16.001 - Materials & Structures
Problem Set #3

Instructors:	Raúl Radovitzky
	Zachary Cordero
Teaching Assistants:	Grégoire Chomette
	Michelle Xu
	Daniel Pickard

Department of Aeronautics & Astronautics
M.I.T.

THIS PSET ONLY HAS TWO PROBLEMS AT THE END FOR GRADE, THE REST OF THE PROBLEMS ARE GIVEN WITH SOLUTIONS AS A LEARNING EXERCISE

Problem M-3.1
(MO: M6)

Consider the pin-jointed cantilever truss shown in Figure 1.

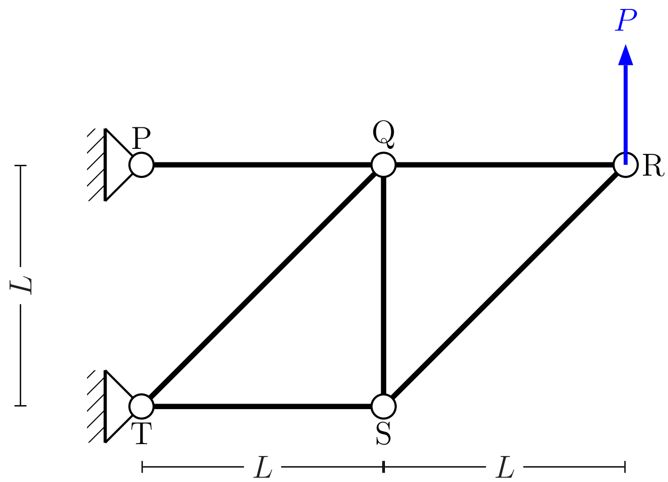
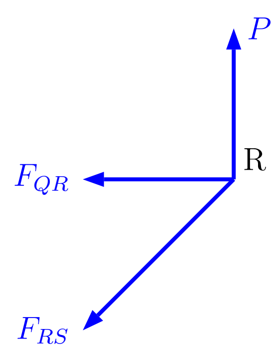


Figure 1: Cantilever Truss

(a) Determine the forces all of the members of the truss due to the applied load P, distinguishing between tension and compression.

Solution: We may use the method of joints to solve for the forces in the members of the truss. Beginning with summation of forces in Joint R,

- **Joint R**



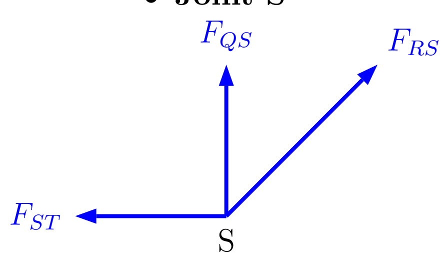
  – Force equilibrium in $\mathbf{e}_2$ direction

$$\sum F_2 = 0$$
$$P - F_{RS}\frac{\sqrt{2}}{2} = 0$$
$$\boxed{F_{RS} = \sqrt{2}P}$$

- Force equilibrium in $\mathbf{e}_1$ direction

$$\sum F_1 = 0$$
$$-F_{QR} - \frac{\sqrt{2}}{2}F_{RS} = 0$$
$$F_{QR} = -\frac{\sqrt{2}}{2}F_{RS} = -\frac{\sqrt{2}}{2}(\sqrt{2}P)$$
$$\boxed{F_{QR} = -P}$$

- **Joint S**



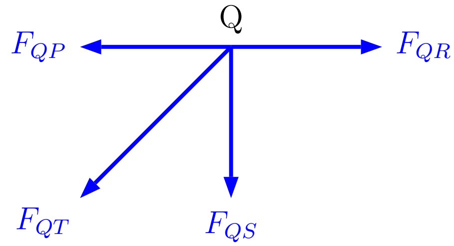
- Force equilibrium in $\mathbf{e}_1$ direction

$$\sum F_1 = 0$$
$$F_{RS}\frac{\sqrt{2}}{2} - F_{ST} = 0$$
$$F_{ST} = F_{RS}\frac{\sqrt{2}}{2} = (\sqrt{2}P)\left(\frac{\sqrt{2}}{2}\right)$$
$$\boxed{F_{ST} = P}$$

