1 Estimation of powered aircraft maximum range: Breguet Range Equation

1.1 Learning Objectives

At the end of this lecture, you will be able to answer the following questions:

• How far can an airplane fly?

• How do the disciplines of structures & materials, aerodynamics and propulsion jointly set the performance of aircraft, and what are the important performance parameters?

• Estimate the performance of aircraft using empirical data and thus begin to develop intuition regarding important aerodynamic, structural and propulsion system performance parameters

1.2 Some operational data
1.3 Historical note about the “Breguet” Range Equation


1.4 Derivation of the Breguet Range Equation

An excellent estimate of the range (maximum distance) that an aircraft can fly (under some important assumptions) is provided by the Breguet Range Equation.

The main consideration is to try and establish a relation between distance traveled \( R \) (m) and the remaining amount or mass of fuel \( W_f \) (N). The equation is obtained by simple considerations of mass, momentum and energy balance.

For simplicity, we will base our estimate assuming level flight (i.e. we ignore take-off, climb, descent and landing).

- Equilibrium or momentum conservation: During level flight, the aircraft travels at a constant ground speed \( V \) (m s\(^{-1}\)) and altitude, see figure:
Under these conditions, the propulsive force or *thrust* \( T \) (N) equals the *aerodynamic drag force* \( D \), and the *aerodynamic lift force* \( L \) (N) equals the total weight \( W \) (N):

\[
T = D, \quad L = W. \tag{1}
\]

You will study in Fluids in the spring that the drag and lift are related and that in level flight, the ratio of the two can be considered constant. This allows us to relate the vertical and horizontal equilibrium equations, and thus the thrust and the weight as follows:

\[
W = L = \frac{L}{D} T, \quad W = \frac{L}{D} T. \tag{2}
\]

Note that \( L/D \) is a measure of the aerodynamic efficiency of the aircraft.

**Mass balance:** At any point during flight, the total weight of the aircraft \( W \) is the addition of the weight of the structure \( W_s \), the weight of the payload \( W_p \), and the weight of the fuel \( W_f \):

\[
W = W_s + W_p + W_f. \tag{3}
\]

The maximum total weight of the aircraft when the fuel tanks are full and under full payload is typically referred to with the acronym *MTOW*, or maximum take-off weight. Also, during flight, the total weight \( W \) changes as the fuel mass is expended. So the total aircraft weight as well as the fuel weight can be thought of as functions of time \( t \) or also distance traveled \( R \). The change can be written mathematically as follows:

\[
\frac{dW}{dt} = \frac{dW_f}{dt} = -\dot{m}_f g, \tag{4}
\]

where \( \dot{m}_f \) (kg s\(^{-1}\)) is the fluid mass flow rate assumed constant in level flight, and \( g \approx 9.81 \) m s\(^{-2}\) is the acceleration of gravity on earth (also assumed constant).

**Energy balance:** The main consideration here is that the fuel energy is expended in producing the thrust which is necessary to counterbalance the drag. The *propulsive power* \( P_p \) is:

\[
P_p = T \times V. \tag{5}
\]

The power is provided by the combustion of the fuel, which has an amount of energy per unit mass \( h_f \) (J kg\(^{-1}\)), and could in principle provide a *fuel power* \( P_f \):

\[
P_f = \dot{m}_f \times h_f. \tag{6}
\]

However, and as you will study in detail in Thermodynamics and Propulsion, a number of losses occur in the process of converting the chemical energy available in the fuel to the final propulsive power. We refer to the fraction of the fuel power effectively contributing to propulsive power as the *total efficiency* \( \eta_0 \):

\[
\eta_0 = \frac{P_p}{P_f} \tag{7}
\]
and our energy balance principle can be written as:

\[ P_p = \eta_0 P_f, \quad \text{or} \quad TV = \eta_0 \dot{m}_f h_f \]  \(\text{(Watt)}\) \hspace{1cm} (8)

