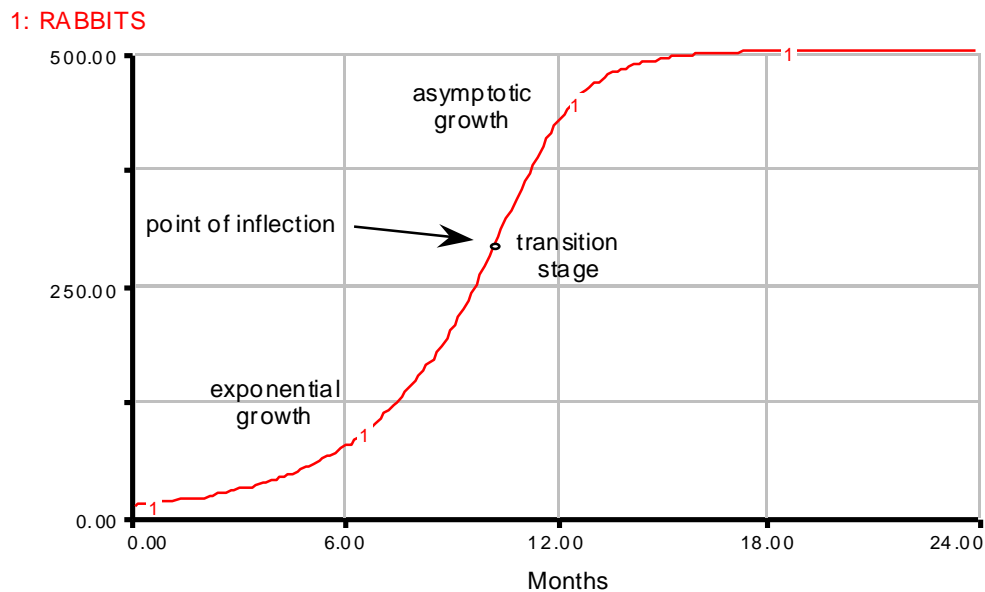
Figure 7: Rabbit population model: stock

Several interesting insights about the nature of S-shaped growth unfold with the examination of the graphs in Figures 7 through 9. First look at the stock, RABBITS, in Figure 7. Notice how RABBITS grows exponentially in the first months. The initial growth is due to the positive feedback loop. Rabbits make rabbits who make more rabbits. Around month 10, however, the curve switches from positive to negative concavity. The rabbit population is starting to feel the constraint of an environment in which resources are limited. Exponential growth has tapered into asymptotic growth. The point on the curve at which the switch occurs is called the "point of inflection." It is sometimes difficult to tell exactly where the point of inflection lies by simply looking at the

graph. Instead of determining the exact value of the point of inflection, it is often practical to simply designate the general region between exponential and asymptotic growth, the "transition stage."

The system's final state is equilibrium. The rabbit population stabilizes as the negative loop generates asymptotic growth. By month 15 the system has almost reached equilibrium.
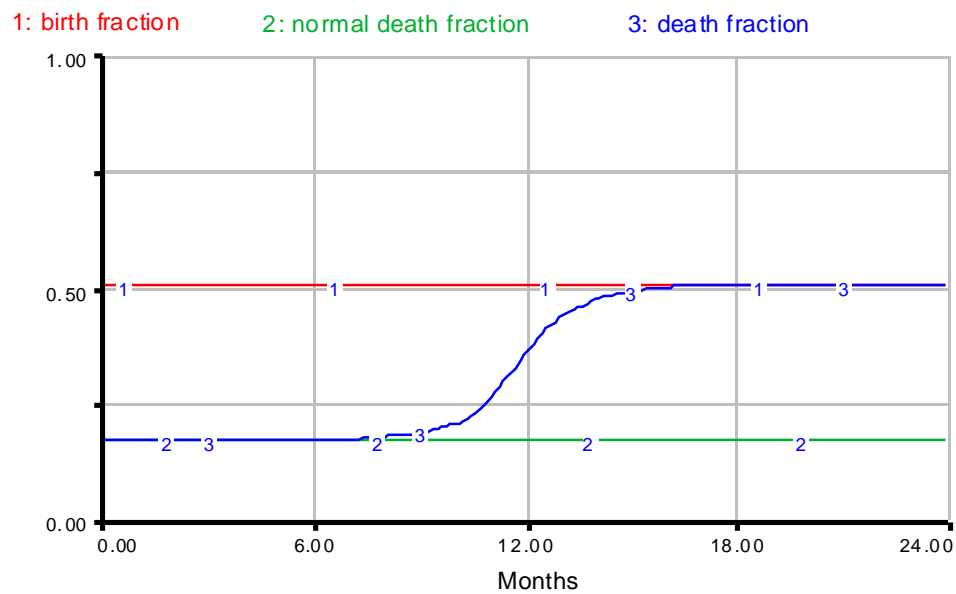

Figure 8: Rabbit population model: fractions
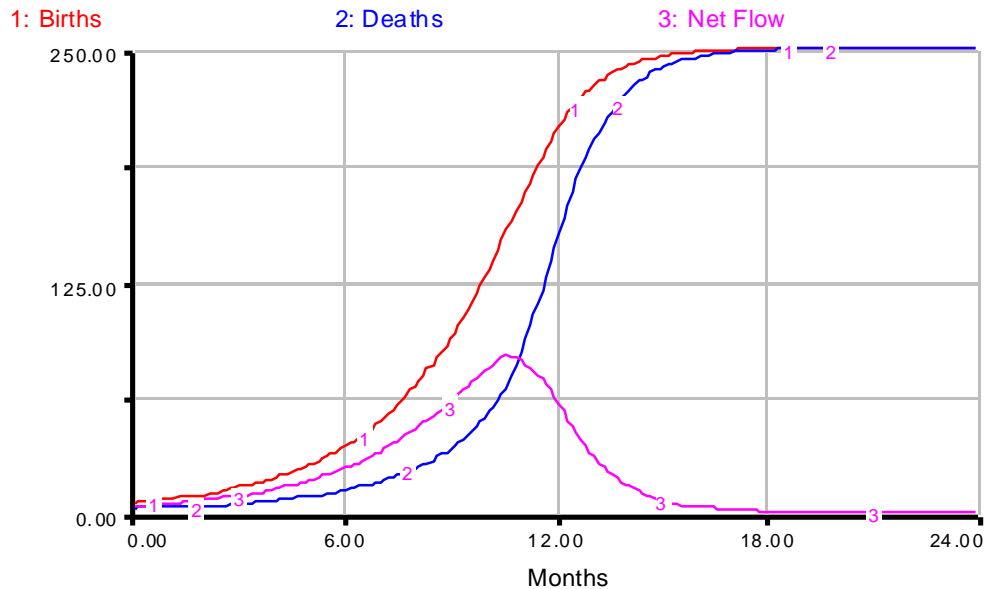
Figure 9: Rabbit population model: flows

Figures 8 and 9 depict the dynamics behind the shift in loop dominance. Examine Figure 8. At first the death_fraction is equal to the normal_death_fraction, so birth_fraction is initially three times the death_fraction. Because the birth_fraction is larger than the death_fraction, the number of Births surpasses the number of Deaths, as seen in Figure 9. Compare the behavior of the Net Flow curve in Figure 9 to the behavior of the rabbit population in Figure 7. When the Net Flow has a positive slope the number of RABBITS grows exponentially.

The death_fraction in Figure 8 starts to increase around month 8. The number of Deaths increases, but the number of Births increases even faster. Growth is still exponential.

Around month 10 a shift occurs. The Net Flow reaches a maximum. Its slope is no longer positive, but zero. When the Net Flow is at a maximum, the gap between Births and Deaths is the largest. Meanwhile, in Figure 7, the growth of the rabbit population experiences a point of inflection. Exponential growth tapers into asymptotic growth. The system switches from positive to negative feedback.

The death_fraction continues to increase in Figure 8 around month 12, approaching the gain_fraction. The number of Deaths is catching up to the number of Births. The Net Flow has a negative slope, so negative feedback causes the number of RABBITS to stabilize towards equilibrium. Although the slope of the Net Flow is negative, the Net Flow itself never goes negative. The number of Deaths never rises higher than the number of Births. So when the Net Flow has a negative slope the rabbit population does not experience decay but stabilizing *growth*.

By month 15 the system is close to equilibrium. The rabbit population stabilizes at equilibrium when the number of Deaths reaches the number of Births. The death_fraction equals the birth_fraction. Births and Deaths cancel each other. The total number of RABBITS remains constant.

There are two possible equilibria in systems that exhibit S-shaped growth. The first occurs before any growth takes place. Visualize the little farm in Provence just before Jean arrived with his rabbits. No rabbits are born. None die. Births and Deaths are both zero, and the system is at equilibrium. Only two rabbits, however, are needed to set the entire system into motion. Twenty months later the system reaches another equilibrium. The number of Births and Deaths each month are no longer zero. They are, however, equal. So the Net Flow of Births minus Deaths is zero. At the second equilibrium, many rabbits are born and just as many die. In all, 500 rabbits roam around Jean's small farm. The first equilibrium is an unstable equilibrium; the second is stable. An *unstable* equilibrium is easily perturbed, easily set into motion, like a penny balanced on its edge. A *stable* equilibrium guarantees that there will be no more motion, in population dynamics, no more growth. A penny lying flat on its side is in stable equilibrium.

The behavior obtained from the rabbit population model illustrates the characteristics which determine growth. Changes in the net flow into a stock change the

growth experienced by the stock. When the net flow has a positive slope, positive feedback loops are dominant and the stock experiences exponential growth. When negative loops dominate, the slope of the net flow becomes negative, and the stock is driven into asymptotic growth. The inflection between exponential and asymptotic growth occurs when the slope of the net flow is zero. At the shift, the net flow is at a maximum. The stock ceases to grow when the net flow is zero.

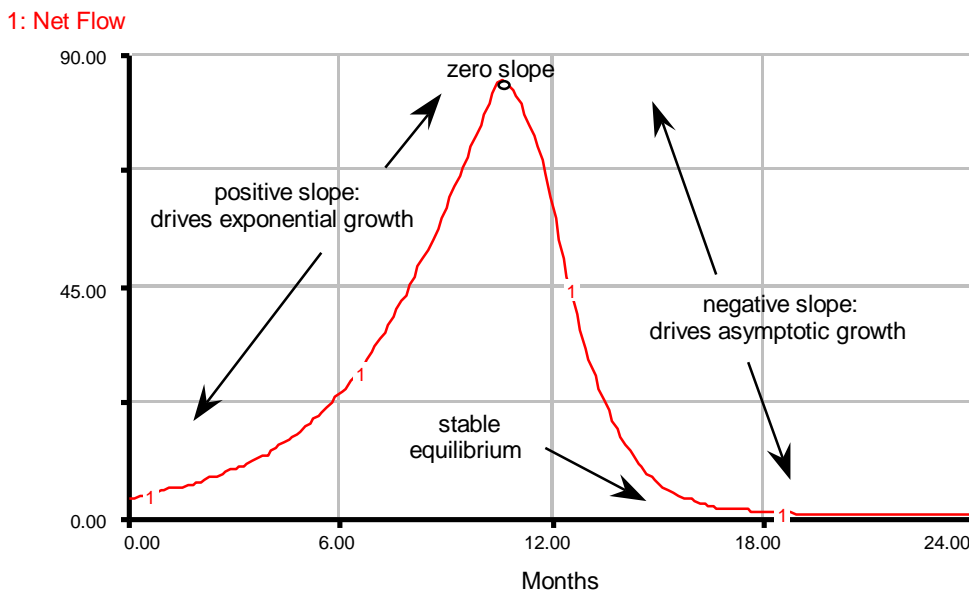The observations on the dynamics of Net Flow are summarized in Figure 10.

Figure 10: Summary of observations

3. Examples

Many different systems exhibit S-shaped growth. The generic structure for S-shaped growth presented in Section 2 helped explore and build intuition about the dynamics behind S-shaped growth. The generic structure explicitly shows the feedback loops and constraint on growth which cause shifting loop dominance. In most systems, however, feedback loops are less apparent and constraints on growth are often hidden or implied. The models in the next three examples will not resemble the generic structure at a first glance. The same underlying dynamics, however, are at play. The following examples will examine the principles of S-shaped growth from different perspectives; emphasizing that the same basic dynamics cause a whole range of different phenomena to produce identical behavior.

3.1 Cabbage Patch Kids

Fads exhibit the characteristics of S-shaped growth. Bell bottoms, charm bracelets, hula hoops, mood rings, Rubik's Cubes, and Cabbage Patch Kids all hit the market with a bang. People rushed to stores to buy the latest fad. All of your friends had charm bracelets, you needed one too. The number of people who would puzzle over Rubik's Cubes grew exponentially, at first. After a while though, the market for such products saturated. You have a Rubik's Cube on your coffee table, another in your office, maybe one in the bathroom. By now you have already resorted to pulling off the colored stickers and gluing them back on so that your centerpiece won't be an embarrassment to your intellect. Your Cabbage Patch Kids have taken over your bed. More mood rings? Well, you have only ten fingers...

Modeling the Cabbage Patch Kids mania will help develop a better understanding of the dynamics behind S-shaped growth. In 1980 Coleco invented Cabbage Patch Kids, fabric dolls designed to look just like their little "parent." In the Babyland General

Hospital, employees dressed in hospital gowns sewed the Cabbage Patch Kids by hand. The dolls were then given names and delivered with birth certificates and adoption papers. The dolls soon became extremely popular. Toy stores were often unable to cope with the demand for Cabbage Patch Kids. Orders that had been on backlog for weeks were greeted by mobs of parents eager to obtain the cherished dolls for their children. After a while though, most children whose parents could afford Cabbage Patch Kids (they often cost over $100) already had them. The fad died. As the small parents grew up, Cabbage Patch Kids were eventually relegated to shelves and toy chests.

Please consider the following questions before continuing with the example.

- What critical variables are involved in the Cabbage Patch Kid system?
  _____

- What constraints are acting on the system?
  _____

- What positive and negative loops determine the system's behavior?
  _____

Some important variables are the number of children who own a Cabbage Patch Kid (CPK), the number of children who have yet to buy a CPK, and how long it takes a child to purchase a doll. To simplify the model, assume that each child only buys one doll.

The constraint, the fixed value, is the market—the total number of children who can afford to buy a doll—approximated at one million children.

The system's behavior is controlled by two major loops. The positive loop represents the popularity of the dolls which grows by word of mouth. The more children own CPKs, the more popular the CPKs become. The negative loop is caused by the constraint of a fixed market. When more children buy CPKs, fewer children remain who have yet to buy a CPK of their own, and fewer purchases are made.