- Force equilibrium in $\mathbf{e}_2$ direction

$$\sum F_2 = 0$$
$$F_{QS} + F_{RS}\frac{\sqrt{2}}{2} = 0$$
$$F_{QS} = -F_{RS}\frac{\sqrt{2}}{2} = -(\sqrt{2}P)(\frac{\sqrt{2}}{2})$$
$$\boxed{F_{QS} = -P}$$

- **Joint Q**



– Force equilibrium in $\mathbf{e}_2$ direction

$$\sum F_2 = 0$$
$$-F_{QT}\frac{\sqrt{2}}{2} - F_{QS} = 0$$
$$F_{QT} = -\sqrt{2}F_{QS} = -\sqrt{2}(-P)$$
$$\boxed{F_{QT} = \sqrt{2}P}$$

– Force equilibrium in $\mathbf{e}_1$ direction

$$\sum F_1 = 0$$
$$-F_{PQ} - F_{QT}\frac{\sqrt{2}}{2} + F_{QR} = 0$$
$$F_{PQ} = -F_{QT}\frac{\sqrt{2}}{2} + F_{QR} = -(-\sqrt{2}P)(\frac{\sqrt{2}}{2}) + (-P)$$
$$\boxed{F_{PQ} = -2P}$$

Using the method of joints and analyzing joints R, S, and Q gives the forces in the 6 members of the truss. In summary, we have that the forces in the rods are:

$\boxed{F_{RS} = \sqrt{2}P}$ tensile

$\boxed{F_{QR} = P}$ compressive

$\boxed{F_{ST} = P}$ tensile

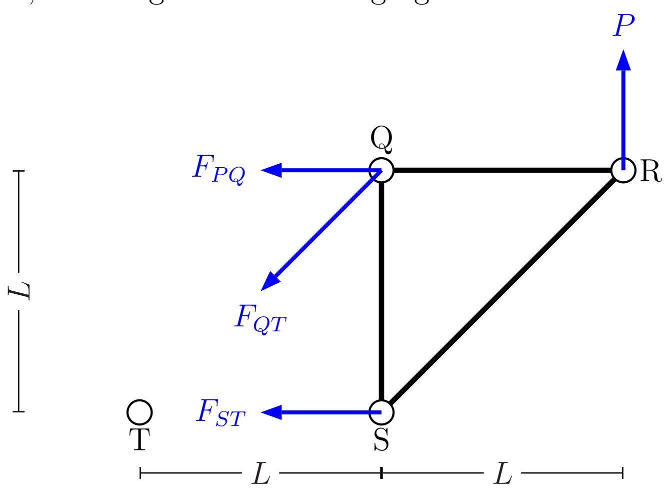
$\boxed{F_{QS} = P}$ compressive

$\boxed{F_{QT} = \sqrt{2}P}$ tensile

$\boxed{F_{PQ} = 2P}$ compressive

(b) Assume now that we were only interested in the force in member PQ. What method could you use to easily find the force in PQ (without knowledge of the internal forces in the other members)? Perform the method and obtain this force.

Solution: We could apply the method of sections in order to determine the internal force in member PQ. Specifically, we may take a cut through members PQ, QT and ST, resulting in the following figure:



Taking moments about the point T (not connected to the newly cut structure, but shown for reference), we obtain the following equation

$$\sum M_T = 0 \rightarrow (2L)(P) + (L)(F_{PQ}) = 0 \quad (1)$$

Solving for F_{PQ} gives $F_{PQ} = -2P$, which agrees with the value found in part (a).

(c) What is the maximum load P the truss may support if the members can only handle a maximum tension or compression of 20 kN?

Solution: To solve this problem, we apply linearity and realize that the forces experienced by individual members of the truss scale with the applied force P. The maximum of these tensile/compressive forces is experienced by rod PQ which experiences a compressive force of $2P$ due to the applied load of P at joint R. Thus

$$2P = 20\ kN$$

$$\boxed{P = 10kN}$$

Thus, the maximum allowable load P such that no member of the truss experiences a force of more than 20 kN in magnitude is $\boxed{10\ kN}$.