Having discussed the important governing principles, we now seek to combine the resulting equations to achieve the desired goal. We start by going back to the initial idea that we are seeking to find a relation between the aircraft weight and the distance traveled. It is clear that by the time the weight of the fuel has vanished, we have reached the maximum range \( R_{\text{max}} \) the aircraft can travel under power. At that point, \( W_f = 0 \), \( W(R_{\text{max}}) = W_0 = W_s + W_p \), i.e. the total weight is the weight of the aircraft with zero fuel \( W_0 \) which is the sum of the structural and payload weight, also known as the operating empty weight (OEW). We note that mass conservation Equation (4) gives us information on how the weight \( W \) changes over time \( t \), not distance traveled \( R \). However, distance traveled and time are related by: \( \frac{dR}{dt} = V \), and we can use the chain rule to write:

\[
\frac{dW}{dt} = \frac{dW}{dR} \frac{dR}{dt} = \frac{dW}{dR} V = -\dot{m}_f g
\]

from (4) \hspace{1cm} (9)

This expression can be combined with energy balance Equation (8), as follows:

from (8):

\[
\frac{\dot{m}_f}{V} = \frac{T}{\eta_0 h_f}
\]

from (9):

\[
\frac{dW}{dR} = -g \frac{\dot{m}_f}{V}
\]

combining:

\[
\frac{dW}{dR} = -g \frac{T}{\eta_0 h_f}
\]

The final step is to now recognize that we haven’t used our equilibrium (or momentum balance) Equation (2), which gives: \( T = \frac{W}{\ell_D} \). Combining this with Equation (12), we get:

\[
\frac{dW}{dR} = -g \frac{W}{\eta_0 h_f \ell_D}
\]

from (10) \hspace{1cm} (13)

This equation has the form:

\[ W'(R) = aW(R) \]

where the constant coefficient: \( a = -\frac{g}{\eta_0 h_f \ell_D} \) \hspace{1cm} (14)

It constitutes a first-order ordinary differential equation with constant coefficients and governs the evolution of the weight of the aircraft \( W \) as a function of distance traveled \( R \). It can be easily integrated by noting that:

\[
(ln f(x))' = \frac{1}{f(x)} f'(x)
\]

\hspace{1cm} (15)
and therefore Equation (14) can be written as:

$$\frac{W'(R)}{W(R)} = (\ln W(R))' = a$$

(16)

integrating:

$$\ln W(R) = aR + C$$

(17)

Now comes the important step of applying the initial condition, or known point in the solution $W(R)$. What we know, is that at the beginning of the flight (distance traveled $R = 0$), the weight of the aircraft is the total weight with full fuel tanks:

$$W(R = 0) = W_{\text{init}} = W_0 + W_{\text{fuel}}$$

(18)

Evaluating Equation (17) at the known solution point:

$$\ln W(R = 0) = a0 + C, \rightarrow C = \ln W_{\text{init}}$$

(19)

Replacing in Equation (17):

$$\ln W(R) - \ln W_{\text{init}} = aR, \rightarrow R(W) = \frac{1}{a} \ln \left( \frac{W}{W_{\text{init}}} \right)$$

(20)

Replacing the value of $a$ from Equation (14):

$$R(W) = \frac{\eta_0 h_f L}{g} \ln \left( \frac{W}{W_{\text{init}}} \right)$$

(21)

$$R(W) = \frac{h_f}{g} \eta_0 \frac{L}{D} \ln \left( \frac{W_{\text{init}}}{W} \right)$$

(22)

We have thus established the sought relation $R(W)$ of the distance traveled in level flight $R$ as a function of the evolving weight of the aircraft $W$. This relation can be inverted to obtain the weight as a function of distance.

A number of remarks and observations are in order in the interpretation of Equation (22):

- the factor $\frac{h_f}{g}$ should define the dimension of the right hand side as all other factors are non-dimensional. Let’s check this using SI units: $h_f$ has units of energy per unit mass, or in SI: J kg$^{-1}$ = kg m s$^{-2}$mkg$^{-1}$ = m$^2$s$^{-2}$, $g$ has units of length per time squared, or in SI: m s$^{-2}$. Then, the ratio $h_f/g$ in SI has units: $\frac{\text{m}^2\text{s}^{-2}}{\text{m} \text{s}^{-2}}$ = m. We conclude that this factor has dimensions of length and gives the dimension of the right hand side.