Figure 11 is a possible model of the Cabbage Patch Kids mania. Documented equations of the model can be found in Appendix 5.2.
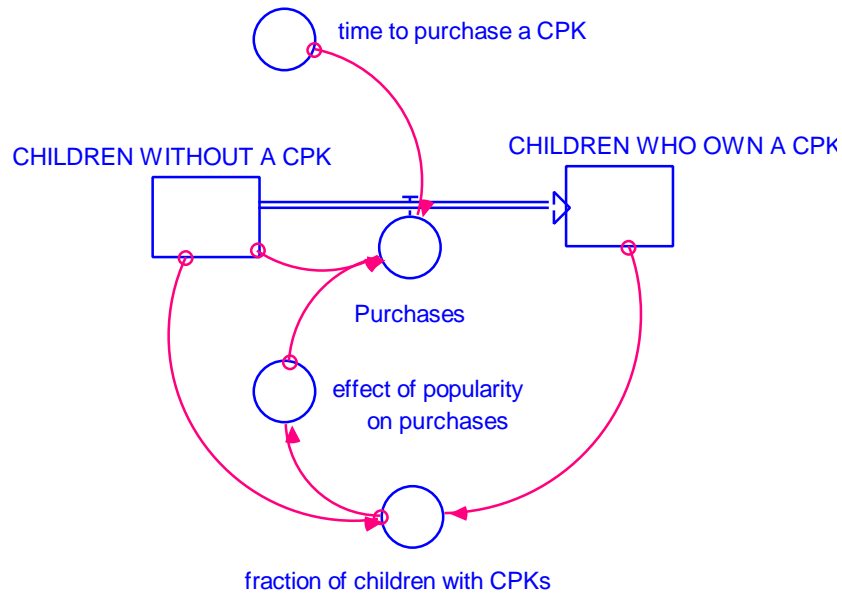
Figure 11: Cabbage Patch Kid model

The positive feedback loop in the Cabbage Patch Kid model is due to the increasing popularity of the dolls. The fraction_of_children_with_CPKs increases proportionally as the number of CHILDREN_WHO_OWN_A_CPK grows. The popularity of the dolls is directly related to the percentage of the population that owns the dolls (excluding real-world perception delays for the simplicity of the model.) An increase in the number of child-owners increases the effect_of_popularity_on_purchases, which is equal to the fraction_of_children_with_CPKs. The popularity multiplier then increases the flow of Purchases, which in turn raises the number of CHILDREN_ WHO_OWN_A_CPK.

The model in Figure 11 emphasizes the flow of children from one status to another. First they long for a doll, then they own one. As the number of CHILDREN_WHO_OWN_A_CPK increases, the number of CHILDREN_WITHOUT_

A_CPK decreases. The flow of Purchases is a fraction of the remaining market over time. The number of CHILDREN_WHO_ OWN_A_CPK determines the popularity of the dolls.   The popularity in turn convinces CHILDREN_WITHOUT_A_CPK to make purchases.   The number of Purchases is calculated by multiplying the effect_of_ popularity_on_purchases multiplier by the number of CHILDREN_WITHOUT_A_CPK and dividing by the time_to_purchase_ a_CPK.   The time_to_purchase_a_CPK is the average time for children to convince their parents to buy the expensive dolls plus the average time for the parents to find a doll which resembles their child.

In order to observe the behavior of the Cabbage Patch Kid phenomena, run the model in Figure 11 over a 24-month time period.   Graph CHILDREN_WHO_ OWN_A_CPK on one graph and Purchases on another.   Then answer the questions below.

? When does the loop dominance shift?

_____

? What is the approximate value of the stock at the point of inflection?

_____

? When does the Cabbage Patch Kid system reach equilibrium?

_____

? How many children own Cabbage Patch Kids at equilibrium?

_____

? How many purchases are made when the system is at equilibrium?
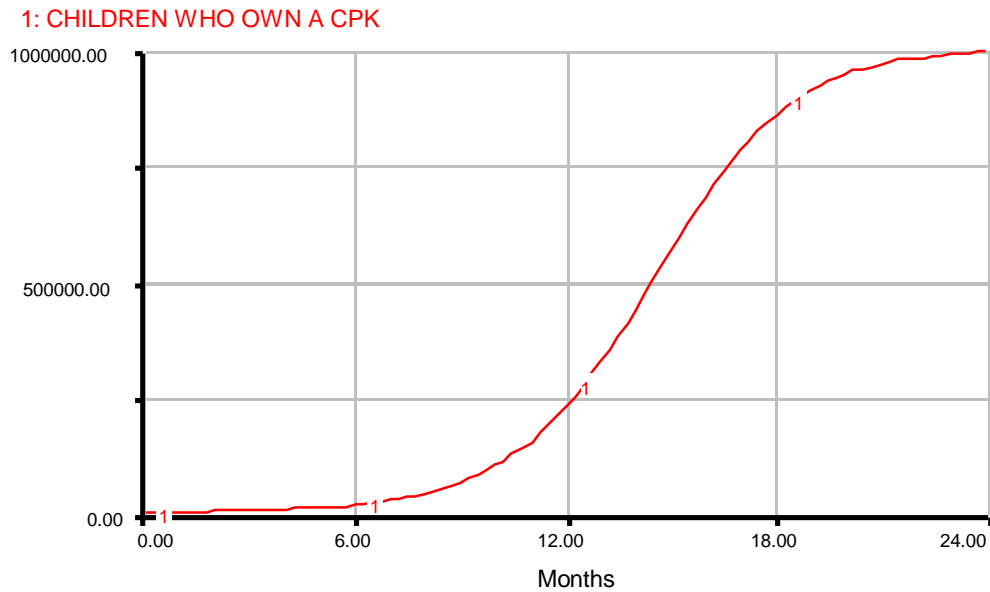
_____

1: CHILDREN WHO OWN A CPK

Figure 12: Cabbage Patch Kids: stock over time

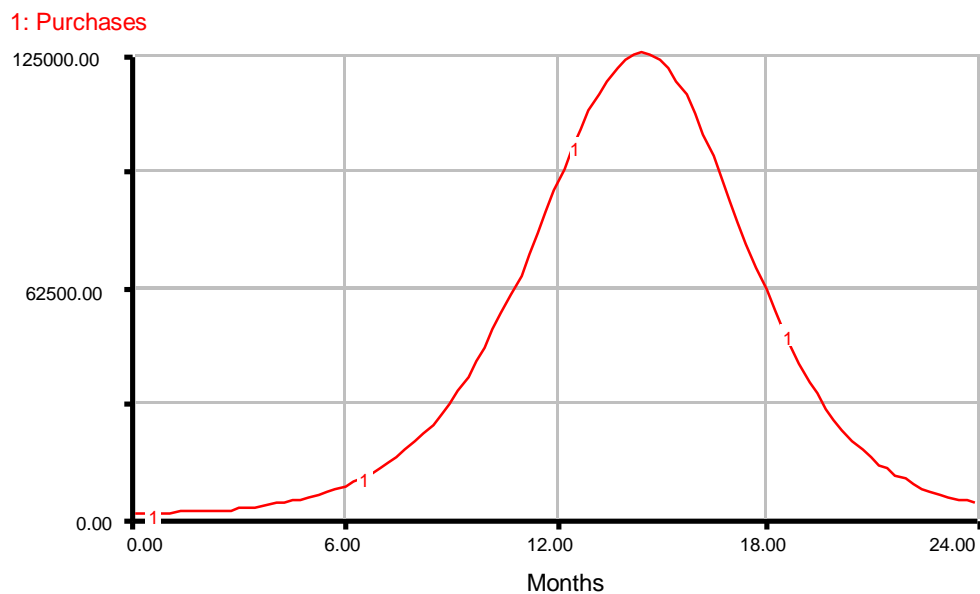1: Purchases

Figure 13: Cabbage Patch Kids: net flow over time

When does the loop dominance shift? When Purchases has a positive slope the system is experiencing positive feedback. On the other hand, when Purchases has a negative slope the system is undergoing negative feedback. The transition occurs when the slope of Purchases is zero, when Purchases reaches a maximum. Note in Figure 13

that Purchases reaches a maximum around month 15. The loop dominance shifts around month 15.

Examine Figure 12 to obtain the value of the stock of CHILDREN_WHO_OWN_ A_CPK at the point of inflection. The curve shifts from exponential to asymptotic growth around month 15. The shift in growth corresponds to the shift in loop dominance. Around month 15, the stock of CHILDREN_WHO_OWN_A_CPK is approximately 500,000 children.

The system approaches its stable equilibrium by month 24. The asymptotic growth in Figure 12 stabilizes as the total number of CHILDREN_WHO_OWN_A_CPK approaches 1 million, exactly the total population of children. At the same time in Figure 13, at month 24, the number of Purchases goes to zero. When the net flow into a stock is zero, the system stabilizes.

3.2 Protestantism

In 1517 Martin Luther posted 95 Theses on the door of a church in a small German town. The Theses were a list of grievances against the Roman Catholic Church. This event marked the beginning of a period in history called the Reformation in which the religion of the Protestants spread across Europe.

Many factors contributed to the rise of Protestantism in the early 16th century. The Roman Catholic Church was corrupt. The Popes had been engaged for the last hundred years in a series of papal wars. At times, several contenders fought for the Papacy. Martin Luther revolted specifically against the practice of indulgences, a process by which penitents could buy forgiveness for their sins.

Protestantism reverted to apostolic poverty and simplicity. Protestants believed in the priesthood of all believers. Martin Luther translated the Bible from Latin (which only the educated people understood) to everyday German so that the Word of God would be accessible to all. Henry VIII, the king of England, severed with the Roman Catholic

Church in 1534 over the issue of divorce and created a branch of Protestantism, the Anglican Church of England.

As more and more people converted to Protestantism, however, the Roman Catholic Church began to fight back. The Counter Reformation, a period of bloody religious civil wars, began. The Inquisition, established in 1542, persecuted the Protestants under the pretense of "suppressing the heretics." Many intellectuals fled to England and Switzerland. In countries such as France, Protestants had to meet in secret, fearing for their lives.

The spread of the Protestant religion exhibited S-shaped growth. Two important stocks are the number of Protestants and the number of non-Protestants.

?   What variables drive the positive feedback loop?

_____

?   What variables drive the negative feedback loops?

_____

The appeal of Protestantism, spread by word of mouth, determines the positive loop. The word of mouth loop is strengthened as the number of Protestants increases, which in turn engenders more conversions. The corresponding negative loop is due to saturation, the constraint of a fixed population. The fewer non-Protestants remaining, the fewer conversions. Persecution accounts for the second negative loop. The threat of Protestantism causes the Roman Catholic Church to retaliate with the Inquisition. Persecution dissuades many people from turning to Protestantism. The more Protestants, the larger the perceived threat, the more persecution, the fewer conversions.
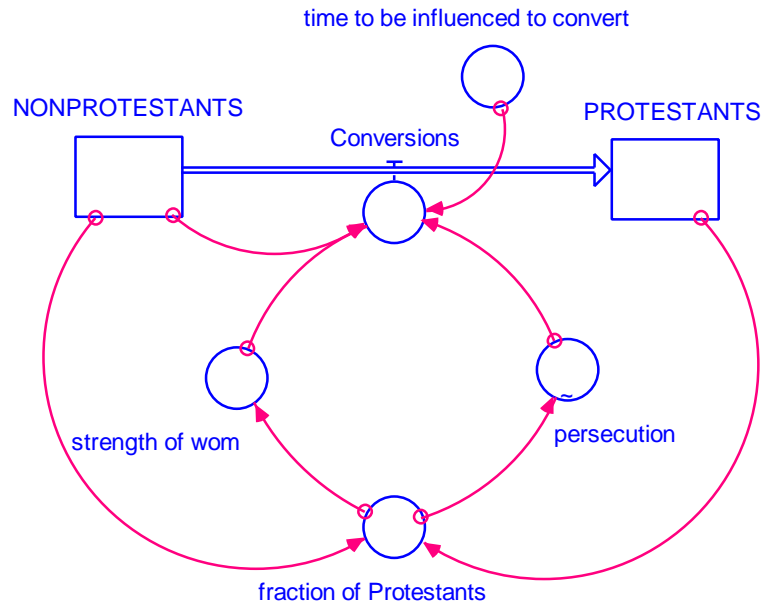
Figure 14: Protestantism model

Figure 14 depicts the Protestantism model. The persecution table function can be found, along with documented equations of the Protestantism model, in Appendix 5.4. Trace through the positive and negative loops in Figure 14. The positive loop is determined by the spread of Protestantism by word of mouth. As the number of PROTESTANTS increases, the fraction_of_Protestants in the total population correspondingly increases. The strength_of_wom (where wom stands for word of mouth) is equal to the fraction_of_Protestants in the total population. The strength_of_wom multiplier increases the number of Conversions, which in turn raises the number of PROTESTANTS.

The outflow from the stock of NONPROTESTANTS, Conversions, depends on the number of NONPROTESTANTS. Here lies a negative saturation loop. When many people are susceptible to being converted, many NONPROTESTANTS, the number of Conversions is large. When no potential converts remain, however, no amount of persuasion will generate new converts. As the number of PROTESTANTS increases, the number of NONPROTESTANTS decreases, and the number of Conversions subsequently decreases.

Persecution also decreases the number of Conversions. The threat of Protestantism increases as the number of PROTESTANTS grows. Persecution and bloody civil war begins when the PROTESTANTS become numerous enough to pose a significant threat to the established regime. In Figure 14, persecution is a multiplier that depends on the fraction_of_Protestants in the population. When the fraction is small, persecution is very low. As the fraction increases, persecution sharply rises. The number of Conversions is negatively affected by the persecution multiplier. The effect of persecution on Conversions is (1 – persecution). So when the number of PROTESTANTS increases, persecution increases, and the number of Conversions decreases.