○ **Problem M-3.2**

A space truss is a three-dimensional truss structure, an example of which is given in Figure 2. The four ball joints are attached to a fixed wall. The three little bars attached to the ball joints can rotate freely. The forces applied on B and C have a magnitude of 2kN each.

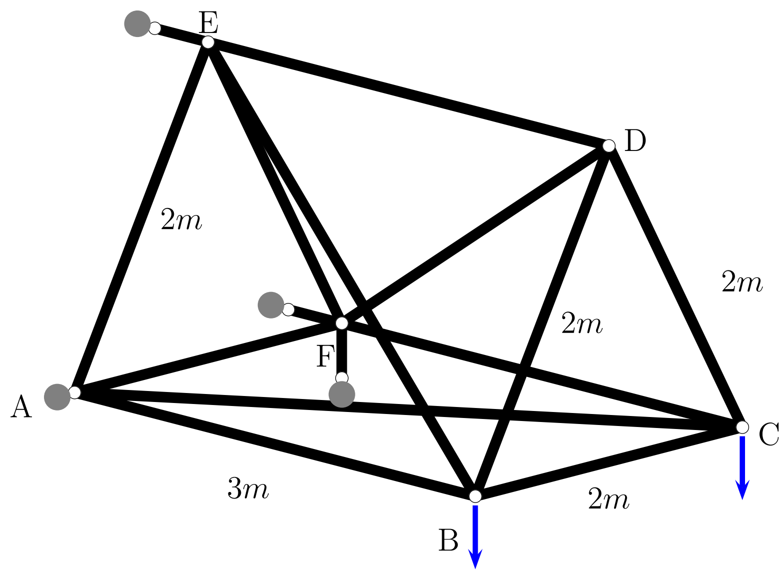
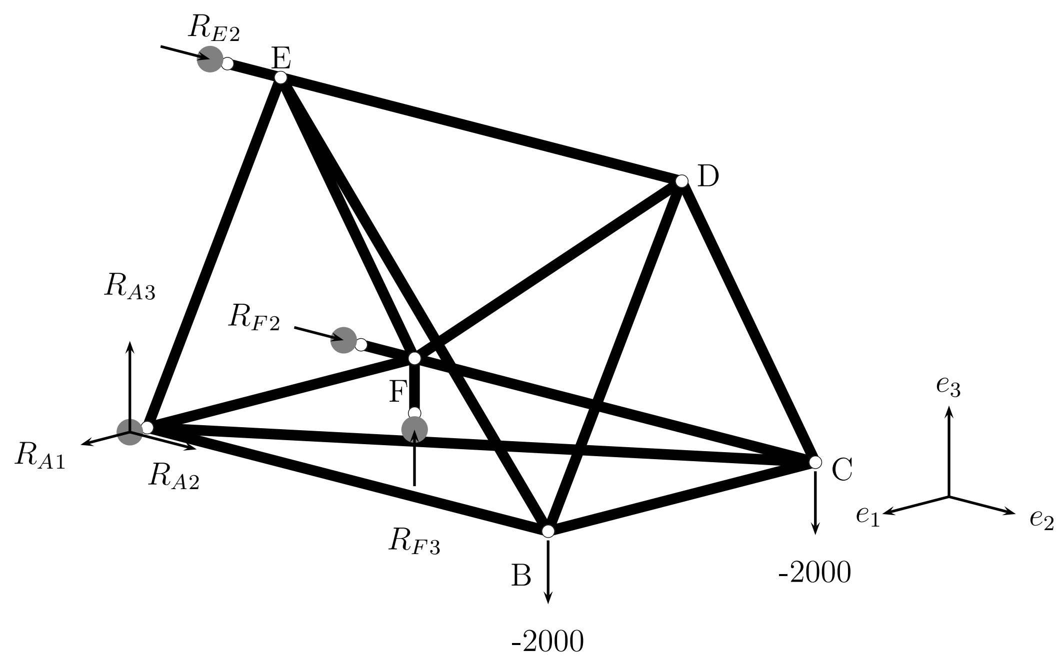


Figure 2: Space truss

(a) Draw a FBD of this problem and determine the reaction at the supports. Is the system statically determinate?