Physically, this factor represents the efficiency of the fuel in terms of the energy density per unit mass. Clearly, a fuel with a higher value of $h_f$ would increase the range, ceteris paribus. Typical values of $h_f$ for jet fuel is around 40MJkg$^{-1}$. The equation also tells us that gravity affects the range in an inversely-proportional manner.
• The factor $\eta_0$ has already been discussed and represents the propulsive efficiency of the engine. Typical values for modern propulsion systems are around $0.2 - 0.4$.

• the factor $\frac{D}{L}$ is non-dimensional and represents the aerodynamic efficiency of the aircraft design. Typical values of $\frac{D}{L}$ in modern aircraft are around $15 - 20$.

• the factor inside the logarithm $\frac{W_{\text{init}}}{W}$ represents the ratio of the sum of the structural $W_s$, payload $W_p$, and initial fuel $W_f$ weights to the current total weight.

The maximum range $R_{\text{max}}$ for a given aircraft is obtained from Equation (22) when the initial weight $W_{\text{init}} = \text{MTOW}$, and all the fuel weight has been expended $W_f = 0$, in which case $W = W_0 = \text{OEW}$

$$R_{\text{max}} = \frac{h_f}{g} \eta_0 \frac{L}{D} \ln \left( \frac{\text{MTOW}}{\text{OEW}} \right)$$

Clearly, $\frac{\text{MTOW}}{\text{OEW}}$ plays the role of a structural efficiency of the aircraft design, and calls for lighter and lighter aircraft where as much as possible of the weight is devoted to the fuel. Typical values of $\frac{\text{MTOW}}{\text{OEW}} \sim 2$.

THE BREGUET RANGE EQUATION

Or equivalently,

$$\text{Range} = \frac{a}{g} \frac{M L/D}{\text{TSFC}} \ln \left( \frac{W_{\text{init}}}{W_{\text{final}}} \right)$$

Thrust Specific Fuel Consumption
TSFC = mass flow rate of fuel per unit thrust

Warning: Watch units of TSFC which are typically kg/s/N or lbm/hr/lbf
1.5 More practical and operational data

**FUEL ENERGY/UNIT MASS**

<table>
<thead>
<tr>
<th>Substance</th>
<th>$\text{kg/g}$</th>
<th>$\text{lb/lb}$</th>
<th>$\text{cal/oz}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prime beef</td>
<td>4.0</td>
<td>9</td>
<td>600</td>
</tr>
<tr>
<td>Meat</td>
<td>4.2</td>
<td>8</td>
<td>2</td>
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<tr>
<td>Whole milk</td>
<td>2.8</td>
<td>0.95</td>
<td>26</td>
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<tr>
<td>Honey</td>
<td>14</td>
<td>4</td>
<td>0.29</td>
</tr>
<tr>
<td>Sugar</td>
<td>15</td>
<td>1</td>
<td>0.07</td>
</tr>
<tr>
<td>Cheese</td>
<td>15</td>
<td>6</td>
<td>0.48</td>
</tr>
<tr>
<td>Beef</td>
<td>29</td>
<td>4</td>
<td>0.14</td>
</tr>
<tr>
<td>Corn meal</td>
<td>15</td>
<td>3.5</td>
<td>0.25</td>
</tr>
<tr>
<td>Peanut butter</td>
<td>27</td>
<td>4</td>
<td>0.15</td>
</tr>
<tr>
<td>Suet</td>
<td>32</td>
<td>4.5</td>
<td>0.14</td>
</tr>
<tr>
<td>Vegetable oil</td>
<td>36</td>
<td>2</td>
<td>0.06</td>
</tr>
<tr>
<td>Roasted</td>
<td>42</td>
<td>0.48</td>
<td>0.02</td>
</tr>
<tr>
<td>Suet oil</td>
<td>42</td>
<td>0.48</td>
<td>0.02</td>
</tr>
<tr>
<td>Gasoline</td>
<td>42</td>
<td>0.40</td>
<td>0.016</td>
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<tr>
<td>Natural gas</td>
<td>45</td>
<td>0.95</td>
<td>0.006</td>
</tr>
</tbody>
</table>

*Note: Values are for 100% of the fuel.