Figure 14 is a model with three feedback loops of varying strengths. Which will be dominant and when? Computer simulation is the only way to correctly estimate a complex model's behavior. Run the model in Figure 14 over a period of 100 years, from 1500 to 1600. Graph the number of PROTESTANTS. Graph Conversions over time. Plot the persecution converter along with the strength_of_wom converter. Make an additional graph which includes both PROTESTANTS and NONPROTESTANTS on the scale.
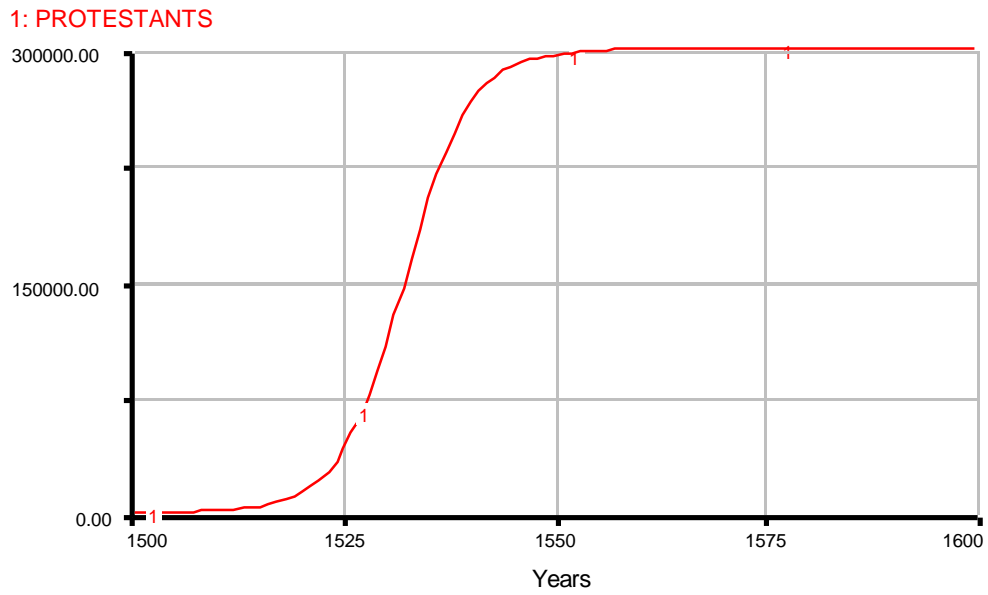
1: PROTESTANTS

Figure 15: Protestantism: stock over time
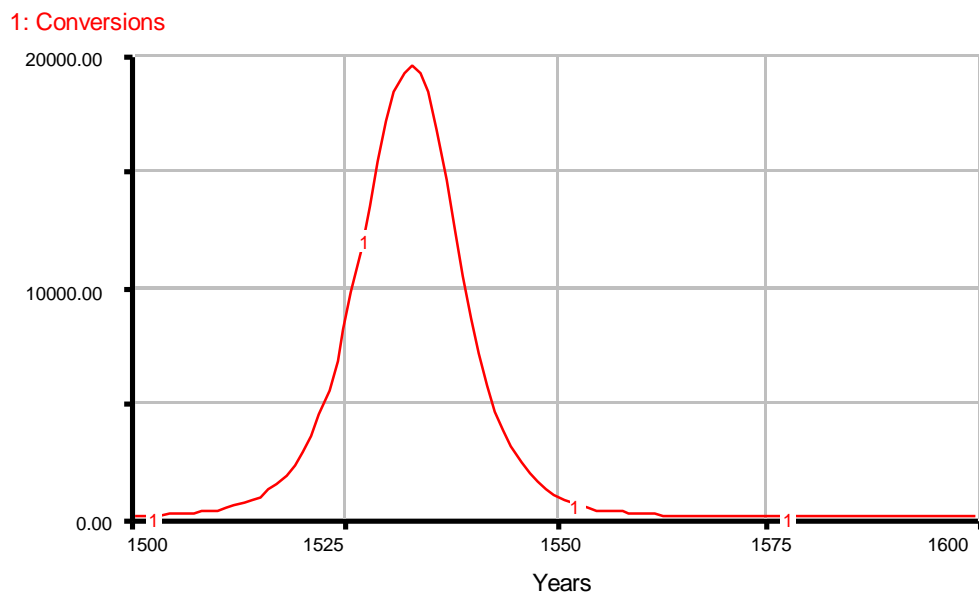
1: Conversions

Figure 16: Protestantism: net flow over time

?   When is Protestantism most popular? (When is the Conversions rate largest?)

Examine Figure 16. Few people converted at the beginning of the century. Lutheran thought was just beginning to take root. As word of mouth causes the intrinsic appeal of Protestantism to entice people to convert, the number of Conversions grows. By 1535 Conversions reaches its maximum of 20,000 people per year. In Figure 15 the number of PROTESTANTS grows exponentially over the first 35 years. The shift from exponential to asymptotic growth occurs when almost a sixth of the population has been converted. At the inflection point the number of Protestants has become large enough to pose a threat to the established Roman Catholic Church.

?   What causes the decrease in the rate of Conversions?

_____

The Inquisition, officially established in 1542, is part of a growing wave of persecution that seriously drops the number of Conversions. Furthermore, with the growth of the number of PROTESTANTS, the number of NONPROTESTANTS is declining. A decrease in the number of NONPROTESTANTS also reduces the rate of conversion. Which negative loop is responsible for the shift in loop dominance seen in Figure 15?
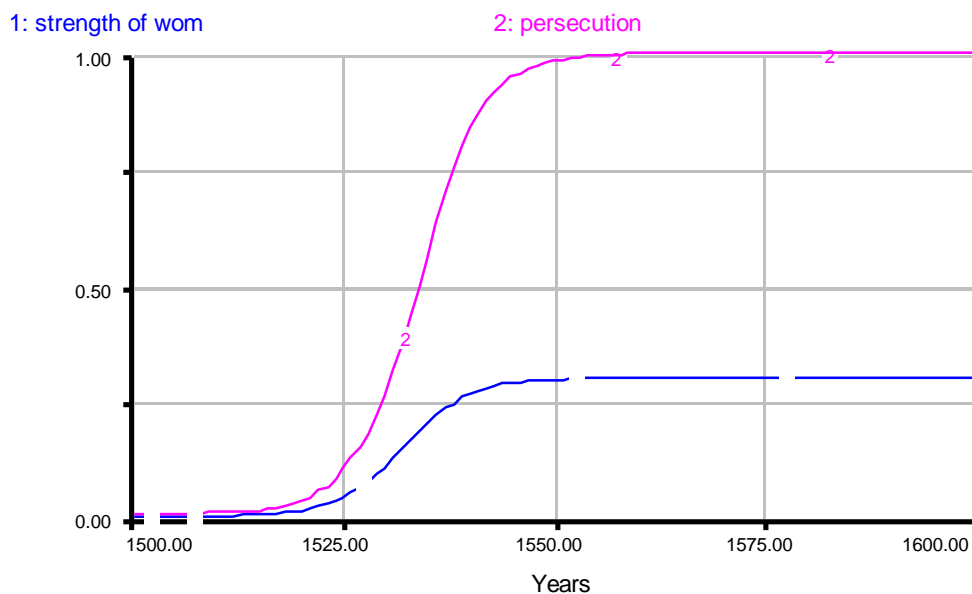
Figure 17: Protestantism: multipliers

Figure 17 sheds some light on the question of which negative loop is responsible for shifting loop dominance.   The effect of persecution is much stronger than the effect_of_wom.   Recall that Conversions is calculated by multiplying the number of NONPROTESTANTS by the strength_of_wom and (1 – persecution) and dividing by a time constant.   In Figure 17, persecution rises sharply around 1535.   As persecution approaches a value of 1, (1 – persecution) approaches zero.   When the number of PROTESTANTS becomes large enough for the fraction_of_Protestants to cause such a threat that persecution reaches a value of one, the Inquisition is in full swing.   Once persecution reaches a value of one, (1 – persecution) equals zero and the rate of Conversions becomes zero.   Because the net flow into the stock of PROTESTANTS has become zero, growth stops.

Look at Figure 18 and notice that the entire European population does not convert to Protestantism, contrary to how the entire potential market bought Cabbage Patch Kids. Persecution drives the rate of Conversions to zero before Protestantism has reached out to the entire population.   The number of PROTESTANTS stabilizes towards a constant number: 300,000 people, almost one third of the entire population.   By 1600, no new conversions take place.   The system has stabilized at equilibrium.

Figure 18: Protestantism: both stocks over time

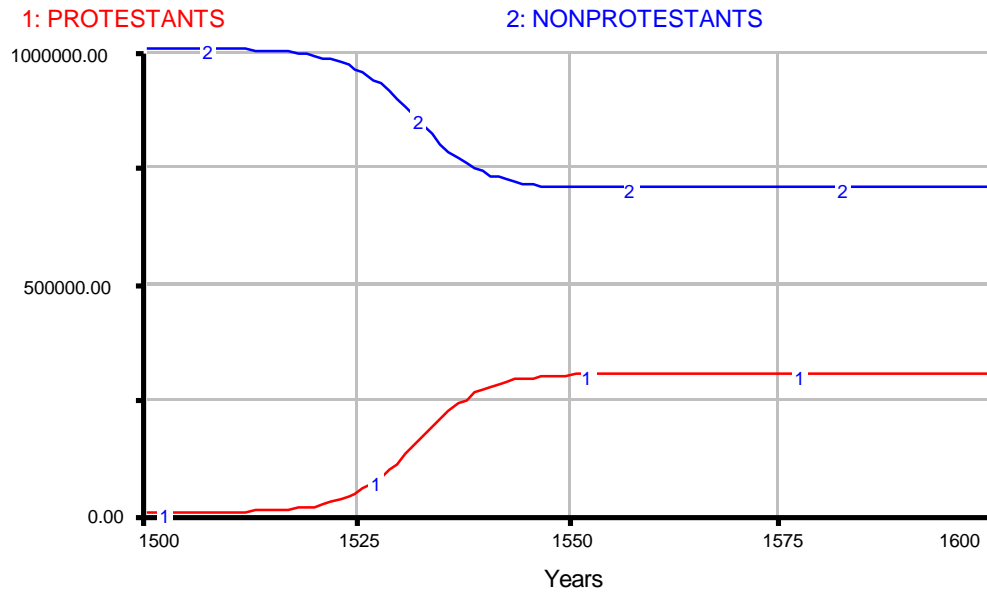In the Protestantism example, more than one negative feedback loop was present. Yet the system still generated S-shaped growth. The number of feedback loops is not a determining factor in the dynamics of S-shaped growth. What is important then? The next example shall address this question.

3.3 Immune Response

By the year 2000 as many as 40 million people worldwide will have become infected with HIV, the human immunodeficiency virus, which causes AIDS: Acquired Immune Deficiency Syndrome. AIDS is caused by a breakdown of the body's immune defense system. This example will first examine the human immune system. Then the model will be slightly modified to describe what happens when HIV impairs the human immune system and leaves it vulnerable to a great variety of otherwise harmless infections.

3.3.1 A Healthy Immune System

The immune system is powerful because of its specificity (cells attack specific invaders) and its memory (cells are ready to launch a rapid attack if the same type of invader returns again). When a foreign cell enters the human body, the intruder begins to multiply and diffuse through the blood stream. Each foreign cell has its own specific antigens. An antigen is a large molecule with a distinct configuration that triggers an immune response. Helper T cells, a specific type of white blood cells, roam around the human body, looking for foreign antigens.

Once a helper T cell recognizes a foreign antigen, it activates B cells, another type of white blood cells, that begin to proliferate rapidly. Most B cells produce plasma cells which secrete antibodies that diffuse into the blood stream. The antibodies bind to the antigens, marking them for destruction. Other B cells are reserved as memory cells. When a similar antigen invades the body a second time, the memory B cells are ready to attack. An immune response which releases antibodies is called an "antibody-mediated" immune response.

If the foreign invader is intracellular (like a virus) rather than extracellular (like bacteria) then helper T cells activate a "cell-mediated" immune response. Once a helper T cell recognizes the foreign intruder, it activates killer T cells, yet another type of white

blood cells, that begin to proliferate rapidly. Killer T cells directly attack and destroy the foreign invaders. Again, memory T cells remain, prepared for any future attacks.

T cells and B cells are also known as white blood cells. In general, white blood cells respond to a foreign invasion by proliferating and producing antigen-specific defenders that destroy the foreign invaders. The number of white blood cells in a human body is indicative of how strong a battle the immune defense system is fighting at that time.

Figure 19 shows a simplified model of the immune system. Like all models, it is a simplification of one small part of a much larger system.
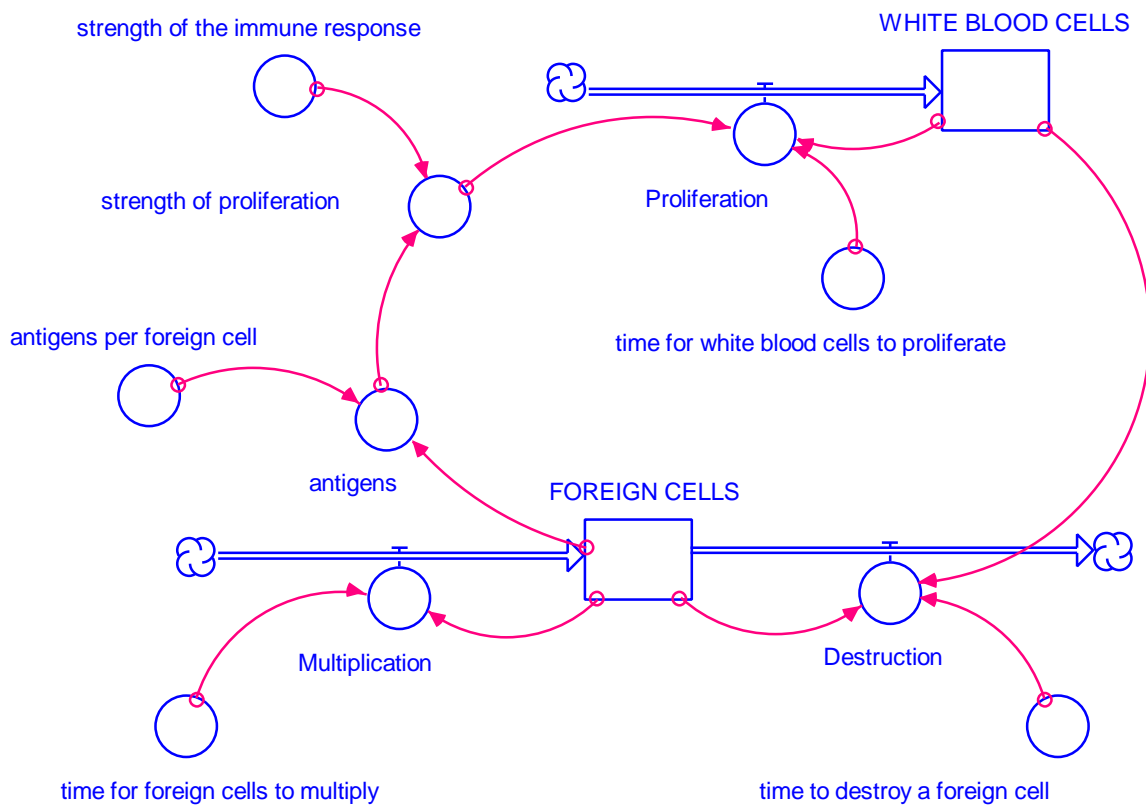
Figure 19: Immune response model

The proliferation of WHITE_BLOOD_CELLS is activated when helper T cells come into contact with antigens. The presence of a very small number of antigens is unlikely to set off a massive immune response. However, FOREIGN_CELLS multiply rapidly, much faster than rabbits. The strength_of_the_immune_response reflects the fraction of antigens that are detected by helper T cells. As the number of antigens grows, more antigens are detected, and the WHITE_BLOOD_CELLS proliferate more rapidly. Increasing Proliferation is the source of the positive loop that drives the growth of the WHITE_BLOOD_CELLS.

A negative loop comes into play as the WHITE_BLOOD_CELLS perform their duty and destroy FOREIGN_CELLS. As the number of WHITE_BLOOD_CELLS grows exponentially, antibodies or killer T cells are produced, and the number of FOREIGN_CELLS is drastically reduced. The immune response senses the absence of FOREIGN_CELLS—helper T cells come into contact with fewer and fewer antigens— and white blood cell Proliferation drops off, leaving the body with a good number of WHITE_BLOOD_CELLS ready to fend off a second possible invasion.