> **Solution:** We begin by establishing our coordinate system. Select F to be the origin, $\mathbf{e_1}$ to align with AF, $\mathbf{e_2}$ to align with FC, and $\mathbf{e_3}$ to point upwards.



$$\sum F_1 = 0 \rightarrow R_{A_1} = 0 \tag{2}$$

$$\sum F_2 = 0 \rightarrow R_{A_2} + R_{F_2} + R_{E_2} = 0 \tag{3}$$

$$\sum F_3 = 0 \rightarrow R_{A_3} + R_{F_3} - 4 = 0 \tag{4}$$

Take moments around point A,

$$\sum \mathbf{M}_A = 0 = \mathbf{r_B} \times \mathbf{P} + \mathbf{r_C} \times \mathbf{P} + \mathbf{r_F} \times \mathbf{R_{F_3}} + \mathbf{r_F} \times \mathbf{R_{F_2}} + \mathbf{r_E} \times \mathbf{R_{E_2}}$$

$$= \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix} + \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}$$

$$+ \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ R_{F_3} \end{bmatrix} + \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ R_{F_2} \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ \sqrt{3} \end{bmatrix} \times \begin{bmatrix} 0 \\ R_{E_2} \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -6 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -6 \\ -4 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 2R_{F_3} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -2R_{F_2} \end{bmatrix} + \begin{bmatrix} -\sqrt{3}R_{E_2} \\ 0 \\ -R_{E_2} \end{bmatrix}$$

$$\sum M_1 = 0 \rightarrow -12 - \sqrt{3}R_{E_2} = 0$$
$$\sum M_2 = 0 \rightarrow -4 + 2R_{F_3} = 0$$
$$\sum M_3 = 0 \rightarrow -2R_{F_2} - R_{E_2} = 0$$

Therefore

$$R_{E_2} = -4\sqrt{3}\text{kN} \quad (5)$$
$$R_{F_3} = 2\text{kN} \quad (6)$$
$$R_{F_2} = -0.5R_{E_2} = 2\sqrt{3}\text{kN} \quad (7)$$

Plug Eq.7 in Eq.4, we have

$$R_{A_1} = 0$$
$$R_{A_2} = -R_{E_2} - R_{F_2} = 2\sqrt{3}\text{kN}$$
$$R_{A_3} = 4 - R_{F_3} = 2\text{kN}$$

(b) Determine the force in members AB, CD, ED, and CF.

Solution: There are several approaches to solve this problem. In this solution, we apply method of joints directly to joints B, C, and D, and solve for the internal forces in the truss members.

Now we apply method of joints to point C, which connects members BC, CD, and CF. In 3D, equilibrium gives us 3 equations at every joint. The force equilibrium equation at joint C results in the following equations:

- **Equilibrium at Joint A**

$$\sum F_1 = 0 \rightarrow R_{A_1} - \frac{1}{2}F_{AE} - F_{AF} - \frac{2}{\sqrt{13}}F_{AC} = 0 \quad (8)$$
$$\sum F_2 = 0 \rightarrow R_{A_2} + F_{AB} + \frac{3}{\sqrt{13}}F_{AC} = 0 \quad (9)$$
$$\sum F_3 = 0 \rightarrow \frac{\sqrt{3}}{2}F_{AE} + R_{A_3} = 0 \quad (10)$$

Therefore we have,

$$F_{AE} = \frac{-4}{\sqrt{3}}\text{kN}$$

- **Equilibrium at Joint E**

$$\sum F_1 = 0 \rightarrow \frac{1}{2}F_{AE} - \frac{1}{2}F_{EF} + \frac{1}{\sqrt{13}}F_{EB} = 0$$
$$\sum F_2 = 0 \rightarrow F_{ED} + \frac{3}{\sqrt{13}}F_{EB} + R_{E_2} = 0$$
$$\sum F_3 = 0 \rightarrow -\frac{\sqrt{3}}{2}F_{AE} - \frac{\sqrt{3}}{2}F_{EF} - \frac{\sqrt{3}}{\sqrt{13}}F_{EB} = 0$$

