16.001 - Materials & Structures
Problem Set #4

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Problems M-4.1

(M.O M7,M8) Consider the rigid bar A-C shown in Figure 1. The bar is subject to a load of intensity P. The hinge support includes a torsional spring which reacts with a moment $M = k_T\theta$, where θ is the angle of rotation of the bar at that point and k_T is the torsional spring constant. The linear springs at the remaining supports have stiffnesses of k and $2k$, respectively.

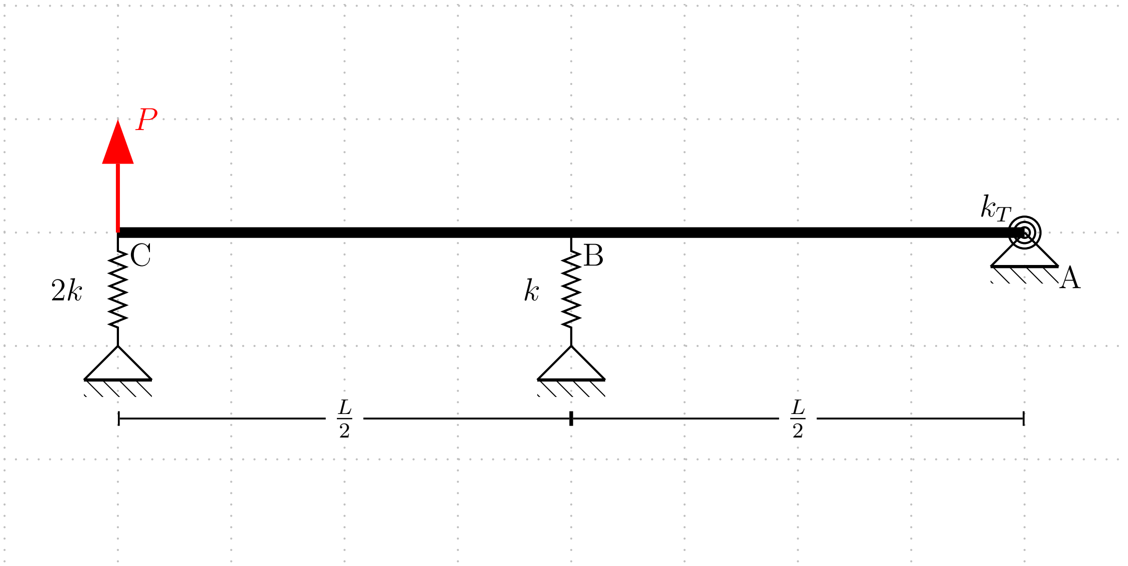

Figure 1

(a) (1 point) Is this structure statically determinate, indeterminate or unstable?

(b) (2 points) Draw the FBD for the whole structure and write the equations of global equilibrium to obtain expressions for the unknown reactions.

(c) (2 points) Draw a schematic of the deformed structure under load, identify appropriate kinematic variables describing the deformation and establish compatibility equations relating these kinematic variables.

(d) (1 point) Complete the system of equations by providing suitable constitutive laws for each of the participating structural components of this structural system. Keep in mind that the bar itself is assumed completely rigid and therefore does not have a constitutive law.

(e) (2 points) Solve the system of equations you have obtained and compute the reaction forces and the deflections of each spring.

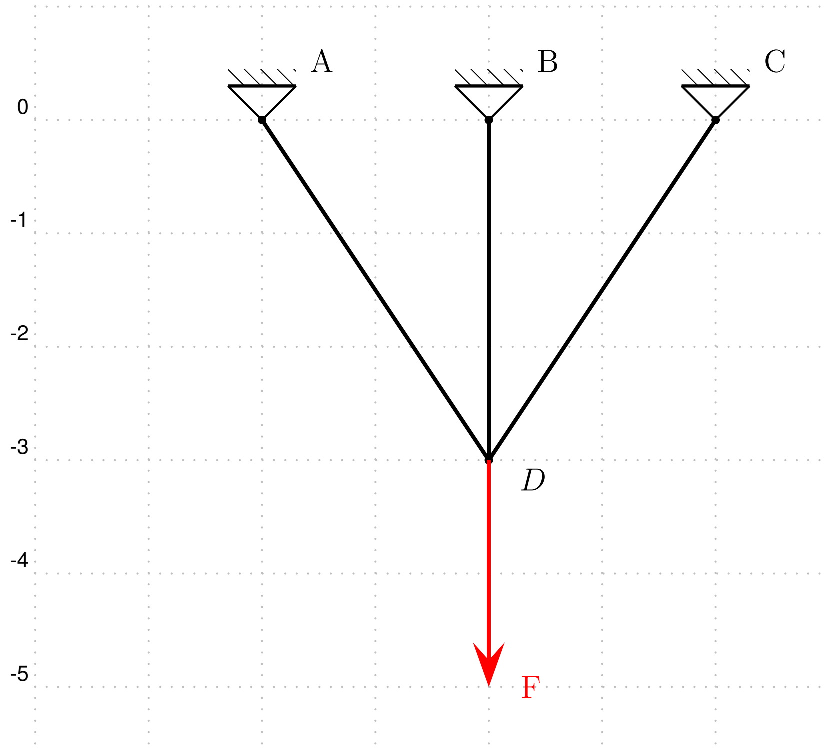

Figure 2

Problems M-4.2

(3 points) (MO: M7,M8) In the structure shown in Figure 2 the bars are made from aluminum tubes of Young's modulus E = 70GPa and cross-sectional area $A = 10mm^2$. The force F = 10KN. Additionally, the grid scale is in meters. Analyze the structure to find the following:

(a) The forces in each bar.

(b) The deflection of point D.

(c) The reactions at the supports.