Run the model in Figure 19 over a period of 32 hours. Graph both WHITE_BLOOD_CELLS and FOREIGN_CELLS on the same plot. Combine the flows Multiplication, Destruction, and Proliferation on one graph. Documented equations for the immune response model can be found in Appendix 5.5.

1: WHITE BLOOD CELLS   2: FOREIGN CELLS
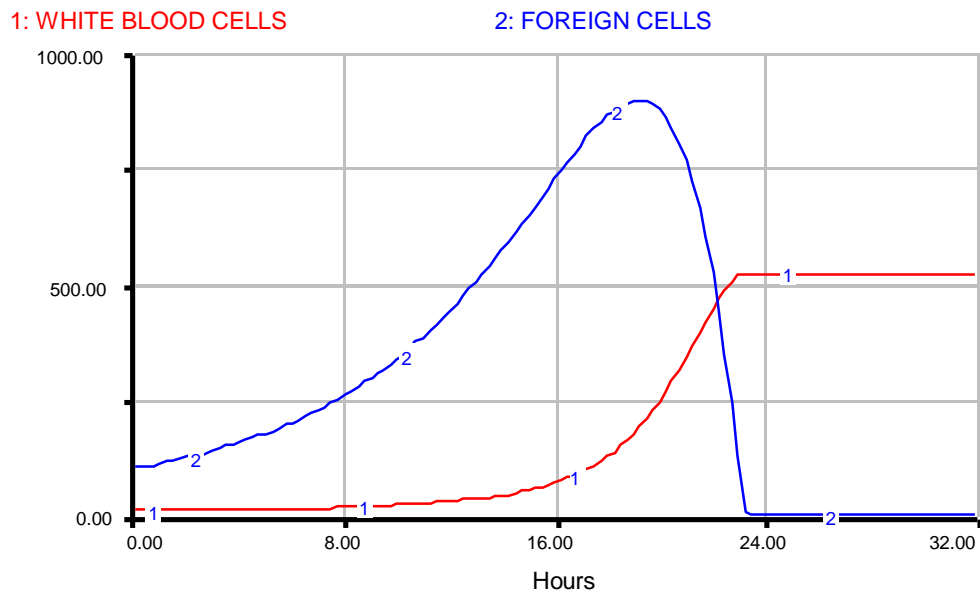
Figure 20: Immune response: stocks over time

Figure 20 depicts the growth of WHITE_BLOOD_CELLS and FOREIGN_ CELLS in an individual with a healthy immune system. Even though FOREIGN_CELLS and WHITE_BLOOD_CELLS have the same doubling time, they do not rise at the same rate because white blood cell Proliferation, unlike foreign cell Multiplication is also dependent on the number of antigens in the system.

The maximum number of FOREIGN_CELLS is larger than the maximum number of WHITE_BLOOD_CELLS because of the delay caused by the strength_of_the_immune_response. Helper T cells only come across approximately 1 out of every 1000 antigens. The WHITE_BLOOD_CELLS need to catch up on the FOREIGN_CELLS' "head start." When the hundred foreign cells per microliter of blood attack, only a few antigen-specific WHITE_BLOOD_CELLS (say, ten per microliter) are present to ward off the invasion. Notice, though, that the stock of WHITE_BLOOD_CELLS does not have an outflow. The white blood cells will eventually die, with the exception of the memory cells which will continue to reproduce, but, during the 24 hours of warfare, their forces are not depleted. The foreign cells, however, are being destroyed. Each white blood cell produces an antigen-specific

antibody or killer T cell that destroys a foreign cell each hour. The number of FOREIGN_CELLS may reach a higher maximum, but the WHITE_BLOOD_CELLS are killing machines.

Notice that the number of FOREIGN_CELLS reaches a maximum before WHITE_BLOOD_CELLS reaches equilibrium. The maximum number of FOREIGN_CELLS corresponds to the point of inflection on the WHITE_BLOOD_ CELLS curve. At the inflection point the number of WHITE_BLOOD_CELLS has become large enough to be effective in destroying FOREIGN_CELLS. As the number of FOREIGN_CELLS shoots down, the number of WHITE_BLOOD_CELLS no longer needs to grow, and their count stabilizes at an equilibrium value.
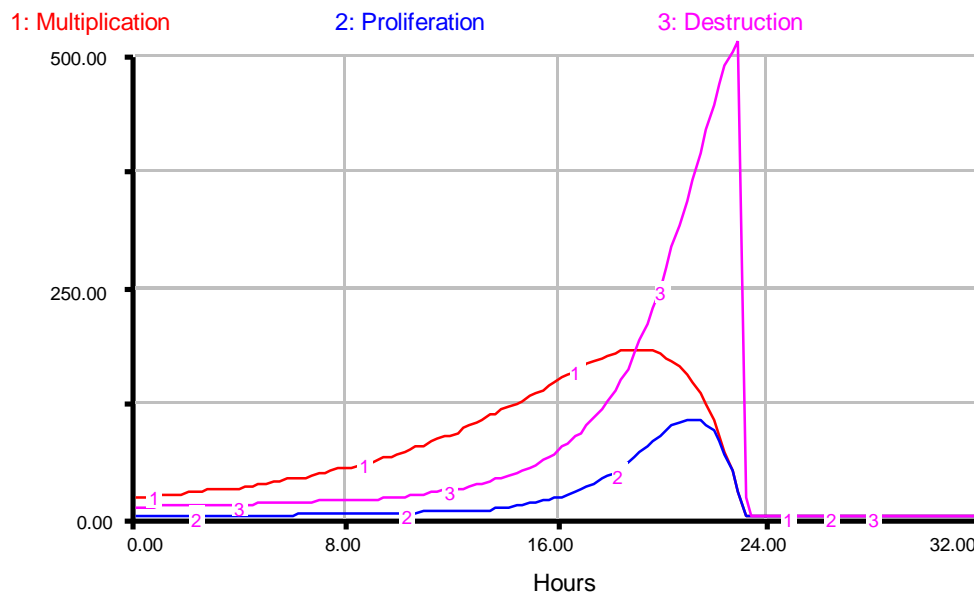

Figure 21: Immune response: net flows versus time

The curves in Figures 20 and 21 do not seem as smooth as the ones from the previous models. Destruction of FOREIGN_CELLS depends only indirectly on the number of FOREIGN_CELLS remaining. Destruction has no exponential smoothing; it continuously destroys FOREIGN_CELLS until none remain.

The immune response system after an invasion of FOREIGN_CELLS is an example of stable equilibrium. If similar invaders were to attempt to enter the body after the immune response, they would be recognized by antigen-specific memory cells and immediately destroyed. Although many of the WHITE_BLOOD_CELLS will eventually die off, memory B cells or memory T cells remain in the blood stream.

The immune response system is in unstable equilibrium before the invasion. Proliferation is zero. Once a few FOREIGN_CELLS enter, however, the system is triggered and a massive immune response ensues. Eventually the equilibrium becomes stable, and any future attacks of that specific type of foreign cell are guaranteed to have little effect.

3.3.2 An Immune System Affected by HIV

Section 3.3.1 discussed the dynamics of an immune response in a healthy immune system. HIV, however, is unlike any normal virus. It attacks the immune system itself.

The effect of HIV on the human immune system can be modeled by changing the strength_of_the_immune_response constant in the model in Figure 19. HIV cripples the immune system by sabotaging the genetic material of helper T cells. The helper T cell population is depleted. In a healthy body, helper T cells come across and recognize, say, one out of every 1000 antigens present per microliter of blood. When the immune system is impaired, however, the ratio goes down. With fewer functioning helper T cells on patrol, antigens are not detected as easily. The foreign cells have time to multiply before the immune response kicks in.

Cut the strength_of_the_immune_response by a factor of 10 and see what happens...
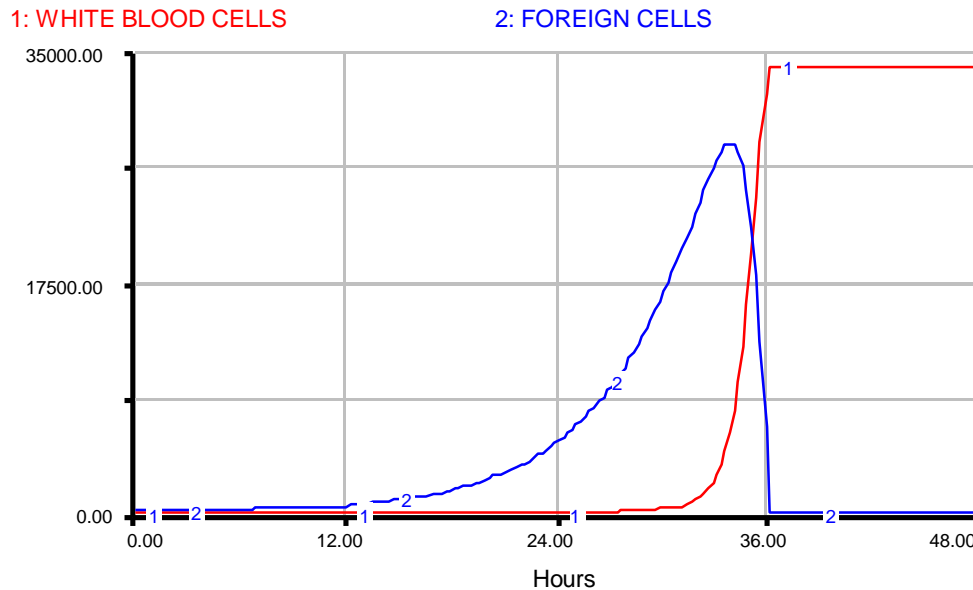
Figure 22: Strength_of_the_immune_response = 1 / 10,000

Figure 22 is a graph of the growth of WHITE_BLOOD_CELLS and FOREIGN_CELLS in an HIV positive individual. A functioning immune system destroys the foreign invasion by hour 24. The body withstands 900 FOREIGN_CELLS per microliter of blood. When the number of helper T cells is depleted by a factor of 10, however, the same 100 FOREIGN_CELLS multiply to become an army of 30,000 FOREIGN_CELLS that it takes WHITE_BLOOD_CELLS 36 hours to destroy!

HIV itself replicates quickly. It spreads through the body until it has impaired the immune system to the point where the body succumbs to any small foreign attack. It is then that an HIV positive patient is diagnosed as having AIDS. When the strength_of_the_immune_response has a value of 1/1500 or 1/2000 the body is still able to ward off invasions. However, as the strength_of_the_immune_response gets smaller and smaller, the effects are disastrous. The body can only stand the presence of a finite number of FOREIGN_CELLS before it succumbs. What would happen if 10,000 FOREIGN_CELLS were the maximum number of FOREIGN_CELLS that a human body could support? The body would collapse by hour 28 before the immune response had the time to produce enough specific antibodies or killer T cells to ward off the invasion.
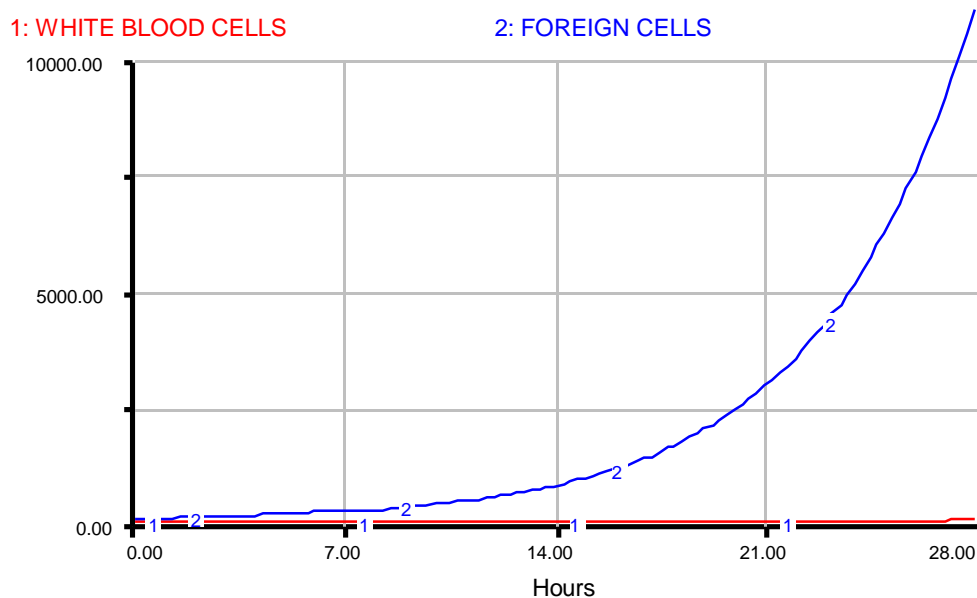
Figure 23: Impaired immune response — detail

Figure 23 shows in detail what happens. The number of FOREIGN_CELLS grows exponentially. The number of WHITE_BLOOD_CELLS also grows exponentially, but at such a slow rate that their presence is almost negligible. When the number of FOREIGN_CELLS reaches 10,000 cells per microliter, the system comes to a halt—the human being infected with AIDS has been destroyed by a foreign invasion. The halt occurs around hour 28. Fewer than 100 WHITE_BLOOD_CELLS have been produced to fight off the invasion.

A victim of AIDS has virtually no immune response system left. A small attack of FOREIGN_CELLS is allowed to grow unhindered because the body is unable to produce enough antibodies or killer T cells in time. How does the behavior of the impaired immune response model reflect the dynamics of S-shaped growth? The positive loop that causes the WHITE_BLOOD_CELLS to proliferate is so weak that the stock never grows large enough to counter the foreign invasion and shift the system into negative feedback. The system collapses under the strain of exponential growth before the natural immune response can act as a constraint. It is not the number of stocks and flows, nor the number

of positive or negative loops that determines the characteristics of S-shaped growth—it is the strength of those loops that matters.

4. Conclusion

Many systems produce S-shaped growth behavior. It is does not matter how many positive or negative feedback loops contribute to a certain system. The number of stocks is also unimportant. A shift in loop dominance is the determining factor for S-shaped growth. When a system shifts from positive to negative feedback, its behavior shifts from exponential to asymptotic growth.   The strength of these loops determines the characteristics of S-shaped growth.

Exponential growth cannot be sustained forever. Constraints will always act as limits to growth.

5. Appendix: Model Equations

5.1 S-Shaped Growth Generic Structure

☐**STOCK(t)** = STOCK(t - dt) + (Inflow - Outflow) * dt
     INIT STOCK = 10
     DOCUMENT: Units: widgets

     ⚕INFLOWS:
     Inflow = gain_fraction*STOCK
     DOCUMENT: Units: widgets/time

     ⚕OUTFLOWS:
     Outflow = loss_fraction*STOCK
     DOCUMENT: Units: widgets/time

○**gain_fraction** = *a constant*
     DOCUMENT: Units: 1/time
     The gain_fraction must be greater than the normal_loss_fraction in order for the
     STOCK to grow initially.