Plug in F_{AE}, we have,

$$F_{EB} = \frac{2\sqrt{13}}{\sqrt{3}}\text{kN}$$
$$\boxed{F_{ED} = 2\sqrt{3}}$$
$$F_{EF} = 0$$

- **Equilibrium at Joint B**

$$\sum F_1 = 0 \rightarrow -\frac{1}{2}F_{BD} - F_{BC} - \frac{1}{\sqrt{13}}F_{BE} = 0$$
$$\sum F_2 = 0 \rightarrow -F_{AB} - \frac{3}{\sqrt{13}}F_{BE} = 0$$
$$\sum F_3 = 0 \rightarrow \frac{\sqrt{3}}{2}F_{BD} + \frac{\sqrt{3}}{\sqrt{13}}F_{BE} - 2 = 0$$

Plug in F_{EB}, we have

$$F_{BD} = 0$$
$$\boxed{F_{AB} = -2\sqrt{3}\text{kN}}$$
$$F_{BC} = \frac{-2}{\sqrt{3}}\text{kN}$$

- **Equilibrium at Joint C**

$$\sum F_1 = 0 \rightarrow F_{BC} + \frac{1}{2}F_{CD} + \frac{2}{\sqrt{13}}F_{AC} = 0$$

$$\sum F_2 = 0 \rightarrow -F_{CF} - \frac{3}{\sqrt{13}}F_{AC} = 0$$

$$\sum F_3 = 0 \rightarrow \frac{\sqrt{3}}{2}F_{CD} - 2 = 0$$

Therefore,

$$\boxed{F_{CD} = \frac{4}{\sqrt{3}}}$$

Plug in $F_{BC} = \frac{-2}{\sqrt{3}}$, we have

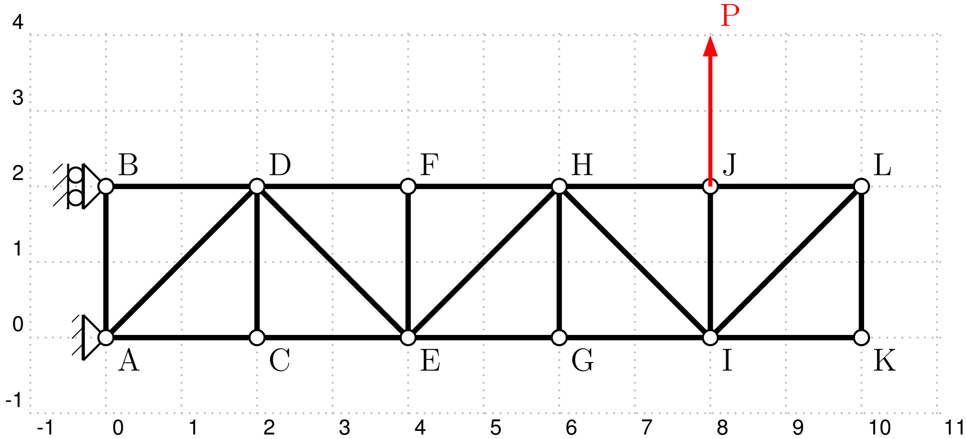
$$F_{AC} = 0\text{kN}$$

Finally we have

$$\boxed{F_{CF} = -\frac{3}{\sqrt{13}}F_{AC} = 0\text{kN}}$$

○ **Problem M-3.3**

For the truss structure in the figure:



(a) (9 points) Can you identify any bars in the structure which carry no internal load for the external load given? Justify your answer.

> **Solution:** AB, CD, EF, GH, KL, JL, IL, IK, HJ

(b) (1 point) Are there any other bars for which the internal load can be inferred without any calculation? Justify your answer.

> **Solution:** Clearly the load in bar IJ is P (tensile).

(c) (3 points) Are there any other bars which you know must carry the same internal load without doing any calculation? Justify your answer.

> **Solution:** $F^{AC} = F^{CE}, F^{DF} = F^{FH}, F^{EG} = F^{GI}$

(d) (3 points) Are the horizontal bars at the top in tension or compression? Compute their internal loads in sequence from the right to the left (this will help you quickly find their values).