○ **Problems M-4.3**

(4 points) Consider the linkage shown in Figure 3 comprised of two steel members ($E = 200$ GPa). An applied force of $P = 10$ kN is applied to the linkage at point C.

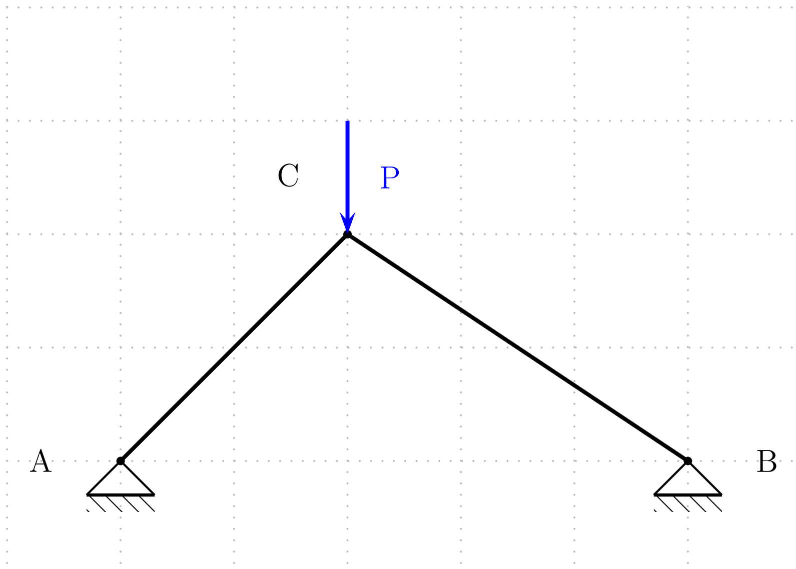

Figure 3: Two Member Steel Linkage

Determine the minimum allowable radius of the steel members such that point C does not have a vertical deflection of more than 1 cm. You may assume that the steel members have circular cross-sections, each of the same area. The grid scale is given in meters.

Problems M-4.4
(thermal loads)

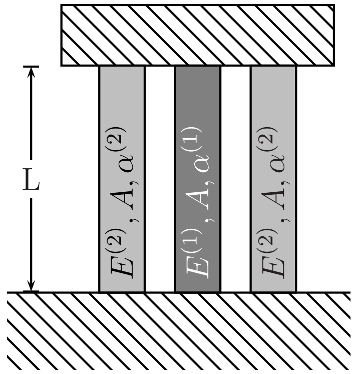

Figure 4: Bar assembly subject to temperature changes

Consider the assembly of bars shown in Figure 4. The center bar is made of a material with Young's modulus $E^{(1)}$, area of the cross section A and coefficient of thermal expansion (CTE) $\alpha^{(1)}$, whereas the respective properties of the left and right bars are $E^{(2)}, A, \alpha^{(2)}$. A rigid block at the top constrans the deformation of the bars but the block is free to move in the vertical direction by an arbitrary value $\bar{\delta}$. However, the block cannot rotate. A temperature change of $\Delta\theta$ is applied after assembly.

(a) (2 points) Express in mathematical form all the relevant principles that apply in this problem both for the individual bars and for the assembly. Make sure you draw Free Body Diagrams as necessary.

(b) (1 point) Identify all the unknowns of the problem. Count the number of equations you have for each principle and comment on the "solvability" of the problem. Is the problem statically determinate or indeterminate?

(c) (3 points) Derive expressions for the forces in the bars and the displacement δ due to the "temperature loading" of the assembly.

(d) (2 points) Show that the deflection of the system $\bar{\delta}$ does not depend on the magnitude of the individual material Young's moduli $E^{(1)}, E^{(2)}$ but only on their relative value $\eta = \frac{E^{(1)}}{E^{(2)}}$. Obtain an expression for the deflection in terms of η.

○ **Problems M-4.5**

The structure in Figure 5 is composed of three bars with the same cross-sectional area $A = 10\ mm^2$ and length $L = 3\ m$, but of materials with different elastic (Young's) moduli: bar AD has $E^{(1)} = 70$ GPa, bar BD has $E^{(2)} = 120$ GPa, bar CD, has $E^{(3)} = 210$ GPa. The corresponding CTEs are $23, 9\,\text{and}\,13 \times 10^{-6}\text{K}^{-1}$. The structure is subjected to a temperature increase $\Delta\theta = 100\text{K}$.

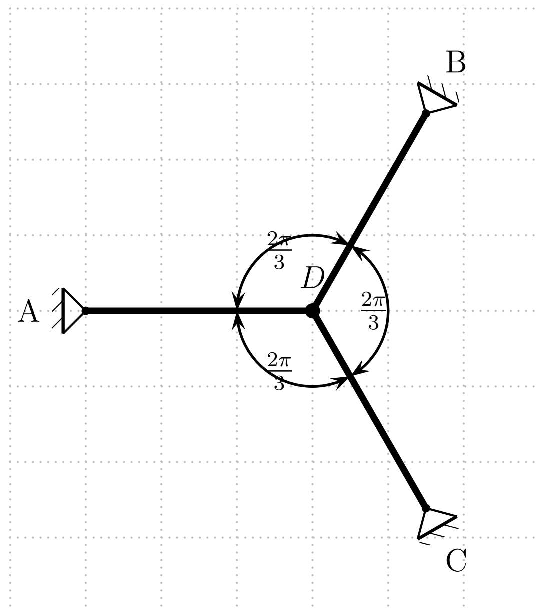

Figure 5: Three bar structure subject to temperature increase

(a) (3 points) Compute the forces in each bar and the deflection of point D caused by the temperature change.

(b) (2 points) Find the load P (magnitude and direction), that needs to be applied at point D to eliminate the displacement produced by the temperature change and return joint D to the original location.