16.001 - Materials & Structures
Problem Set #8

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Question	Points
1	5
2	5
3	8
4	3
5	3
Total:	24

○ **Problems M-8.1** [5 points]

You've been tasked with selecting the material for the grid fins on SpaceX's next launch vehicle, Starship. These grid fins will be significantly larger than the ones on the Falcon 9 (7x3 m^2 vs. 2x1.2 m^2), making cost a much bigger concern. The grid fins should be light, cheap, and capable of surviving multiple exposures to high temperatures (>400 °C). They also need to be stiff so that they don't deflect during reentry.

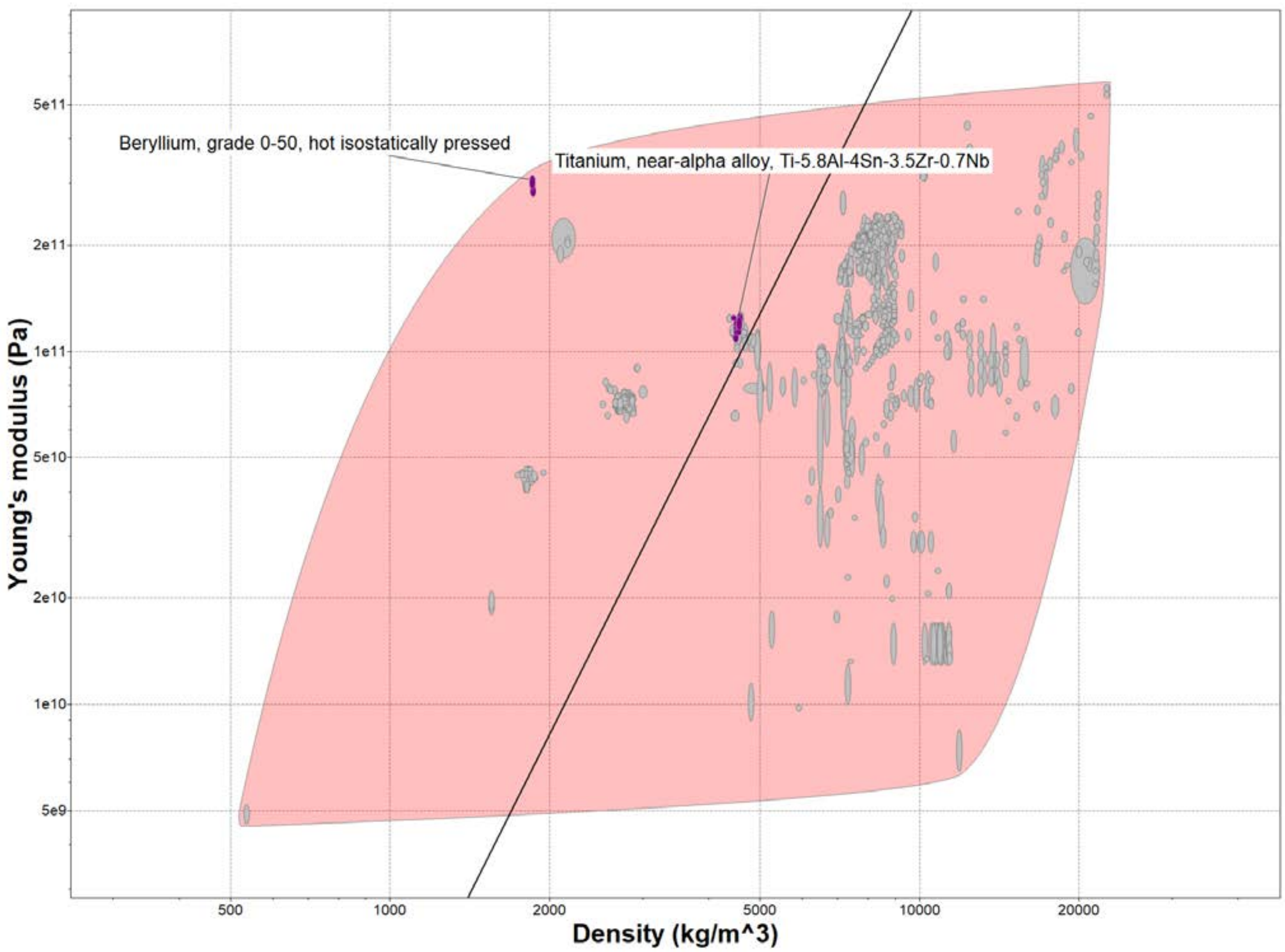
In each of the following materials selection problems, list the function, objective(s), constraints, and materials indices with which ranked the different materials. You can use this reference to determine the appropriate materials index. Show the Property Diagram(s) that you used to make your decisions with the appropriate materials index contour overlaid. Indicate on the property diagram the best 2 or 3 materials options using the labeling function.