> **Solution:** All in compression, except for HJ and JL which have no load. To compute F^{FH}, cut the bar, expose the internal load, take moments wrt E:
>
> $$F^{FH} \cdot 2\text{m} + P \cdot 4\text{m} = 0, \rightarrow \boxed{F^{FH} = -2P = F^{FD}}$$
>
> To compute F^{BD}, cut the bar, expose the internal load, take moments wrt A:
>
> $$F^{BD} \cdot 2\text{m} + P \cdot 8\text{m} = 0, \rightarrow \boxed{F^{BD} = -4P}$$

(e) (4 points) Conduct the same analysis for all the horizontal bars at the bottom. Are they in tension or compression?

Solution: All in tension, except for IK which has no load. To compute F^{GI}, cut the bar, expose the internal load, take moments wrt H:

$$-F^{GI} \cdot 2\text{m} + P \cdot 2\text{m} = 0, \rightarrow \boxed{F^{GI} = P = F^{EG}}$$

To compute F^{CE}, cut the bar, expose the internal load, take moments wrt D:

$$-F^{CE} \cdot 2\text{m} - P \cdot 6\text{m} = 0, \rightarrow \boxed{F^{CE} = 3P = F^{AC}}$$

(f) (4 points) Conduct this analysis one more time for all the diagonal bars.

Solution: Cutting through with a vertical line between HJ HI and GI, drawing the FBD, exposing F^{HI} and doing sum of forces in the vertical direction:

$$F^{HI}\frac{\sqrt{2}}{2} + P = 0, \rightarrow \boxed{F^{HI} = -\sqrt{2}P}$$

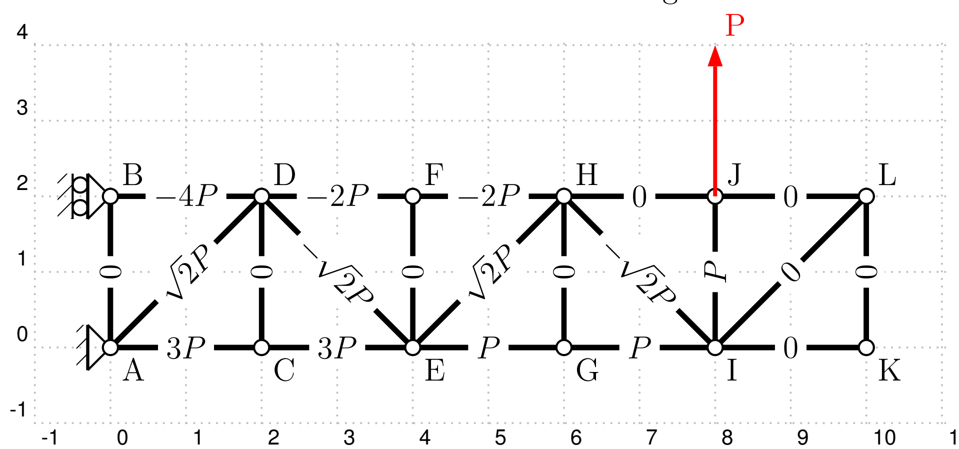
Doing the same throug FH, EH, EG:

$$-F^{EH}\frac{\sqrt{2}}{2} + P = 0, \rightarrow \boxed{F^{EH} = \sqrt{2}P}$$

Similarly, find:

$$\boxed{F^{ED} = -\sqrt{2}P}, \boxed{F^{AD} = \sqrt{2}P}$$

All the internal loads in the bars are shown in the figure:



○ **Problem M-3.4**

Consider the truss shown in Figure 3. The structure is subject to a load with a magnitude $\bar{F} = 600\,\mathrm{N}$ at joint C.