○**loss_fraction** = normal_loss_fraction*effect_of_stock
     DOCUMENT: Units: 1/time
     The loss_fraction is a normal_loss_fraction times a multiplier which depends on
     the relative size of the STOCK. The loss fraction, as it grows, determines the
     shifting loop dominance.

○**normal_stock** = *a constant*
      DOCUMENT: Units: widgets

○**normal_loss_fraction** = *a constant*
      DOCUMENT: Units: 1/time
      The normal_loss_fraction must be less than the gain_fraction in order for the
      STOCK to grow initially. The normal_loss_fraction represents the initial
      loss_fraction. If no constraints were acting upon the system, the loss_fraction
      would equal the normal_loss_fraction.

⊘**effect_of_stock** = GRAPH(STOCK/normal_stock)
      (0.00, 1.00), (0.2, 1.40), (0.4, 1.80), (0.6, 2.20), (0.8, 2.60), (1.00, 3.00), (1.20,
      3.40), (1.40, 3.80), (1.60, 4.20), (1.80, 4.60), (2.00, 5.00)
      DOCUMENT: Units: dimensionless
      The simplest curve that generates S-shaped growth is a straight line that is linearly
increasing. When the STOCK is 0, the multiplier is 1.
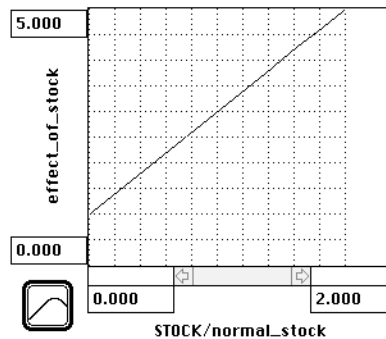
5.2 Rabbit Population Model

☐**RABBITS(t)** = RABBITS(t - dt) + (Births - Deaths) * dt
      INIT RABBITS = 10
      DOCUMENT: Units: rabbits

      ⌖INFLOWS:
      Births = birth_fraction*RABBITS
      DOCUMENT: Units: rabbits/month

      ⌖OUTFLOWS:
      Deaths = death_fraction*RABBITS
      DOCUMENT: Units: rabbits/month

○**birth_fraction** = 0.5
    DOCUMENT: Units: 1/month
    The birth fraction is the rate at which each rabbit can reproduce.

○**death_fraction** = normal_death_fraction*effect_of_crowding
    DOCUMENT: Units: 1/month
    The death fraction is the actual rate at which each rabbit dies.

○**normal_rabbit_population** = 500
    DOCUMENT: Units: rabbits
    The maximum rabbit population is the maximum number of rabbits for which Jean
    can provide water. It is fixed (a constant) because Provence has a dry, arid climate
    and Jean only has a fixed supply of water available to him. (In the story Jean de
    Florette by Marcel Pagnol, however, Jean's supply of water was not fixed but
    depletable. So as the number of rabbits grew, the water was all used up and all the
    rabbits died. The dynamics of overshoot and collapse will be explored in Road
    Maps 9.)

○**normal_death_fraction** = (0.5)/3
    DOCUMENT: Units: 1/month
    The normal_death_fraction is the rate at which each rabbit would die if it had an
    unlimited supply of food and water. It is three times less than the birth_fraction.

⊘**effect_of_crowding** = GRAPH(RABBITS/normal_rabbit_population)
    (0.00, 1.00), (0.2, 1.00), (0.4, 1.10), (0.6, 1.30), (0.8, 2.00), (1.00, 3.00), (1.20,
    4.20), (1.40, 5.62), (1.60, 7.80), (1.80, 11.1), (2.00, 15.0)
    DOCUMENT: Units: dimensionless
    Crowding has no effect on the death fraction as long as the number of rabbits on
    Jean's farm does not reach more than half the number that he can support. As the
    number of rabbits reaches the maximum rabbit population, the multiplier
    exponentially increases from 1 to 3.
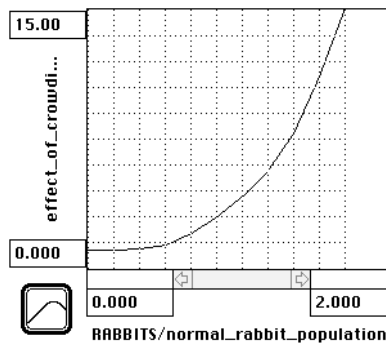
5.3 Cabbage Patch Kids Model

☐**CHILDREN_WHO_OWN_A_CPK(t)** = CHILDREN_WHO_OWN_A_CPK(t - dt)
    + (Purchases) * dt
    INIT CHILDREN_WHO_OWN_A_CPK = 1000
    DOCUMENT: Units: children

    ⚲INFLOWS:
    Purchases = effect_of_popularity_on_purchases *
    CHILDREN_WITHOUT_A_CPK / time_to_purchase_a_CPK
    DOCUMENT: Units: children/month

☐**CHILDREN_WITHOUT_A_CPK(t)** = CHILDREN_WITHOUT_A_CPK(t - dt) + (-
    Purchases) * dt
    INIT CHILDREN_WITHOUT_A_CPK = 1,000,000
    DOCUMENT: UNITS: children
    The total population considered here is the total population that is susceptible to
    buying a doll. (ie. it excludes children who can't afford the dolls)

    ⚲OUTFLOWS:
    Purchases = effect_of_popularity_on_purchases *
    CHILDREN_WITHOUT_A_CPK / time_to_purchase_a_CPK
    DOCUMENT: Units: children/month

○**effect_of_popularity_on_purchases** = fraction_of_children_with_CPKs
    DOCUMENT: Units: dimensionless
    As fraction of children with dolls increases, a corresponding fraction representing
    the effect_of_popularity_on_purchases increases from 0 to 1.

○**fraction_of_children_with_CPKs** = CHILDREN_WHO_OWN_A_CPK /
    (CHILDREN_WHO_OWN_A_CPK + CHILDREN_WITHOUT_A_CPK)
    DOCUMENT: Units: dimensionless
    The fraction_of_children_with_CPKs is the percentage of the total population
    suceptible to buying dolls that actually owns a doll.

○**time_to_purchase_a_CPK** = 2
    DOCUMENT: Units: months
    The time_to_purchase_a_CPK is the average time for children to convince their
    parents to buy a doll plus the average time for the parents to find a doll which
    resembles their child.

5.4 Protestantism Model

☐**NONPROTESTANTS(t)** = NONPROTESTANTS(t - dt) + (- Conversions) * dt

INIT NONPROTESTANTS = 1000000
DOCUMENT: Units: people

✤OUTFLOWS:
Conversions = NONPROTESTANTS * strength_of_wom * (1 - persecution) / time_to_be_influenced_to_convert
DOCUMENT: Units: people/year
Conversions is rate at which NonProtestants, usually Roman Catholics, converted to become Protestant. Conversions is the product of the number of people left who have yet to convert, the fraction representing the strength of word of mouth, the opposite of the fraction representing the strength of persecution, all over the time it takes to convert.

☐**PROTESTANTS(t)** = PROTESTANTS(t - dt) + (Conversions) * dt
INIT PROTESTANTS = 100
DOCUMENT: Units: people
The initial value of 100 people represents the original converts (and friends) of Martin Luther.

✤INFLOWS:
Conversions = NONPROTESTANTS*strength_of_wom * (1 - persecution) / time_to_be_influenced_to_convert
DOCUMENT: Units: people/year

○**fraction_of_Protestants** = PROTESTANTS / (PROTESTANTS + NONPROTESTANTS)
DOCUMENT: Units: dimensionless
The fraction_of_Protestants is the percentage of Protestants in the total population

○**strength_of_wom** = fraction_of_Protestants
DOCUMENT: Units: dimensionless
The strength of word of mouth is equal to the number of Protestants spreading that word of mouth.

○**time_to_be_influenced_to_convert** = 4
DOCUMENT: Units: years
The time_to_be_influenced_to_convert is the time for a nonProtestant to hear about Protestantism and decide to adopt its principles.

⊘**persecution** = GRAPH(fraction_of_Protestants)
(0.00, 0.005), (0.1, 0.235), (0.2, 0.545), (0.3, 1.00), (0.4, 1.00), (0.5, 1.00), (0.6, 1.00), (0.7, 1.00), (0.8, 1.00), (0.9, 1.00), (1, 1.00)
DOCUMENT: Units: dimensionless
Persecution is negligeable when there are relatively few Protestants. As the number of Protestants becomes threatening, it shoots up to the point of cutting off

future conversions. When the Inquisition and religious civil wars started in Europe, there were very few new conversions.
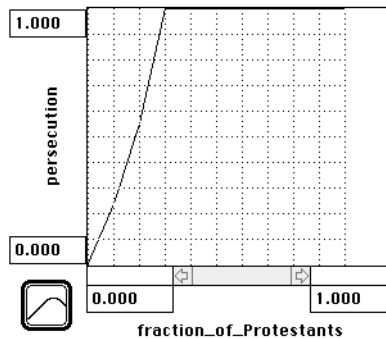
5.5 Immune Response Model

☐**FOREIGN_CELLS(t)** = FOREIGN_CELLS(t - dt) + (Multiplication-Destruction)*dt
   INIT FOREIGN_CELLS = 100
   DOCUMENT: Units: foreign cells
   100 is an approximate number of foreign cells that invade one microliter of blood.

   ♂INFLOWS:
   Multiplication = FOREIGN_CELLS/time_for_foreign_cells_to_multiply
   DOCUMENT: Units: foreign cells/hour

   ♂OUTFLOWS:
   Destruction = IF (FOREIGN_CELLS = 0) THEN 0 ELSE
   (WHITE_BLOOD_CELLS / time_to_destroy_a_foreign_cell)
   DOCUMENT: Units: foreign cells/hour
   Destruction is the average rate at which white blood cells destroy foreign cells.

☐**WHITE_BLOOD_CELLS(t)** = WHITE_BLOOD_CELLS(t - dt) + (Proliferation)*dt
   INIT WHITE_BLOOD_CELLS = 10
   DOCUMENT: Units: white blood cells
   Approximately 10 antigen-specific white blood cells are present in one microliter of blood before an antigen invasion is recognized. White blood cells is the name given to the cells that constitute the human immune response: helper T cells, killer T cells, B cells, memory cells...

   ♂INFLOWS:
   Proliferation = strength_of_the_immune_response * WHITE_BLOOD_CELLS / time_for_white_blood_cells_to_proliferate
   DOCUMENT: Units: white blood cells/hour

Proliferation is determined by multiplying a normal proliferation rate (white blood cells/time for white blood cells to proliferate) by the variable strength of proliferation.

○**antigens** = FOREIGN_CELLS * antigens_per_foreign_cell
    DOCUMENT: Units: antigens
    Antigens are macromolecules on foreign cells that, when identified by helper T cells, trigger an immune response .

○**antigens_per_foreign_cell** = 2
    DOCUMENT: Units: antigens/foreign cell
    Approximately 2 antigens are attached to each foreign cell.

○**strength_of_proliferation** = strength_of_the_immune_response * antigens
    DOCUMENT: Units: dimensionless
    Strength_of_proliferation is a multiplier that increases the proliferation rate as the number of recognized antigens increases.

○**strength_of_the_immune_response** = 1/1000
    DOCUMENT: Units: dimensionless
    The strength_of_the_immune_response is a fraction representing the ability of helper T cells to recognize antigens and activate the proliferation of white blood cells. When someone is infected with HIV, the human immune system is crippled and the strength_of_the_immune_response is severely weakened.

○**time_for_foreign_cells_to_multiply** = 5
    DOCUMENT: Units: hours

○**time_for_white_blood_cells_to_proliferate** = 5
    DOCUMENT: Units: hours

○**time_to_destroy_a_foreign_cell** = 1
    DOCUMENT: Units: (white blood cells/foreign cell) * hours

Vensim Examples:
Exploring S-Shaped Growth
Lei Lei & Nathaniel Choge
October 2001

S-Shaped Growth Generic Structure
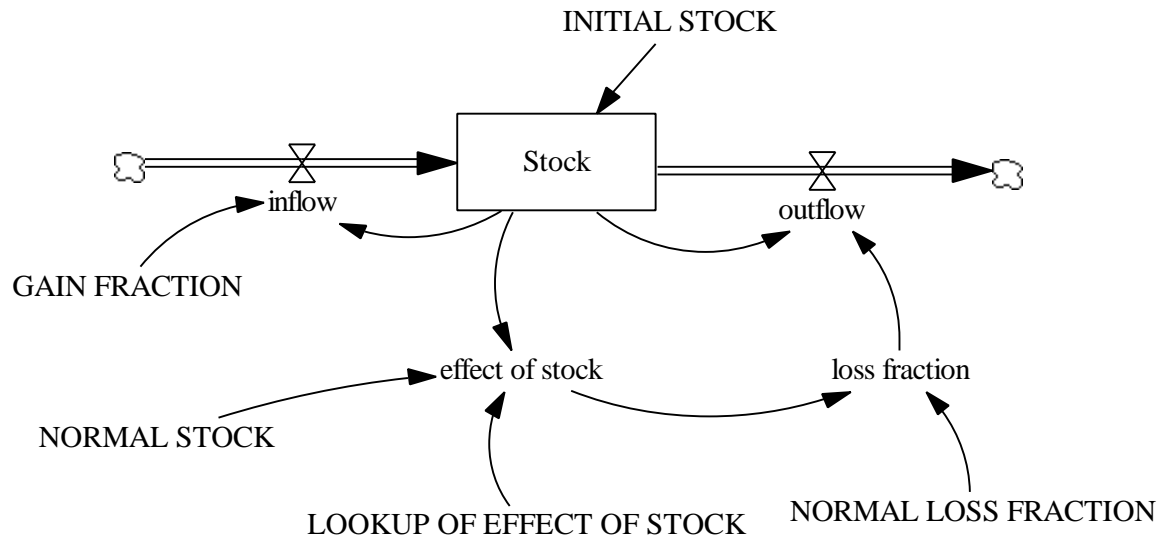
Figure 24: Vensim Equivalent of Figure 3: Generic Structure that produces S-shaped Growth

Documentation for S-Shaped Generic Structure Model

(01)   effect of stock = LOOKUP OF EFFECT OF STOCK (Stock/NORMAL STOCK)
          Units: dmnl

(02)   FINAL TIME = 100
          Units: time
          The final time for the simulation.

(03)   GAIN FRACTION = a constant
          Units: 1/time
          The GAIN FRACTION must be greater than the normal loss fraction in order for the Stock to grow initially.

(04)   inflow = GAIN FRACTION* Stock
          Units: widgets/time

(05)   INITIAL STOCK = 10
        Units: widgets

(06)   INITIAL TIME = 0
        Units: time
        The initial time for the simulation.

(07)   LOOKUP OF EFFECT OF STOCK
        ([(0,0)-(10,10)],(0,1),(0.2,1.4),(0.4,1.8),(0.6,2.2),(0.8,2.6),(1,3),
        (1.2,3.4),(1.4,3.8), (1.6,4.2),(1.8,4.6),(2,5))
        Units: dmnl
        The simplest curve that generates S-shaped growth is a straight line that is linearly
        increasing. When the Stock is 0, the multiplier is 1.