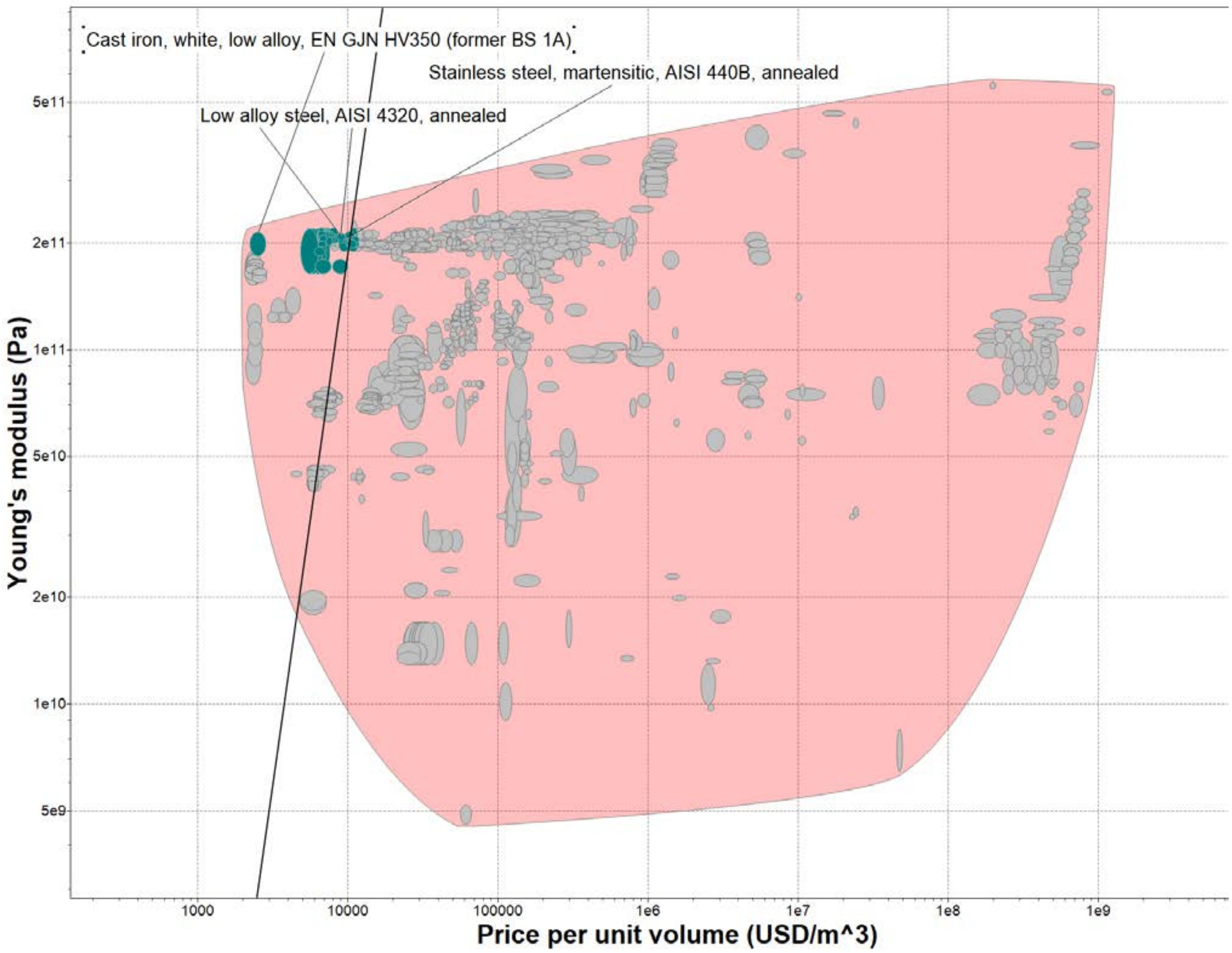
Solution: Function: Grid fins on F9 booster

