16.001 Unified Engineering
Materials and Structures

Structural equilibrium: supports, general requirements and classification of structures
Internal forces

Reading assignments: CDL 1.7, 1.8

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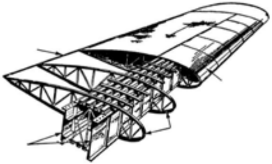
Structural supports and reactions



Structural supports are devices that restrict the translational or rotational motion of the structure or part of the structure that is subject of our analysis. The structure unloads the loads it carries on the supports. Conversely, the supports restrict motion by their ability to exert a resistive or *reaction* force (or moment) in the degree of freedom against which they act.

Degree of freedom constrained

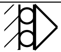
- displacement x_1
- displacement x_2
- rotation θ_3

Reaction force produced

- Force R_1 in direction x_1
- Force R_2 in direction x_2
- Moment M_3

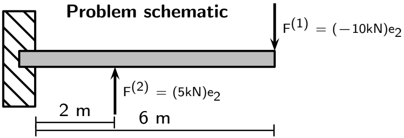
Supports (cont'd)

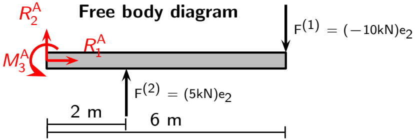
We idealize different types of structural supports with symbols that suggest which is (are) the constrained degree(s) of freedom. Most common support idealizations in 2D:

Name of support	Symbol	Constrained dof	Reaction
Horizontal Roller		Vertical displacement u_2	Vertical force R_2
Vertical Roller		? Horizontal displacement u_1	? Horizontal force R_1
Pin or hinge		? Both displacements u_1, u_2	? R_1, R_2
Fixed, built-in or clamp		? Both displacements u_1, u_2 and rotation θ_3	? R_1, R_2, M_3

Important

The choice of support in structural analysis is an idealization that might have important practical implications: can the physical realization really provide the reactions assumed?





Equilibrium equations:

$$\sum_k F_1^{(k)} = 0 \rightarrow \boxed{R_1^A = 0}$$

$$\sum_k F_2^{(k)} = 0 \rightarrow R_2^A + 5\text{kN} - 10\text{kN} = 0$$

$$\boxed{R_2^A = 5\text{kN}}$$

$$\sum_k M_3^A = 0 \rightarrow M_3^A + 5\text{kN} \times 2\text{m} - 10\text{kN} \times 6\text{m} = 0\text{k}$$

$$\boxed{M_3^A = 50\text{kNm}}$$

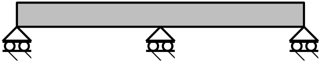
Requirements for structural equilibrium

A structure is said to be stable if all the rigid body motions are prevented regardless of the loads applied.

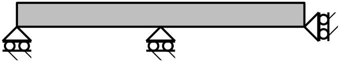
- In 3D, six motion constraints must be provided: three translations and three rotation.
- In 2D, three motion constraints must be provided: two translations and one rotation.

When selecting the types of support to constrain these motions at different points of the structure, choose their orientation and point of application carefully, as there are configurations that will not prevent all of the necessary motions.
For example, in 2D, three roller supports can prevent both translations and rotation, but only if the constrained directions do not intersect at one point. Clearly, in that case the structure could rotate with respect to that point.

Unstable support arrangement

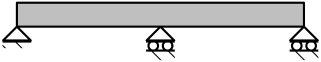


Unstable support arrangement



Stable support arrangement

Stable support arrangement



Stable support arrangement (but overconstrained)

Requirements for structural equilibrium

Supports (displacement constraints)

- 3D: Six motion restraints: three translations, three rotations
- 2D: Three motion restraints, e.g.:
  - two non-parallel translations, one rotation

Finding reactions:

When all degrees of freedom are properly and minimally constrained:

- draw FBD without unknown reactions
- use equilibrium equations to determine reaction values

STATICALLY-DETERMINED system

Mechanism

- Too few constraints
- More equilibrium equations than unknowns. Over-determined system with NO SOLUTION.
- The structure is **UNSTABLE**

Overconstrained structure

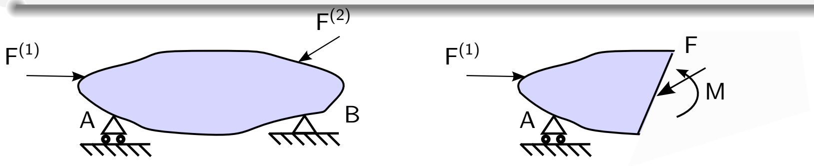
- More constraints than needed for static equilibrium
- More unknown reactions than equations. Equilibrium equations are under-determined, INFINITE SOLUTIONS
- The structure is **STATICALLY INDETERMINATE**

Internal forces

Subject to external loads, a structure deforms at each point of its material points. The deformation is resisted by internal stresses which are in equilibrium.

Definition

Internal forces and moments on a given cut plane going through the structure constitute the net (equipollent) effect of these internal stresses acting on that plane



After reactions are determined, the internal forces can be computed by applying the equilibrium equations to the FBD of substructure (either side). Clearly, the internal forces change with the choice of cut plane. We will study internal forces in structures when we discuss beams