(08)   loss fraction = NORMAL LOSS FRACTION* effect of stock
        Units: 1/time
        The loss fraction is a NORMAL LOSS FRACTION times a multiplier that
        depends on the relative size for the Stock. The loss fraction, as it grows,
        determines the shifting loop dominance.

(09)   NORMAL LOSS FRACTION = a constant
        Units: 1/time
        The NORMAL LOSS FRACTION must be less than the GAIN FRACTION in
        order for the Stock to grow initially. The NORMAL LOSS FRACTION represents
        the initial loss fraction. If no constraints were acting upon the system, the loss
        fraction would equal the NORMAL LOSS FRACTION.

    (10) NORMAL STOCK = a constant
        Units: widgets

(11)   outflow = loss fraction* Stock
        Units: widgets/time

(12)   SAVEPER = TIME STEP
        Units: time
        The frequency with which output is stored.

(13)   Stock = INTEG (inflow-outflow, INITIAL STOCK)
        Units: widgets

(14)   TIME STEP = 1
        Units: time
        The time step for the simulation.

Applying the Generic Structure to the Rabbit Population Model
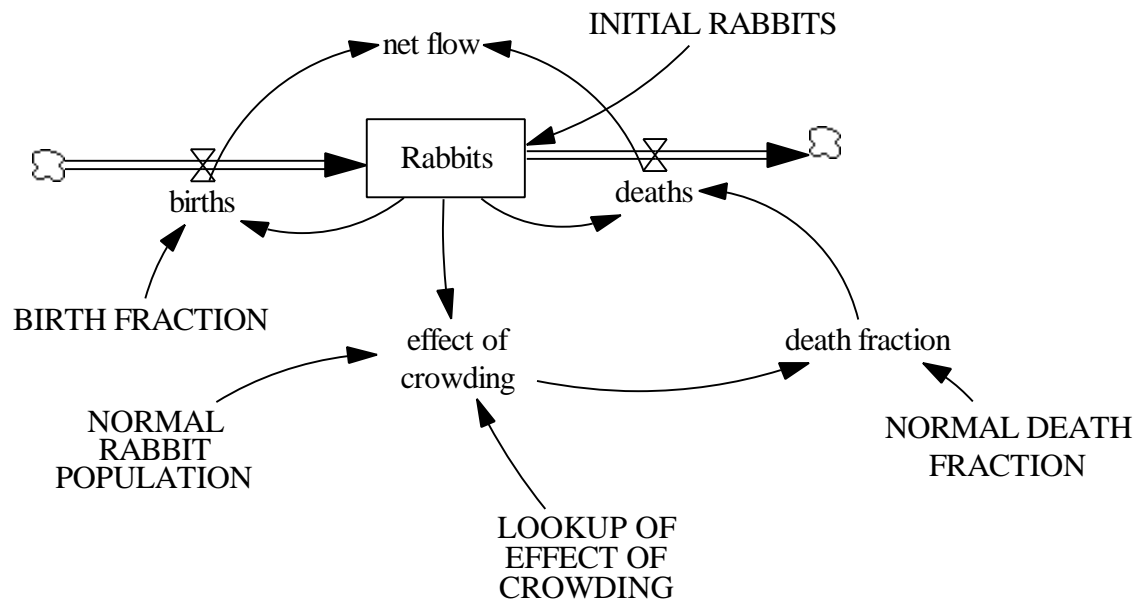
Figure 25: Vensim Equivalent of Figure 5: Stock and flow diagram of the rabbit
            Population

Documentation for Rabbit Population Model

(01)   BIRTH FRACTION = 0.5
        Units: 1/Month
        The BIRTH FRACTION is the rate at which each rabbit can reproduce.

(02)   births = BIRTH FRACTION* Rabbits
        Units: rabbits/Month

(03)   death fraction = NORMAL DEATH FRACTION* effect of crowding
        Units: 1/Month
        The death fraction is the actual rate at which each rabbit dies.

(04)   deaths = death fraction* Rabbits
        Units: rabbits/Month

(05)   effect of crowding = LOOKUP OF EFFECT OF CROWDING
        (Rabbits/NORMAL RABBIT POPULATION)
        Units: dmnl

This is the equation that shows that the effect of crowding is a function of the rabbit population.

(06)   FINAL TIME = 24
        Units: Month
        The final time for the simulation.

(07)   INITIAL RABBITS = 10
        Units: rabbits

(08)   INITIAL TIME = 0
        Units: Month
        The initial time for the simulation.

(08)   LOOKUP OF EFFECT OF CROWDING
        ([(0,0)-(2,15)],(0,1),(0.2,1),(0.4,1.1),(0.6,1.3),(0.8,2),(1,3),(1.2,4.2),
        (1.4,5.62),(1.6,7.8),(1.8,11.1),(2,15))
        Units: dmnl
        Crowding has no effect on the death fraction as long as the number of rabbits on Jean's farm does not reach more than half the number that he can support. As the number of Rabbits reaches the maximum rabbit population, the multiplier exponentially increases from 1 to 3.

(10)   net flow = births-deaths
        Units: rabbits/Month
        This is the net flow of rabbits/month.

(11)   NORMAL DEATH FRACTION = 0.167
        Units: 1/Month
        The NORMAL DEATH FRACTION is the rate at which each rabbit would die if it had an unlimited supply of food and water. It is three time less than the birth fraction.

(12)   NORMAL RABBIT POPULATION = 500
        Units: rabbits
        The maximum rabbit population is the maximum number of rabbits for which Jean can provide water. It is fixed (a constant) because Provence has a dry, arid climate and Jean only has a fixed supply of water available to him. (In the story Jean de Florette by Marcel Pagnol, however, Jean's supply of water was not fixed but depletable. So as the number of rabbits grew, the water was all used up and all the rabbits died. The dynamics of overshoot and collapse will be explored in Road Maps 9.)

(13)   Rabbits = INTEG (births-deaths, INITIAL RABBITS)

Units: rabbits

(14)   SAVEPER = TIME STEP
        Units: Month
        The frequency with which output is stored.
(15)   TIME STEP = 0.0625
        Units: Month
        The time step for the simulation.

Graph of birth, normal death, and death fraction
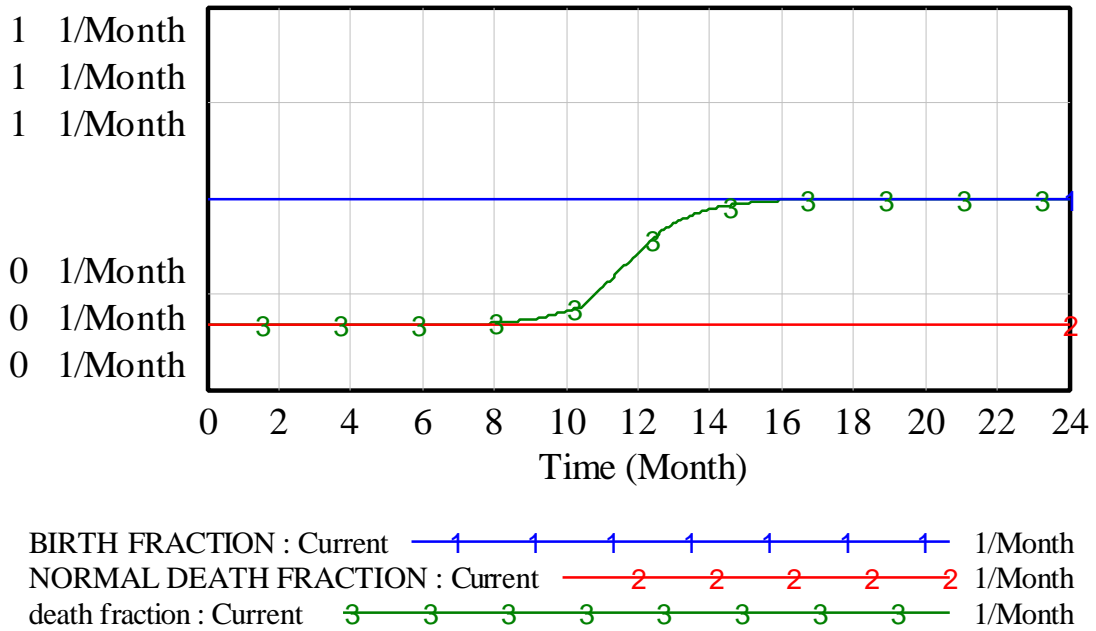
Figure 26: Vensim Equivalent of Figure 8: Rabbit population model: fractions

Graph of births, deaths, and net flow

250	rabbits/Month
250	rabbits/Month
250	rabbits/Month
0	rabbits/Month
0	rabbits/Month
0	rabbits/Month

births : Current ——1——1——1——1——1——1——1——1—— rabbits/Month
deaths : Current ——2——2——2——2——2——2——2——2—— rabbits/Month
net flow : Current ——3——3——3——3——3——3——3—— rabbits/Month

Figure 27: Vensim Equivalent of Figure 9: Rabbit population model: flows

Cabbage Patch Kids
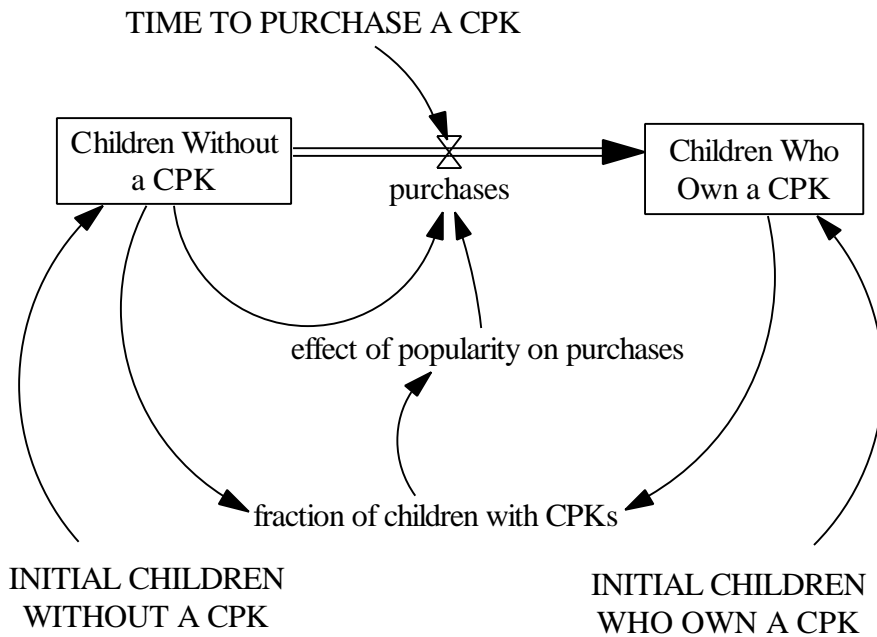
Figure 28: Vensim Equivalent of Figure 11: Cabbage Patch Kid Model

Documentation for Cabbage Patch Kids Model

(01)   Children Who Own a CPK = INTEG (+purchases, INITIAL CHILDREN WHO OWN A CPK)
Units: children
Number of children who own a Cabbage Patch Kid.

(02)   Children Without a CPK = INTEG (-purchases, INITIAL CHILDREN WITHOUT A CPK)
Units: children
The total population considered here is the total population that is susceptible to buying a doll. (i.e. it excludes children who can't afford the dolls)

(03)   effect of popularity on purchases = fraction of children with CPKs
Units: dmnl
As fraction of children with dolls increases, a corresponding fraction representing the effect or popularity on purchases increases from 0 to 1.

(04)   FINAL TIME = 24
Units: Month
The final time for the simulation.

(05)   fraction of children with CPKs =
Children Who Own a CPK/(Children Who Own a CPK+Children Without a CPK)
Units: dmnl
The fraction of children with CPKs is the percentage of the total population susceptible to buying dolls that actually owns a doll.

(06)   INITIAL CHILDREN WHO OWN A CPK = 1000
Units: children

(07)   INITIAL CHILDREN WITHOUT A CPK = 1e+006
Units: children

(08)   INITIAL TIME = 0
Units: Month
The initial time for the simulation.

(09)   purchases = effect of popularity on purchases* Children Without a CPK/TIME TO PURCHASE A CPK
Units: children/Month

(10)   SAVEPER = TIME STEP

Units: Month
The frequency with which output is stored.

(11)   TIME STEP = 0.0625
Units: Month
The time step for the simulation.

(12)   TIME TO PURCHASE A CPK = 2
Units: Month
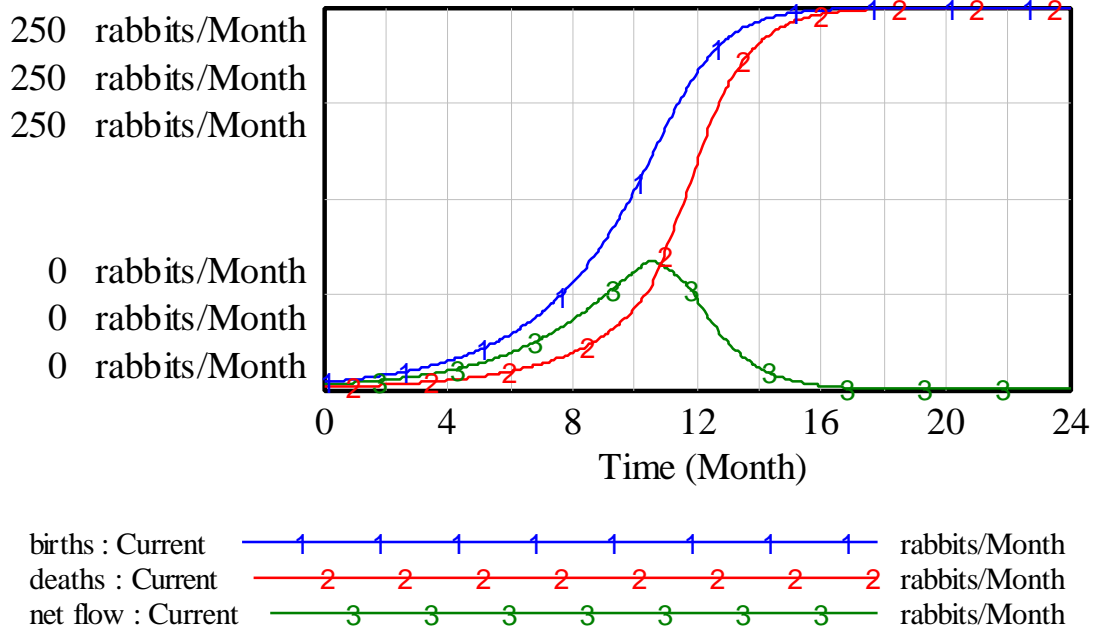The time to purchase a CPK is the average time for children to convince their parents to buy a doll plus the average time for the parents to find a doll that resembles their child.