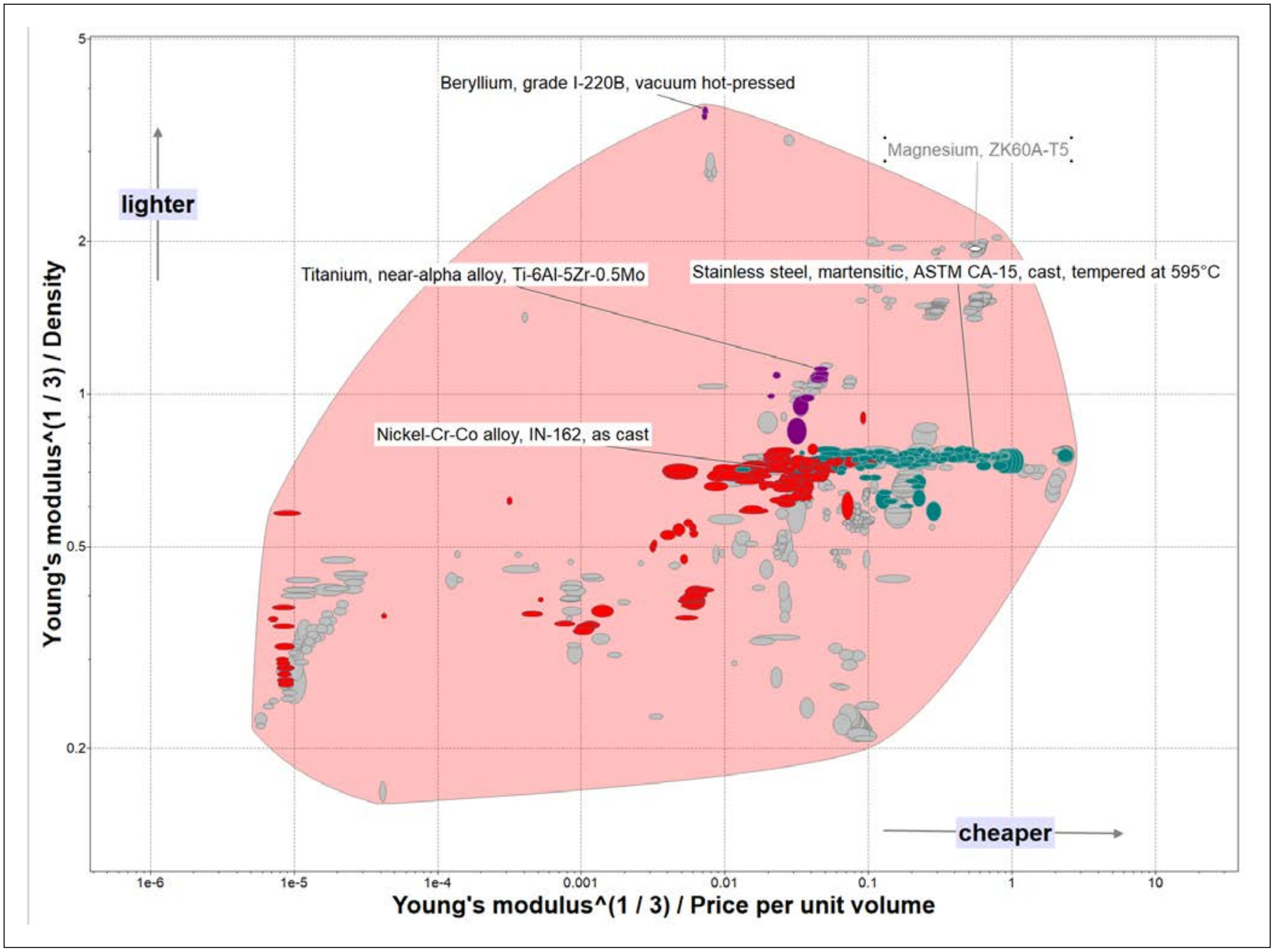
Objective(s): Minimize cost and mass

Constraints: • Must have a maximum service temperatures greater than 500 °C • Should be stiff to minimize elastic deflection • Easy to process into bulk forms

Notes + materials indices: We are trying to minimize two objectives in this problem – cost and mass. These objectives often conflict, since stiff, low-density materials also tend to be expensive. Ashby gives a thorough discussion of how to handle such multi-objective materials selection problems in this excellent manuscript. The constraint on processability immediately suggests focusing on metals, which tend to be easy to form into large shapes. As discussed in recitation, grid fins can be approximated as panels. The relevant material indices are therefore $E^{1/3}/\rho$ and $E^{1/3}/(\rho Cm)$. Consider the property diagrams E vs. ρ, E vs. ρCm, and $E^{1/3}/\rho$ vs $E^{1/3}/(\rho Cm)$ shown below. The relevant material indices have been overlaid on the E vs. ρ and E the vs. ρCm diagrams. Beryllium and titanium alloys are attractive candidates on a mass basis if cost is not an issue. Steels become the obvious choice if cost is a concern (Note: cast iron, a high carbon ferrous alloy, is way too brittle for this application, while stainless steels offer a good combination of ductility, high temperature strength, and oxidation resistance). The property diagram $E^{1/3}/\rho$ vs $E^{1/3}/(\rho Cm)$ shows the envelope of material indices for all metals, highlighting the tradeoff between cost and mass. Low alloy steels and stainless steels seem like they have a good balance of high stiffness, low cost, and low density. You will have to dig deeper into the documentation before making a final selection. If temperature wasn't a constraint, then magnesium alloys would be attractive grid fin materials. However, magnesium tends to burn aggressively in high temperature oxidizing environments which is why it is used in fireworks.


Beryllium, grade 0-50, hot isostatically pressed
Titanium, near-alpha alloy, Ti-5.8Al-4Sn-3.5Zr-0.7Nb
Young's modulus (Pa)
5e11
2e11
1e11
5e10
2e10
1e10
5e9
500
1000
2000
5000
10000
20000
Density (kg/m^3)


Cast iron, white, low alloy, EN GJN HV350 (former BS 1A)
Stainless steel, martensitic, AISI 440B, annealed
Low alloy steel, AISI 4320, annealed
Young's modulus (Pa)
5e11
2e11
1e11
5e10
2e10
1e10
5e9
1000
10000
100000
1e6
1e7
1e8
1e9
Price per unit volume (USD/m^3)


Beryllium, grade I-220B, vacuum hot-pressed
Magnesium, ZK60A-T5
lighter
Titanium, near-alpha alloy, Ti-6Al-5Zr-0.5Mo
Stainless steel, martensitic, ASTM CA-15, cast, tempered at 595°C
Nickel-Cr-Co alloy, IN-162, as cast
cheaper
Young's modulus^(1 / 3) / Density
Young's modulus^(1 / 3) / Price per unit volume
5
2
1
0.5
0.2
1e-6
1e-5
1e-4
0.001
0.01
0.1
1
10

Problems M-8.2 [5 points]
The extensional and shear strains at a point of a loaded structure have been measured with respect to a particular set of cartesian basis vectors. The measured values are

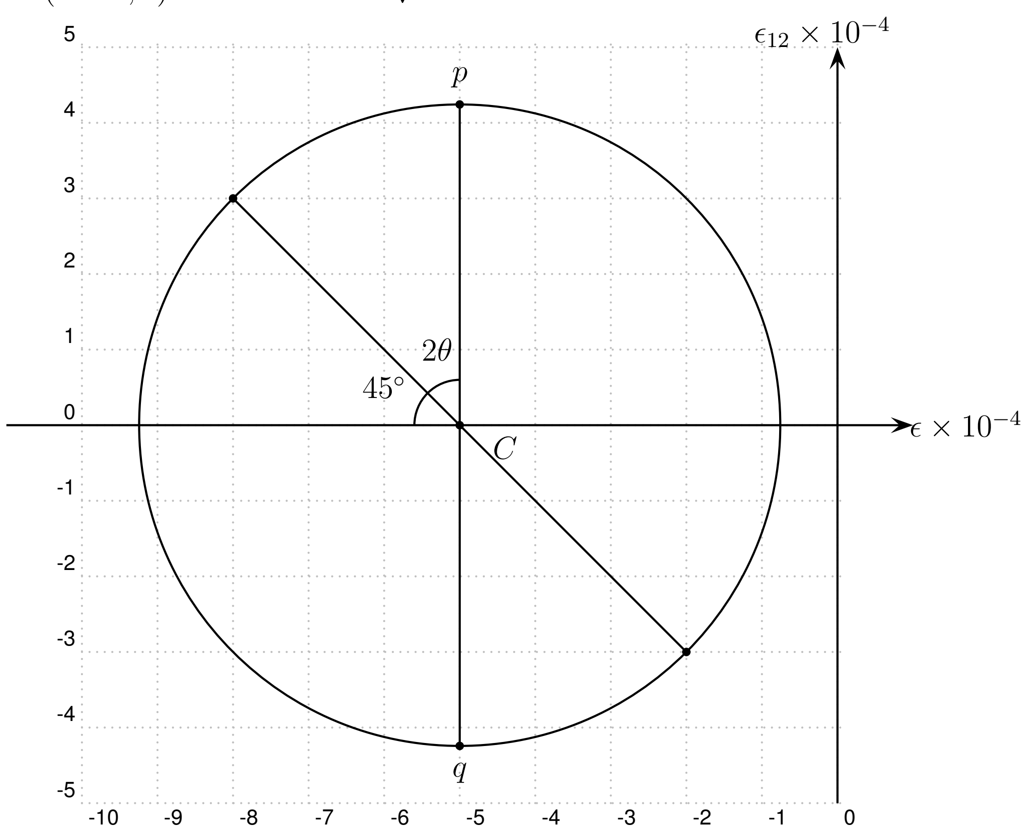
$$\epsilon_{11} = -800 \times 10^{-6} \quad (1)$$