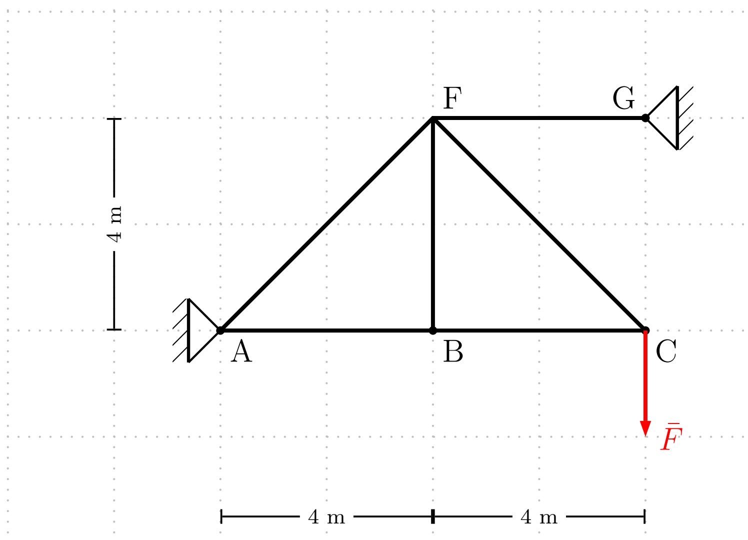


Figure 3: Analysis of a truss structure.

(a) (1 point) Is the system SD, SI or unstable. Justify your answer.

(b) (3 points) Compute the reactions at the supports.

(c) (5 points) Determine the axial forces in the members of the truss.

(d) (5 points) The support G can be moved at your discretion while preserving the length of bar FG, that is, rotating bar FG counterclockwise by an angle α around joint F. Find the optimal angle α_{opt} for the location of support G such that the maximum load magnitude in all of the bars is minimized. Is there an angle α_{bad} for which you find a problem with the structure? What problem is it and how do you explain it?

○ **Problem M-3.5**
(MO: M5, M6) A Fink roof truss structure is shown in Figure 4.

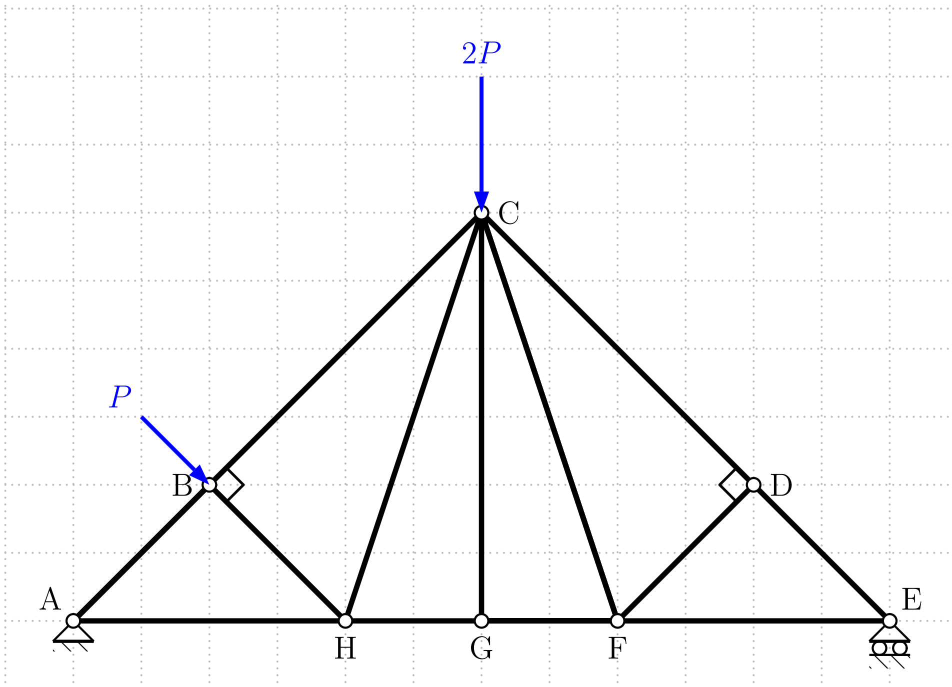


Figure 4: Fink Truss

(a) (5 points) Find the reaction forces at points A and E.

(b) (5 points) Determine the internal force in bar DC using the method of sections.

(c) (5 points) Determine which bars in the structure carry no internal load for the external loads given.