**WEIGHT & GEOMETRY**

<table>
<thead>
<tr>
<th>Wildlife</th>
<th>$W$ (kg)</th>
<th>$S$ ($\text{m}^2$)</th>
<th>$b$ (m)</th>
<th>$a$</th>
<th>$r$</th>
<th>$F = L/D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>House sparrow</td>
<td>0.28</td>
<td>0.009</td>
<td>0.23</td>
<td>5</td>
<td>4</td>
<td>1</td>
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<tr>
<td>Swift</td>
<td>0.36</td>
<td>0.016</td>
<td>0.42</td>
<td>11</td>
<td>10</td>
<td>1.2</td>
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<tr>
<td>Common tern</td>
<td>1.3</td>
<td>0.056</td>
<td>0.83</td>
<td>12</td>
<td>12</td>
<td>1.0</td>
</tr>
<tr>
<td>Kestrel (sparrow hawk)</td>
<td>1.8</td>
<td>0.055</td>
<td>0.74</td>
<td>9</td>
<td>9</td>
<td>1.0</td>
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<tr>
<td>Common crow</td>
<td>3.5</td>
<td>0.12</td>
<td>0.78</td>
<td>5</td>
<td>5</td>
<td>1.0</td>
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<tr>
<td>Common buzzard</td>
<td>8.6</td>
<td>0.22</td>
<td>2.25</td>
<td>7</td>
<td>10</td>
<td>1.0</td>
</tr>
<tr>
<td>Peregrine falcon</td>
<td>6.1</td>
<td>0.13</td>
<td>1.96</td>
<td>9</td>
<td>10</td>
<td>1.0</td>
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<tr>
<td>Heron</td>
<td>14</td>
<td>0.36</td>
<td>1.73</td>
<td>8</td>
<td>9</td>
<td>1.0</td>
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<tr>
<td>White albatross</td>
<td>34</td>
<td>0.95</td>
<td>2.90</td>
<td>8</td>
<td>10</td>
<td>1.0</td>
</tr>
<tr>
<td>Wandering albatross</td>
<td>85</td>
<td>0.67</td>
<td>3.40</td>
<td>19</td>
<td>19</td>
<td>1.0</td>
</tr>
<tr>
<td>Hang glider</td>
<td>1000</td>
<td>15</td>
<td>10</td>
<td>7</td>
<td>8</td>
<td>1.0</td>
</tr>
<tr>
<td>Para-gliding</td>
<td>1000</td>
<td>25</td>
<td>5</td>
<td>2.6</td>
<td>4</td>
<td>1.0</td>
</tr>
<tr>
<td>Powered paragliding</td>
<td>1700</td>
<td>35</td>
<td>10</td>
<td>2.7</td>
<td>4</td>
<td>1.0</td>
</tr>
<tr>
<td>Ultralight (hang glider)</td>
<td>2000</td>
<td>15</td>
<td>10</td>
<td>7</td>
<td>8</td>
<td>1.0</td>
</tr>
</tbody>
</table>

(from The Simple Science of Flight, by H. Tennekes)
AERODYNAMIC EFFICIENCY TRENDS

OVERALL PROPULSION SYSTEM EFFICIENCY

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ENGINE EFFICIENCY TRENDS
Turboprops, Regional Jets, Large Aircraft

Babikian, Raffi, The Historical Fuel Efficiency Characteristics of Regional Aircraft From Technological, Operational, and Cost Perspectives, SM Thesis, Massachusetts Institute of Technology, June 2001

STRUCTURAL EFFICIENCY TRENDS
Turboprops, Regional Jets, Large Aircraft

Babikian, Raffi, The Historical Fuel Efficiency Characteristics of Regional Aircraft From Technological, Operational, and Cost Perspectives, SM Thesis, Massachusetts Institute of Technology, June 2001

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