$$\epsilon_{22} = -200 \times 10^{-6} \quad (2)$$

$$\gamma_{12} = -600 \times 10^{-6} \quad (3)$$

2.1 (1 point) Draw Mohr's circle for this state of strain

Solution: The center and radius of the circle are $C = \frac{\epsilon_{11}+\epsilon_{22}}{2} = -500$ and $R = \sqrt{(\frac{\epsilon_{11}-\epsilon_{22}}{2})^2 + \epsilon_{12}^2}$ respectively, where $\epsilon_{12} = \frac{\gamma_{12}}{2}$. Thus, Mohr's circle for this strain state is a circle centered at $(-500, 0)$ with radius $300\sqrt{2}$.



2.2 (2 points) Find the principal strains and principal directions. Show also the deformed shape of an element which originally was a parallelepiped with its faces parallel to these axes

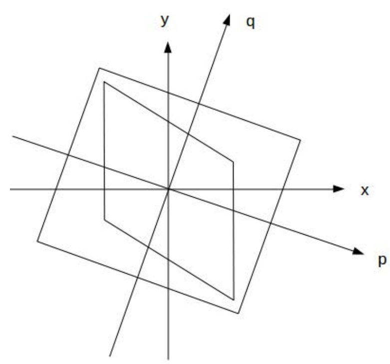
Solution: The principal strains are

$$\epsilon_I = -75.7 \times 10^{-6}, \epsilon_{II} = -924.3 \times 10^{-6} \tag{4}$$

The principal directions are $2\alpha_p = 45°$, $2\alpha_p = 225° \rightarrow \alpha_p = 22.5°$, $\alpha_p = 112.5°$

2.3 (2 points) Find the maximum shear strains and corresponding directions. Show also the deformed shape of an element which originally was a parallelepiped with its faces parallel to these axes

Solution: pq are the maximum shear axes. The maximum shear strain are $\gamma_s = \pm 2 \times 424.3 \times 10^{-6} = \pm 848.6 \times 10^{-6}$. The maximum shear directions are $2\theta = -45°$, $2\theta = 135°$ $\rightarrow \theta = -22.5°$, $\theta = 67.5°$ The deformed shape of an element which originally was a parallelepiped with its faces parallel to these axes is shown as follows,



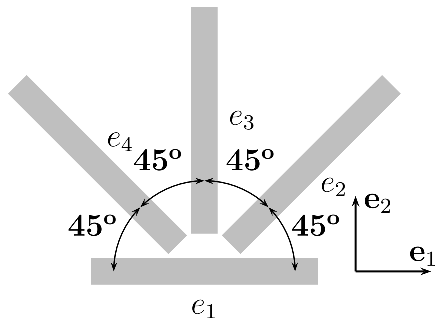


Figure 1: T-V rosette strain gauge

○ **Problems M-8.3** [8 points]
Consider the T-V rosette shown in Figure 1. The measured strains along the directions of the individual strain gauges are respectively $e_1 = 910\mu$, $e_2 = 990\mu$, $e_3 = 310\mu$, and $e_4 = 190\mu$.

3.1 (2 points) Use the equations of transformation of strain components in 2D as many times as needed, to relate the measured strain components and those in the cartesian system $\mathcal{E} = (\mathbf{e}_1, \mathbf{e}_2)$

Solution: The measured data e_1, e_2, e_3 and e_4 correspond respectively to the values of the strains ε_{11}^* for the angles $\theta_1 = 0°$, $\theta_2 = 45°$, $\theta_3 = 90°$ and $\theta_4 = -45°$. We can use the expression for transforming strain components to the axial ε_{11}^* component in the new axis repeatedly for each datum and its corresponding angle, which leads to the following system of equations:

$$\begin{aligned}
e_1 &= \varepsilon_{11}^*(\theta_1) = \frac{\varepsilon_{11}+\varepsilon_{22}}{2} + \left(\frac{\varepsilon_{11}-\varepsilon_{22}}{2}\right)\cos(0) + \varepsilon_{12}\sin(0) \\
e_2 &= \varepsilon_{11}^*(\theta_2) = \frac{\varepsilon_{11}+\varepsilon_{22}}{2} + \left(\frac{\varepsilon_{11}-\varepsilon_{22}}{2}\right)\cos(90) + \varepsilon_{12}\sin(90) \\
e_3 &= \varepsilon_{11}^*(\theta_3) = \frac{\varepsilon_{11}+\varepsilon_{22}}{2} + \left(\frac{\varepsilon_{11}-\varepsilon_{22}}{2}\right)\cos(180) + \varepsilon_{12}\sin(180) \\
e_4 &= \varepsilon_{11}^*(\theta_4) = \frac{\varepsilon_{11}+\varepsilon_{22}}{2} + \left(\frac{\varepsilon_{11}-\varepsilon_{22}}{2}\right)\cos(-90) + \varepsilon_{12}\sin(-90),
\end{aligned}$$

which reduces to

$$\begin{aligned}
e_1 &= \varepsilon_{11} \\
e_2 &= \frac{1}{2}\varepsilon_{11} + \frac{1}{2}\varepsilon_{22} + \varepsilon_{12} \\
e_3 &= \varepsilon_{22} \\
e_4 &= \frac{1}{2}\varepsilon_{11} + \frac{1}{2}\varepsilon_{22} - \varepsilon_{12},
\end{aligned}$$

