SAMPLE PROBLEM:

Problem 1

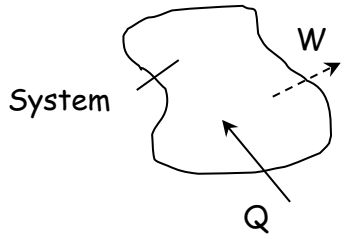
During a reversible process executed by a closed system, the pressure increases from 345 kPa to 1,380 kPa in accordance with pV = C, and the internal energy increases by 22,575 J. The initial volume is V_1 = 85 l. Find the heat transferred.

SAMPLE SOLUTION:

Problem 1 Tim Target

Concepts used to solve problem:

- 1st law of thermodynamics for a control mass
- Work and heat
- State changes



Known: Initial state p_1 = 345 kPa Final state p_2 = 1,380 kPa

V_1 = 85 l = 0.085 m^3

Find: Heat transfer Q given a known change in internal energy ΔU = 25,575 J

Using the 1st law of thermodynamics and neglecting potential and kinetic energies yields:

$$\Delta U = Q - W$$

where Q is the heat transfer to the system and W is the work done by the system. So the heat transfer becomes

$$Q = \Delta U + W \qquad (1)$$

The work done by the system is: (note that work is path dependent)

Knowing that pV = C or p = C/V, the work becomes:

$$W = C = C \ln (V_2 / V_1)$$

The constant C can be found from the initial state:

$$p_1 V_1 = C$$

and the volume ratio can be expressed in terms of the pressure ratio as:

$$V_2 / V_1 = p_1 / p_2$$

such that combining these gives:

$$W = p_1 V_1 \ln (p_1 / p_2)$$

From the 1st law (equation 1), one can thus find the heat transfer:

$$Q = \Delta U + p_1 V_1 \ln (p_1 / p_2)$$

Use the specific values for this problem for the generic variables in the previous equation to acquire:

$$Q = 25{,}575 \text{ J} + (345 \text{ kPa})(0.085 \text{ m}^3) \ln (345 \text{ kPa}/1380 \text{ kPa})$$

Working through the details (including the units) gives:

$$Q = 25{,}575 \text{ J} + (345\ 10^3 \text{N/m}^2)(0.085 \text{ m}^3) \ln (1/4)$$

so that $Q = 25{,}575 \text{ J} + (345)\ (8.5)\ [\text{N} \cdot \text{m}]\ (-1.386)$

giving $Q = 25{,}575 \text{ J} - 40{,}653 \text{ J}$

With the final result that:

$$\underline{Q = -15{,}078 \text{ J}}$$

NOTE: The negative sign indicates that heat is rejected by the system.

16.001 Unified Engineering: Materials and Structures
Fall 2021