Graph for Children Who Own a CPK
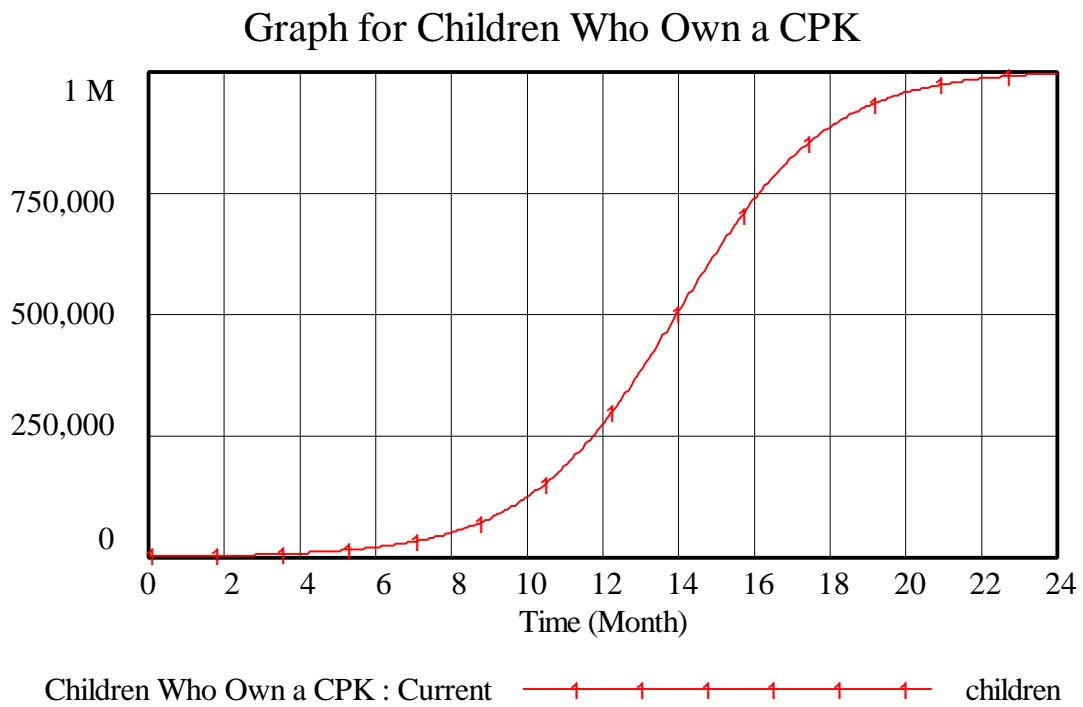
Figure 29: Vensim Equivalent of Figure 12: Cabbage Patch Kids: stock over time

Graph of purchases
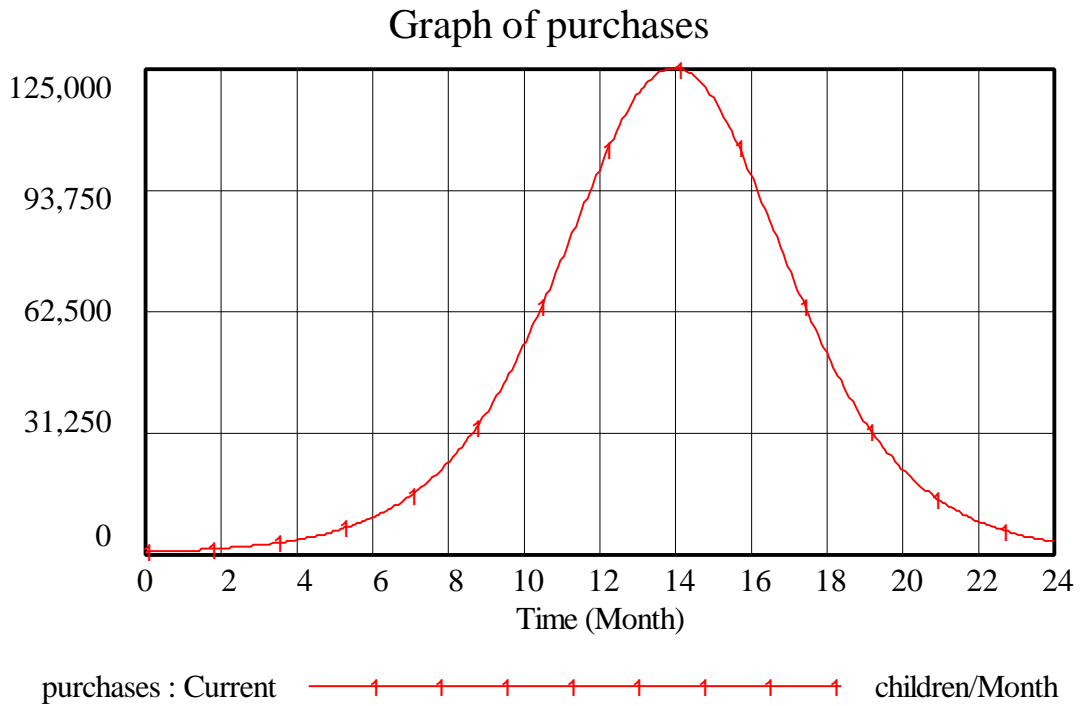

purchases : Current —————— children/Month

Figure 30: Vensim Equivalent of Figure 13: Cabbage Patch Kids: net flow over time
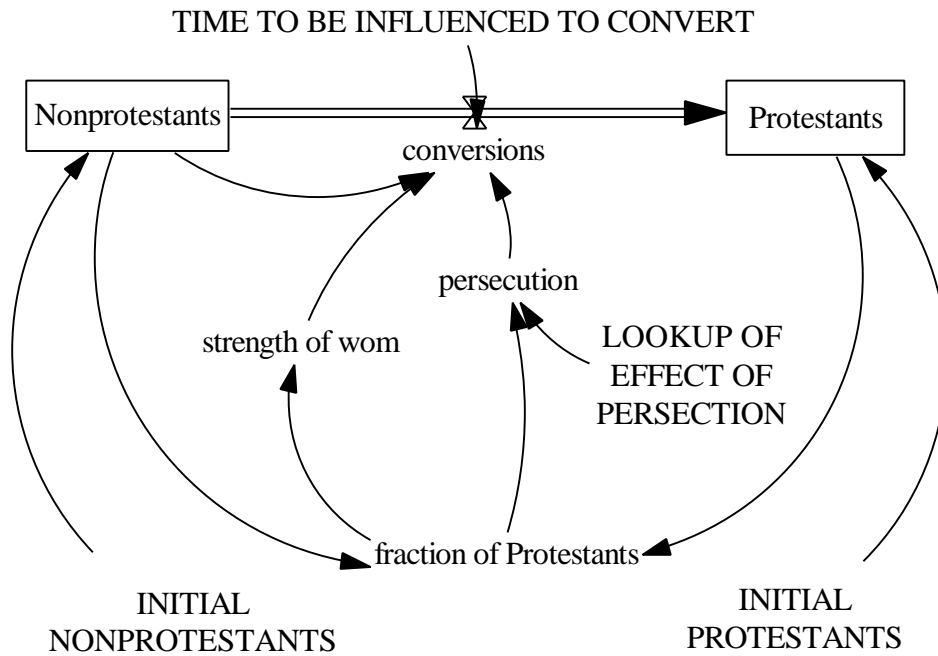
Protestantism

Figure 31: Vensim Equivalent of Figure 14: Protestantism model

Documentation for Protestantism Model

(01)   conversions = Nonprotestants * strength of wom *
        (1-persecution)/TIME TO BE INFLUENCED TO CONVERT
        Units: people/year
        Conversions is rate at which Nonprotestants, usually Roman Catholics, converted
        to become Protestant. Conversions is the product of the number of people left who
        have yet to convert, the fraction representing the strength of word of mouth, the
        opposite of the fraction representing the strength of persecution, all over the time
        it takes to convert.

(02)   FINAL TIME = 1600
        Units: year
        The final time for the simulation.

(03)   fraction of Protestants = Protestants/(Nonprotestants+Protestants)
        Units: dmnl
        The fraction of Protestants is the percentage of Protestants in the total population.

(04)   INITIAL NONPROTESTANTS = 1e+006
        Units: people

(05)   INITIAL PROTESTANTS = 100
        Units: people

(06)   INITIAL TIME = 1500
        Units: year
        The initial time for the simulation.

(06)   LOOKUP OF EFFECT OF PERSECTION
        ([(0,0)-(1,1)],(0,0.005),(0.1,0.235),(0.2,0.545),(0.3,1),(0.4,1),(0.5,1),
        (0.6,1),(0.7,1),(0.8,1),(0.9,1),(1,1))
        Units: dmnl
        Persecution is negligible when there are relatively few Protestants. As the number
        of Protestants becomes threatening, it shoots up to the point of cutting off future
        conversions. When the Inquisition and religious civil wars started in Europe, there
        were very few new conversions.

(08)   Nonprotestants = INTEG (-conversions, INITIAL NONPROTESTANTS)
        Units: people

(09)   persecution = LOOKUP OF EFFECT OF PERSECTION (fraction of Protestants)
        Units: dmnl
        This equation shows that persecution is a function of the fraction of Protestants.

(10)   Protestants = INTEG (conversions, INITIAL PROTESTANTS)

Units: people

The initial value of 100 people represents the original converts (and friends) of Martin Luther.

(11)   SAVEPER = TIME STEP
        Units: year
        The frequency with which output is stored.

(12)   strength of wom = fraction of Protestants
        Units: dmnl
        The strength of word of mouth is equal to the number of Protestants spreading that word of mouth.

(13)   TIME STEP = 1
        Units: year
        The time step for the simulation.

(14)   TIME TO BE INFLUENCED TO CONVERT = 4
        Units: year
        The time to be influenced to convert is the time for a Nonprotestant to hear about Protestantism and decide to adopt its principles.

Graph of Protestants

Protestants : Current ———————————— people

Time (year)
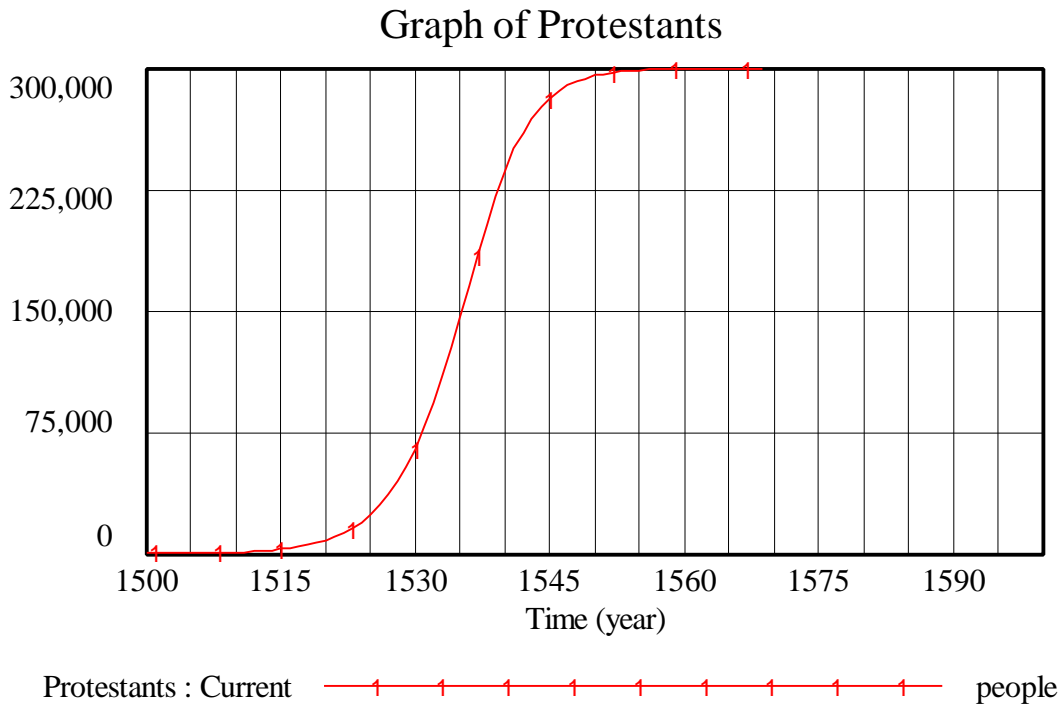
Figure 32: Vensim Equivalent of Figure 15: Protestantism: stock over time

Graph for conversions
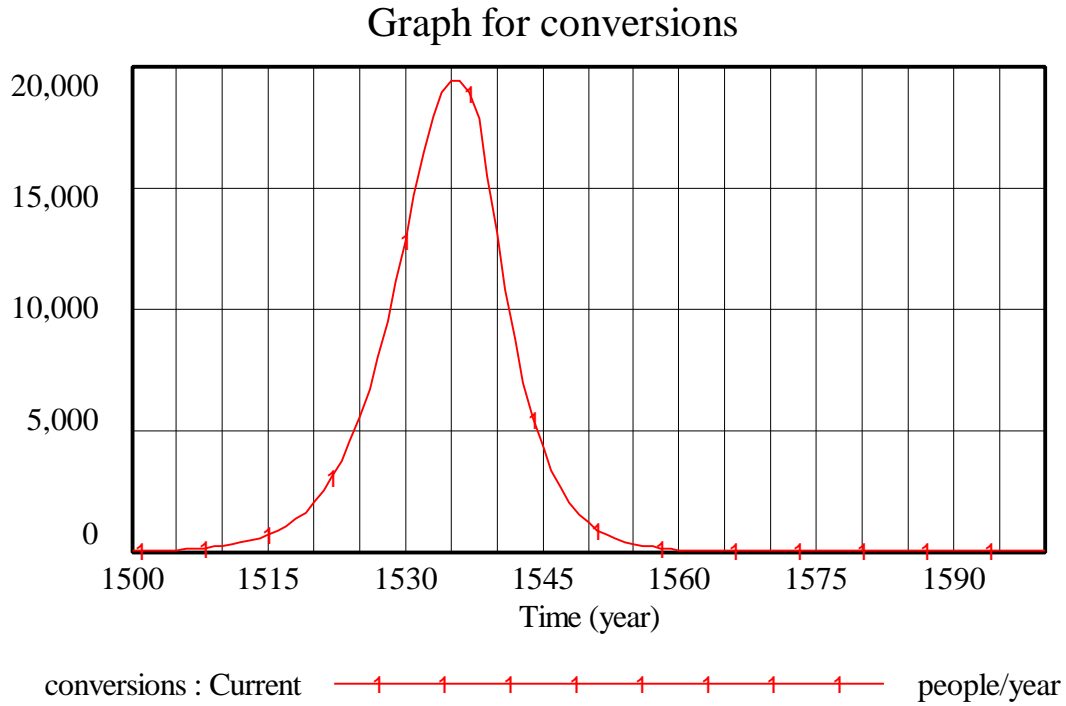
Figure 33: Vensim Equivalent of Figure 16: Protestantism: net flow over time