3.2 (2 points) Can you determine the strain components $\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{12}$ from these equations? Do you have insufficient or redundant information? How can this be useful from the experimental standpoint?

Solution: The system is clearly overdetermined as it has three unknowns and four equations. This overdetermination provides some means of reducing the uncertainty of the experimental measurements: instead of finding the exact solution to the system which most likely won't exist unless the four values are mutually consistent, one can try to find the best approximation to the quantities of interest from the given data.

It makes a lot of sense to have more measurements than the minimum required. For instance, if one of the gauges is damaged, the extra measurements are very useful (and maybe essential) to determine the state of strain. If all the gauges are working, the redundant information can be used to compensate the experimental errors.

3.3 (3 points) Use a least-squares approach to obtain the "best approximation" to the strain components $\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{12}$ in terms of the measured data. Hint: as it name indicates, the least squares method finds a solution of the overdetermined system by minimizing the sum of the square of the errors incurred in the satisfaction of each equation.

Solution: The sum of the square of the errors incurred in the satisfaction of each equation reads:

$$S = (e_1 - \varepsilon_{11})^2 + \left[e_2 - \left(\frac{1}{2}\varepsilon_{11} + \frac{1}{2}\varepsilon_{22} + \varepsilon_{12}\right)\right]^2 +$$
$$(e_3 - \varepsilon_{22})^2 + \left[e_4 - \left(\frac{1}{2}\varepsilon_{11} + \frac{1}{2}\varepsilon_{22} - \varepsilon_{12}\right)\right]^2.$$

Computing the derivatives of S with respect to $\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{12}$ and setting them to zero we obtain:

$$\begin{aligned} -e_2 - e_4 - 2(e_1 - \varepsilon_{11}) + \varepsilon_{11} + \varepsilon_{22} &= 0 \\ -e_2 - e_4 + \varepsilon_{11} - 2(e_3 - \varepsilon_{22}) + \varepsilon_{22} &= 0 \\ -2\left(e_2 - \frac{1}{2}\varepsilon_{11} - \varepsilon_{12} - \frac{1}{2}\varepsilon_{22}\right) + 2\left(e_4 - \frac{1}{2}\varepsilon_{11} + \varepsilon_{12} - \frac{1}{2}\varepsilon_{22}\right) &= 0, \end{aligned}$$

which simplifies to:

$$\begin{aligned} 3\varepsilon_{11} + \varepsilon_{22} &= 2e_1 + e_2 + e_4 \\ \varepsilon_{11} + 3\varepsilon_{22} &= e_2 + 2e_3 + e_4 \\ 2\varepsilon_{12} &= e_2 - e_4, \end{aligned}$$

and results in:

$$\varepsilon_{11} = \frac{1}{4}(3e_1 + e_2 - e_3 - e_4)$$
$$\varepsilon_{22} = \frac{1}{4}(-e_1 + e_2 + 3e_3 + e_4)$$
$$\varepsilon_{12} = \frac{1}{2}(e_2 - e_4).$$

After replacing the measured values: $e_1 = 910\mu$, $e_2 = 990\mu$, $e_3 = 310\mu$, and $e_4 = 190\mu$, we obtain:

$$\boxed{\varepsilon_{11} = 900\mu, \varepsilon_{22} = 300\mu, \varepsilon_{12} = 400\mu}$$

More generally, the overdetermined system of equations can be written in matrix form as

$$\begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1/2 & 1 \\ 0 & 1 & 0 \\ 1/2 & 1/2 & -1 \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} \end{bmatrix} = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix}.$$

If we have an over-determined system $\mathbf{Ax} = \mathbf{b}$, where $dim(\mathbf{A}) = M \times N$, $dim(\mathbf{x}) = N \times 1$, $dim(\mathbf{b}) = M \times 1$, and $M > N$, we can find the solution $\mathbf{x}$ that minimizes the square of the norm of the error (least-squares approach) by solving the system $\left(\mathbf{A}^T\mathbf{A}\right)\mathbf{x} = \mathbf{A}^T\mathbf{b}$. Note that in this new system of equations $\mathbf{A}^T\mathbf{A}$ is a nonsingular matrix with $dim(\mathbf{A}^T\mathbf{A}) = N \times N$. For our problem the system of equations becomes

$$\begin{bmatrix} 3 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 2 & 0 & -2 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix}.$$

By inverting the matrix $\mathbf{A}^T\mathbf{A}$ we are able to calculate directly the solution of the problem as $\mathbf{x} = \left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{b}$, which for this specific case takes the form

$$\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 3 & 1 & -1 & 1 \\ -1 & 1 & 3 & 1 \\ 0 & 2 & 0 & -2 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix}.$$

3.4 (1 point) Find the orientation of the principal strain directions, and the principal strains

Solution: The principal strains and strain directions are respectively given by the eigenvalues and the eigenvectors of this tensor, i.e. $\varepsilon^* = \mathbf{R}^T \varepsilon \mathbf{R}$, where

$$\varepsilon^* = \begin{bmatrix} \varepsilon^*_{11} & 0 \\ 0 & \varepsilon^*_{22} \end{bmatrix} \approx \begin{bmatrix} 1100\mu & 0 \\ 0 & 100\mu \end{bmatrix},$$

and

$$\mathbf{R} \approx \begin{bmatrix} -0.8944 & 0.4472 \\ -0.4472 & -0.8944 \end{bmatrix}.$$

The values of the strain components can be verified through the invariants of the strain tensor, i.e. $I_1 = I_1^* = 1200\mu$ and $I_2 = I_2^* = 110000\mu^2$.

Problems M-8.4 [3 points]
(M.O. M11)

The state of strain in a composite is determined by a rectangular strain gauge rosette attached to the surface, as shown in Figure 2. The three strain gauges (a, b, & c) are arranged at angles $\alpha_a = 0^o, \alpha_b = 45^o, \& \ \alpha_c = 90^o$. The gauges read $\epsilon_a = 20 \times 10^{-6}, \epsilon_b = 55 \times 10^{-6}, \epsilon_c = -60 \times 10^{-6}$. The composite is a polymer matrix reinforced with unidirectional fibers that are aligned at 120^o from horizontal.

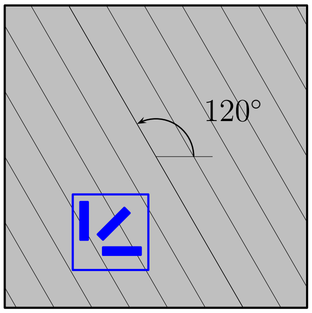


Figure 2: Composite material with 3 strain gauges

Determine the normal and shear strain components in the directions aligned and perpendicular to the fibers.

Solution:

There are two different ways to approach this problem. The first is to transform the strain such that $\mathbf{e_1}$ and $\mathbf{e_2}$ are horizontal and vertical, respectively. The second approach is to transform the stress directly to the orientation of the fibers.

Method 1

First, transform the strain so that $\mathbf{e_1}$ and $\mathbf{e_2}$ are aligned to the horizontal and vertical. Use the equations below.

$$\epsilon_a = \frac{\epsilon_{11} + \epsilon_{22}}{2} + \frac{\epsilon_{11} - \epsilon_{22}}{2}\cos(2\alpha_a) + \epsilon_{12}\sin(2\alpha_a)$$
$$\epsilon_b = \frac{\epsilon_{11} + \epsilon_{22}}{2} + \frac{\epsilon_{11} - \epsilon_{22}}{2}\cos(2\alpha_b) + \epsilon_{12}\sin(2\alpha_b)$$
$$\epsilon_c = \frac{\epsilon_{11} + \epsilon_{22}}{2} + \frac{\epsilon_{11} - \epsilon_{22}}{2}\cos(2\alpha_c) + \epsilon_{12}\sin(2\alpha_c)$$

Simplifying the equations with the given angles.

$$\epsilon_a = \epsilon_{11}$$
$$\epsilon_b = \frac{\epsilon_{11} + \epsilon_{22}}{2} + \epsilon_{12}$$
$$\epsilon_c = \epsilon_{22}$$

Solving the system of equations yields

$$\epsilon_{11} = 20 \times 10^{-6}$$
$$\epsilon_{22} = -60 \times 10^{-6}$$
$$\epsilon_{12} = 75 \times 10^{-6}$$

Next, transform the strain so that it is aligned with fiber using the equations

$$\tilde{\epsilon}_{11} = \frac{\epsilon_{11} + \epsilon_{22}}{2} + \frac{\epsilon_{11} - \epsilon_{22}}{2}\cos(2\alpha_1) + \epsilon_{12}\sin(2\alpha_1)$$
$$\tilde{\epsilon}_{22} = \frac{\epsilon_{11} + \epsilon_{22}}{2} - \frac{\epsilon_{11} - \epsilon_{22}}{2}\cos(2\alpha_1) - \epsilon_{12}\sin(2\alpha_1)$$
$$\tilde{\epsilon}_{12} = -\frac{\epsilon_{11} - \epsilon_{22}}{2}\sin(2\alpha_1) + \epsilon_{12}\cos(2\alpha_1)$$

where $\alpha_1 = 120^o$. This results in

$$\tilde{\epsilon}_{11} = -104.95 \times 10^{-6}$$
$$\tilde{\epsilon}_{22} = 64.95 \times 10^{-6}$$
$$\tilde{\epsilon}_{12} = -2.86 \times 10^{-6}$$

To transform the stress to be aligned perpendicular to fibers change α_1 to be 30^o in the equations for strain transformation above. They will result in

$$\tilde{\epsilon}_{11} = 64.95 \times 10^{-6}$$
$$\tilde{\epsilon}_{22} = -104.95 \times 10^{-6}$$
$$\tilde{\epsilon}_{12} = 2.86 \times 10^{-6}$$

<u>Method 2</u>
To compute the direct transformation start with the equations

$$\epsilon_a = \frac{\tilde{\epsilon}_{11} + \tilde{\epsilon}_{22}}{2} + \frac{\tilde{\epsilon}_{11} - \tilde{\epsilon}_{22}}{2}\cos(2\alpha_1) + \tilde{\epsilon}_{12}\sin(2\alpha_1)$$
$$\epsilon_b = \frac{\tilde{\epsilon}_{11} + \tilde{\epsilon}_{22}}{2} + \frac{\tilde{\epsilon}_{11} - \tilde{\epsilon}_{22}}{2}\cos(2\alpha_2) + \tilde{\epsilon}_{12}\sin(2\alpha_2)$$
$$\epsilon_c = \frac{\tilde{\epsilon}_{11} + \tilde{\epsilon}_{22}}{2} + \frac{\tilde{\epsilon}_{11} - \tilde{\epsilon}_{22}}{2}\cos(2\alpha_3) + \tilde{\epsilon}_{12}\sin(2\alpha_3)$$