Graph of strength of wom and persecution
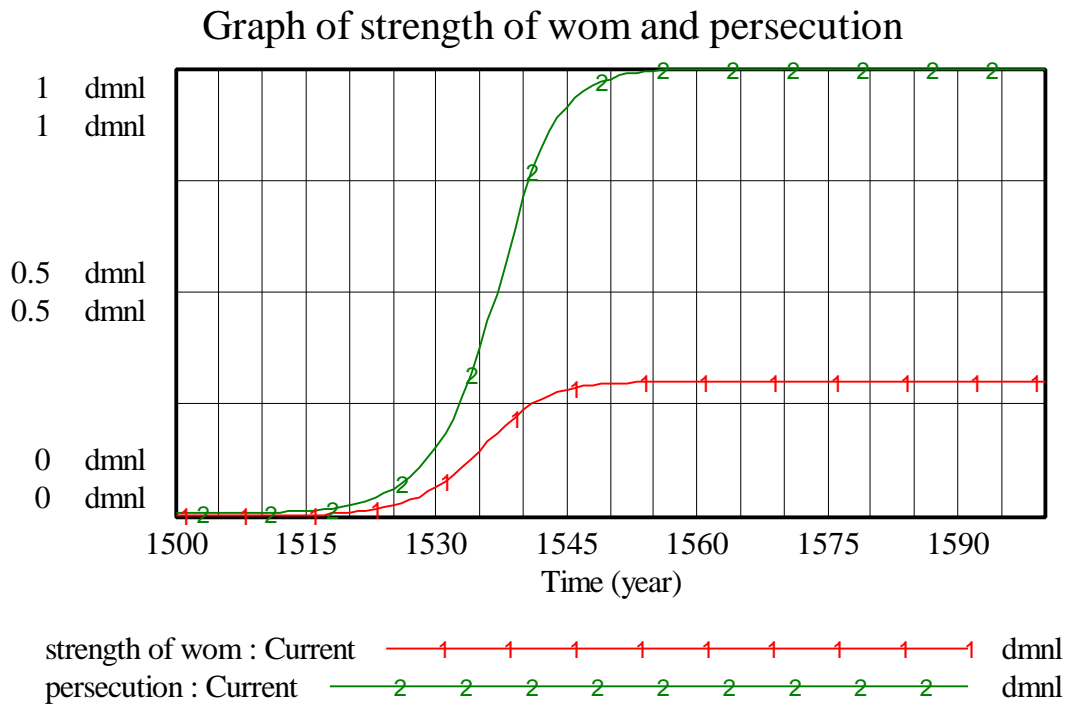
Figure 34: Vensim Equivalent of Figure 17: Protestantism: multipliers

Graph of Protestants and Nonprotestants

Figure 35: Vensim Equivalent of Figure 18: Protestants: both stocks over time

Immune Response

Figure 36: Vensim Equivalent of Figure 19: Immune response model

NOTE: Differences between the Stella and Vensim versions of the Immune System Model are described below the model documentation.

Documentation for Immune Response Model

(01)   antigens = Foreign Cells*ANTIGENS PER FOREIGN CELL
         Units: antigens
         Antigens are macromolecules on foreign cells that, when identified by helper T cells, trigger an immune response.

(02)   ANTIGENS PER FOREIGN CELL = 2
         Units: antigens/foreign cells
         Approximately 2 antigens are attached to each foreign cell.

(03)   cell destruction multiplier = LOOKUP FOR EFFECT OF RELATIVE DENSITY
          ON DESTRUCTION(relative white blood cell density)
          Units: dmnl
          A dimensionless multiplier produced by the lookup table and the ratio of foreign
          cells to white blood cells. This multiplier shows the effect of relative white blood
          cell density on the number of foreign cells destroyed.

(04)   destruction = Foreign Cells/TIME TO DESTROY A FOREIGN CELL*cell
          destruction multiplier*NORMAL DESTRUCTION
          Units: foreign cells/hour
          Destruction is the average rate at which white blood cells destroy foreign cells.

(05)   FINAL TIME = 48
          Units: hour
          The final time for the simulation.

(06)   Foreign Cells= INTEG (+multiplication-destruction, INITIAL FOREIGN CELLS)
          Units: foreign cells
          100 is an approximate number of foreign cells that invade one microliter of blood.

(07)   INITIAL FOREIGN CELLS =100
          Units: foreign cells

(08)   INITIAL TIME = 0
          Units: hour
          The initial time for the simulation.

(09)   INITIAL WHITE BLOOD CELLS = 10
          Units: white blood cells

(10)   LOOKUP FOR EFFECT OF RELATIVE DENSITY ON DESTRUCTION(
                [(0,0)-(2,2)],(0,0),(0.4,0.625),(1,1.125),(1.5,1.25),(2,1.25))
          Units: dmnl
          This lookups gives the relationship between the ratio of white blood cells to
          foreign cells (relative white blood cell density) and the number of foreign cells
          destroyed. As the ratio gets higher, the number of foreign cells destroyed
          increases. However, when the ratio is above one, the multiplier is at the value that
          produces (cell destruction multiplier*NORMAL DESTRUCTION) = 1, since we
          cannot destroy more foreign cells than are present in the blood.

(11)   multiplication = Foreign Cells/TIME FOR FOREIGN CELLS TO MULTIPLY
          Units: foreign cells/hour

(12)   NORMAL ANTIGENS = 1

Units: antigens
This is the normal number of antigens in the blood. i.e. without any foreign cells invading.

(13)   NORMAL DESTRUCTION = 0.8
Units: dmnl
This is a multiplier that gives the proportion of Foreign Cells that are found and destroyed by White Blood Cells when the "relative white blood cell density" is equal to 1. i.e. the proportion of Foreign Cells killed at normal white blod cell concentration.

(14)   NORMAL RATIO OF WHITE BLOOD CELLS TO FOREIGN CELLS = 1
Units: white blood cells/foreign cells
This gives the normal number of white blood cells for each red blood cells in blood uninfected by antigens.

(15)   proliferation = strength of proliferation*White Blood Cells/TIME FOR WHITE BLOOD CELLS TO PROLIFERATE
Units: white blood cells/hour
Proliferation is determined by multiplying a normal proliferation rate (white blood cells/time for white blood cells to proliferate) by the variable strength of proliferation.

(16)   ratio of white blood cells to foreign cells = White Blood Cells/Foreign Cells
Units: white blood cells/foreign cells
Ratio of White Blood Cells to Foreign Cellls in the blood.

(17)   relative white blood cell density = ratio of white blood cells to foreign cells/NORMAL RATIO OF WHITE BLOOD CELLS TO FOREIGN CELLS
Units: dmnl
This gives a measure of the density of white blood cells with respect to the normal proportions of foreign cells and white blood cells.

(18)   SAVEPER = TIME STEP
Units: hour
The frequency with which output is stored.

(19)   STRENGTH OF IMMUNE RESPONSE = 0.0001
Units: dmnl
The strength of the immune response is a fraction representing the ability of helper T cells to recognize antigens and activate the proliferation of white blood cells. When someone is infected with HIV, the human immune system is crippled and the strength of the immune response is severely weakened.

(20) strength of proliferation = STRENGTH OF IMMUNE RESPONSE*antigens/NORMAL ANTIGENS
Units: dmnl
Strength of proliferation is a multiplier that increases the proliferatin rate as the number of recognized antigens increases.

(21) TIME FOR FOREIGN CELLS TO MULTIPLY = 5
Units: hour
Time it takes for white blood cells to multiply after detection of the antigens.

(22) TIME FOR WHITE BLOOD CELLS TO PROLIFERATE = 5
Units: hour

(23) TIME STEP = 0.0625
Units: hour
The time step for the simulation.

(24) TIME TO DESTROY A FOREIGN CELL = 1
Units: hour
Time it takes to destroy a foreign cell.

(25) White Blood Cells= INTEG (proliferation, INITIAL WHITE BLOOD CELLS)
Units: white blood cells
Approximately 10 antigen-specific white blood cells are present in one microliter of blood before an antigen invasion is recognized. White blood cells is the name given to the cells that constitute the human immune response: helper T cells, killer T cells, B cells, memory cells....

Differences between the Vensim and Stella Immune System Models:

1. The differences between the Vensim and Stella versions all centre around the Foreign cell destruction. In the Stella version the destruction of foreign cells was limited only by the fact that the stock of "Foreign Cells" was set to **non-negative.** There was no first order control on the number of foreign cells being killed, since in any timestep it was possible to kill a number of foreign cells higher than the actual number of foreign cells present in the blood. This is incorrect modeling practise.
2. To fix the problem in the Vensim version, we have introduced new structure to ensure that the number of foreign cells destroyed is always less than or equal to the number of foreign cells present in the blood. The ratio of White Blood Cells to Foreign Cells gives a relative cell density that is the input to a lookup table. The lookup table shows how the relative cell density affects Foreign Cells destruction through the LOOKUP FOR EFFECT OF RELATIVE DENSITY ON DESTRUCTION. The output multiplier from this lookup table multiplied with the NORMAL DESTRUCTION and "Foreign Cells/TIME TO DESTROY A FOREIGN CELL" gives a rate of destruction that depends directly on the ratio of white blood cells to foreign cells for relative cell densities less than 1. However, for relative cell densities greater

than 1 i.e. more white blood cells than foreign cells in the blood, the table function begins to limit the destruction of foreign cells destroyed as they become more scarce, producing a model that more acurately represents the real system.

Graph of White Blood Cells and Foreign Cells

Figure 37: Vensim Equivalent of Figure 20: Immune response: stocks over time

Graph of Multiplication, Proliferation and Destruction

Figure 38: Vensim Equivalent of Figure 21: Immune response: net flows versus time

Graph with Immune Response Strength of 1/10,000

White Blood Cells : aids ——1——1——1——1——1—— white blood cells
Foreign Cells : aids ——2——2——2——2——2——2—— foreign cells
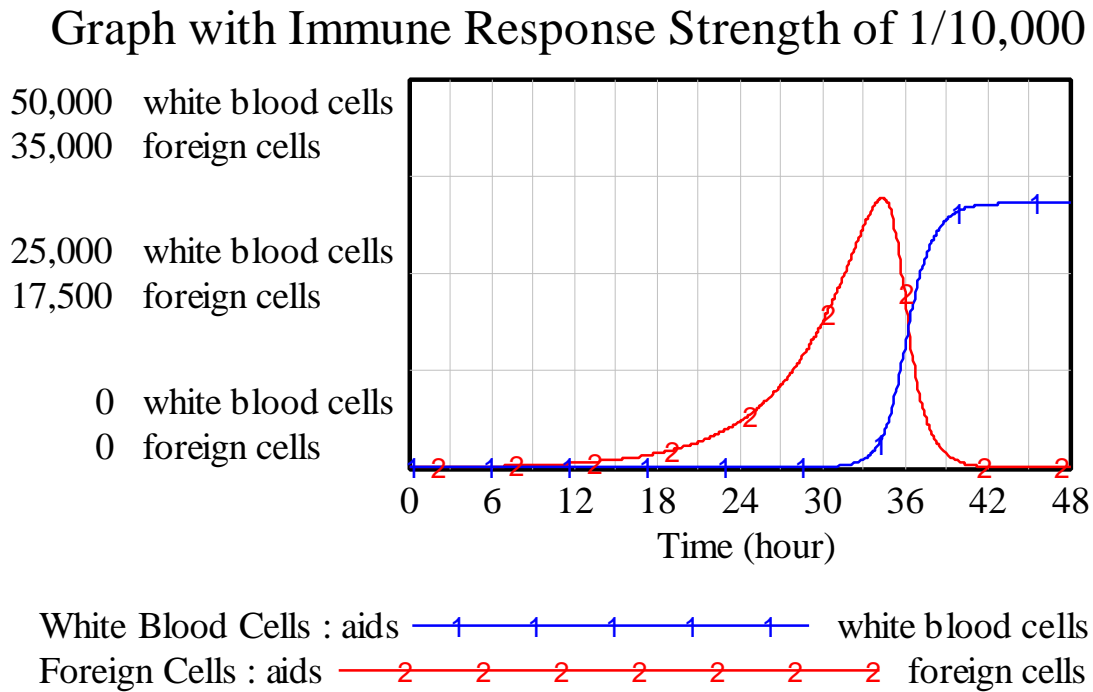
Figure 39: Vensim Equivalent of Figure 22: Strength of the immune response = 1/10,000

Graph with Impaired Immune Response - Detail

White Blood Cells : aids ——1——1——1——1——1—— white blood cells
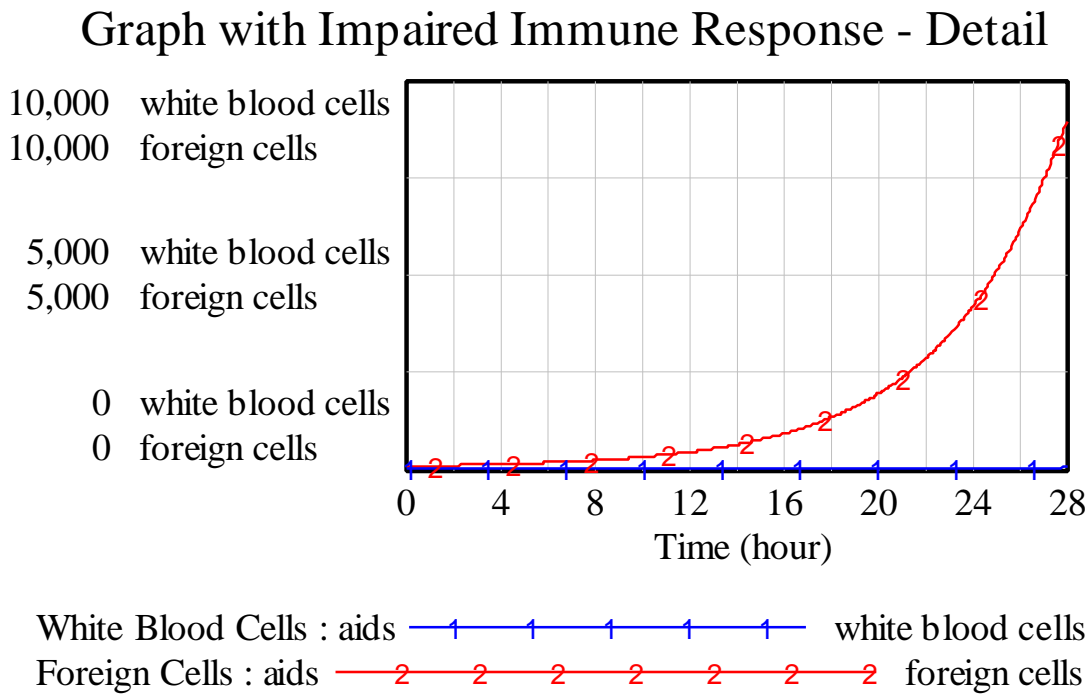Foreign Cells : aids ——2——2——2——2——2——2—— foreign cells

Figure 40: Vensim Equivalent of Figure 23: Impaired immune response - detail