In the case of when the strain is aligned with the fibers the angles are

$$\alpha_1 = -120^o$$
$$\alpha_2 = -75^o$$
$$\alpha_3 = -30^o$$

The angles are measured from the orientation of $\tilde{\mathbf{e}}_1$ (120^o in the first case and 30^o in the second) to each gauge in the rosette. Plugging these angles into the equations above and solving the system of equations results in

$$\begin{aligned}\tilde{\epsilon}_{11} &= -104.95 \times 10^{-6}\\ \tilde{\epsilon}_{22} &= 64.95 \times 10^{-6}\\ \tilde{\epsilon}_{12} &= -2.86 \times 10^{-6}\end{aligned}$$

To compute the components of strain when aligned perpendicular to the fibers, the angles are

$$\begin{aligned}\alpha_1 &= -30^o\\ \alpha_2 &= 15^o\\ \alpha_3 &= 60^o\end{aligned}$$

which result in

$$\begin{aligned}\tilde{\epsilon}_{11} &= 64.95 \times 10^{-6}\\ \tilde{\epsilon}_{22} &= -104.95 \times 10^{-6}\\ \tilde{\epsilon}_{12} &= 2.86 \times 10^{-6}\end{aligned}$$

Notice that both methods result in the same solution. Also, notice the two scenarios have a difference in orientation of 90^o. When you go between the the two cases $\tilde{\epsilon}_{11}$ and $\tilde{\epsilon}_{22}$ switch and the the sign of $\tilde{\epsilon}_{12}$ switches.

○ **Problems M-8.5** [3 points]
(M.O. M11)
The state of strain at a point in an aluminum component of the fuselage of an airplane is measured with a delta strain gauge rosette (See Figure 3, where each gauge is a side of the triangle) of three strain gauges a, b, c arranged at angles $\alpha_a = 0, \alpha_b = 60, \alpha_c = 120°$. The strain gauges read $\epsilon_a = 15 \times 10^{-6}, \epsilon_b = 60 \times 10^{-6}, \epsilon_c = 80 \times 10^{-6}$.

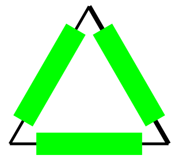


Figure 3: Delta Rosette strain gauge

Determine:

5.1 (1 point) All the components of strain in cartesian axes $\mathbf{e}_1, \mathbf{e}_2$ respectively aligned with the horizontal and vertical direction

Solution:
Using the equations below we can solve for the state of strain where the Cartesian axes are align with the horizontal and vertical directions:

$$\epsilon_a = \frac{\epsilon_{11} + \epsilon_{22}}{2} + \frac{\epsilon_{11} - \epsilon_{22}}{2}\cos(2\alpha_a) + \epsilon_{12}\sin(2\alpha_a)$$
$$\epsilon_b = \frac{\epsilon_{11} + \epsilon_{22}}{2} + \frac{\epsilon_{11} - \epsilon_{22}}{2}\cos(2\alpha_b) + \epsilon_{12}\sin(2\alpha_b)$$
$$\epsilon_c = \frac{\epsilon_{11} + \epsilon_{22}}{2} + \frac{\epsilon_{11} - \epsilon_{22}}{2}\cos(2\alpha_c) + \epsilon_{12}\sin(2\alpha_c)$$

Substituting in the values above

$$15 \times 10^{-6} = \epsilon_{11}$$
$$60 \times 10^{-6} = .25\epsilon_{11} + .75\epsilon_{22} + .866\epsilon_{12}$$
$$80 \times 10^{-6} = .25\epsilon_{11} + .75\epsilon_{22} - .866\epsilon_{12}$$

Solving the system of equations yields

$$\epsilon_{11} = 15 \times 10^{-6}$$
$$\epsilon_{22} = 88.3 \times 10^{-6}$$
$$\epsilon_{12} = -11.55 \times 10^{-6}$$

5.2 (1 point) The principal strains $\epsilon_{I,II}$, their directions $\alpha_{I,II}$

Solution:
The equations below find the principal strains and the orientation of the element.

$$tan2\alpha_{I,II} = \frac{2\epsilon_{12}}{\epsilon_1 - \epsilon_2}$$
$$\epsilon_{I,II} = \frac{\epsilon_1 + \epsilon_2}{2} \pm \sqrt{(\frac{\epsilon_1 - \epsilon_2}{2})^2 + \epsilon_{12}^2}$$

substituting in the given parameters yields

$$\alpha_{I,II} = 8.74^o$$
$$\epsilon_{12} = 0$$
$$\epsilon_I = 13.22 \times 10^{-6}$$
$$\epsilon_{II} = 90.1 \times 10^{-6}$$

Alternatively, this problem could be completed using Mohr's circle for strain. Mohr's circle can be defined using the equations below(see Figure 4).

$$R = \sqrt{\left(\frac{\epsilon_{11} - \epsilon_{22}}{2}\right)^2 + \epsilon_{12}^2} = 38.426 \times 10^{-6}$$
$$Center: \quad C(\frac{\epsilon_{11} + \epsilon_{22}}{2}, 0) = (51.65 \times 10^{-6}, 0)$$

Using Mohr's circle, the principal stress can be found by

$$\epsilon_I, \epsilon_{II} = Center \pm R$$
$$\epsilon_{12} = 0$$

So the stresses are

$$\epsilon_I = 13.22 \times 10^{-6}$$
$$\epsilon_{II} = 90.1 \times 10^{-6}$$
$$\epsilon_{12} = 0$$

The orientation is calculated from the same equation as above, and thus it is the also the same.

5.3 (1 point) The maximum shear strains γ^{max} and their directions α_s

Solution:

The equations below find the maximum shear strain and the orientation of the element.

$$tan2\alpha_s = -\frac{\epsilon_1 - \epsilon_2}{2\epsilon_{12}}$$
$$\frac{\gamma_{max}}{2} = \pm\sqrt{(\frac{\epsilon_1 - \epsilon_2}{2})^2 + \epsilon_{12}^2}$$
$$\epsilon_{avg} = \frac{\epsilon_1 + \epsilon_2}{2}$$

Substituting the given parameters yield

$$\alpha_s = -36.26^o$$
$$\gamma_{max} = \pm 76.85 \times 10^{-6}$$
$$\epsilon_{avg} = 51.65 \times 10^{-6}$$

Once again this problem could be completed using Mohr's circle for strain (see Figure 5).

$$\epsilon_{avg} = Center$$
$$\frac{\gamma_{max}}{2} = \pm R$$

plugging in the given values results in

$$\epsilon_{avg} = 51.65 \times 10^{-6}$$
$$\frac{\gamma_{max}}{2} = -38.43 \times 10^{-6}$$

which evaluate to

$$\epsilon_{avg} = 51.65 \times 10^{-6}$$
$$\gamma_{max} = -76.85 \times 10^{-6}$$

The orientation angle from Mohr's circle is calculated from the same equation above, so the orientation angle is the same.

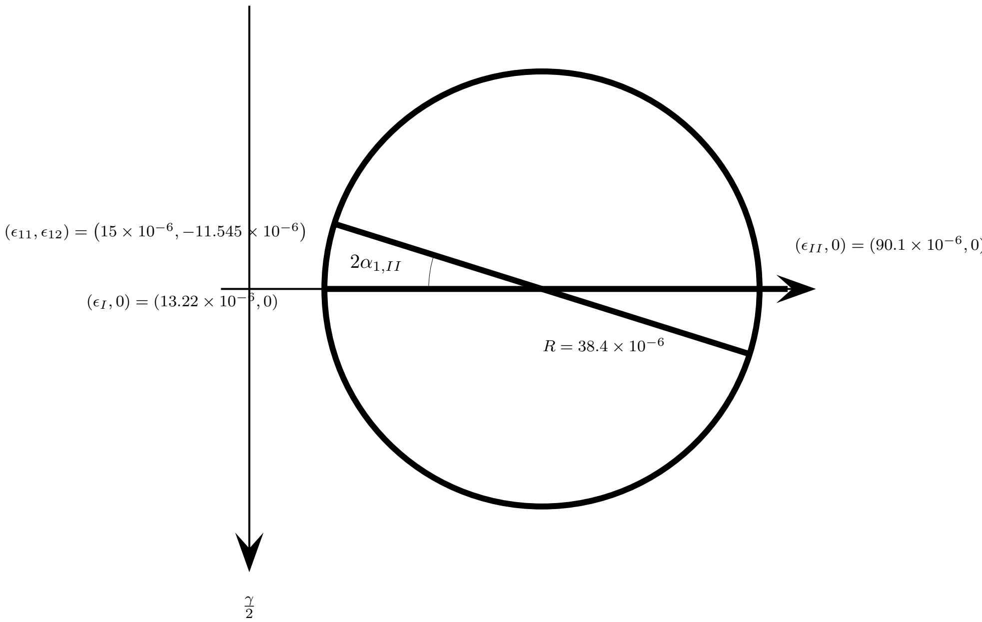


Figure 4: Mohr's Circle, principal state

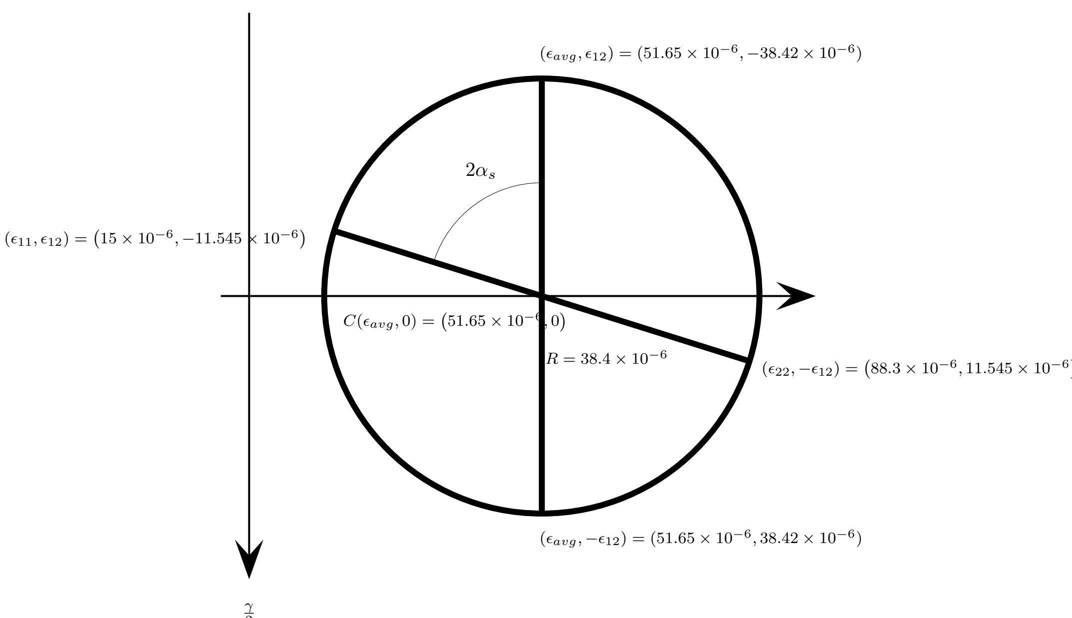


Figure 5: Mohr's Circle at Maximum Shear State

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