Department of Aeronautics & Astronautics, M.I.T. 16.001 - Materials & Structures

Quiz No. 5

Instructor: Raúl Radovitzky

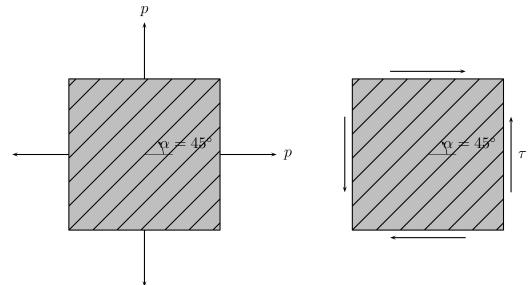
Student's name:_____

Question	Points	Score
1	20	
2	20	
Total:	40	

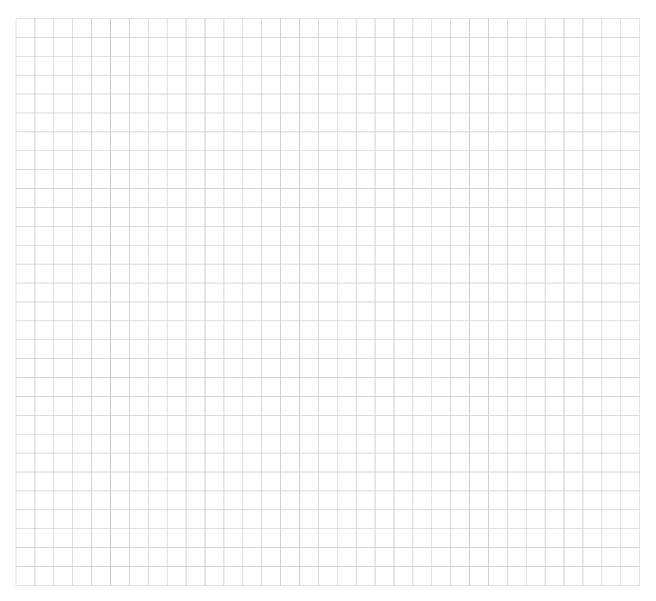
Letter grade: _____

Question 1 [20 points]

An orthotropic composite material with elastic constants $E_1, E_2, \nu_{12}, G_{12}$ is subject to two different states of stress: 1) hydrostatic, 2) and pure shear, as shown in the figures below. The dominant fiber direction (E_1) is at an angle $\alpha = 45^{\circ}$ with respect to the \mathbf{e}_1 axis.



1.1 (10 points) Hydrostatic state of stress: Compute the state of strain in some axes of your choosing. Then find the principal strains and directions. In addition, prove that the maximum shear strains are independent of the Poisson ratios ν_{12}, ν_{21}

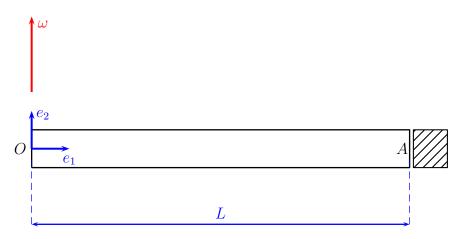


1.2 (10 points) Pure shear state of stress. Compute the state of strain in some axes of your choosing. Then find the principal strains and directions. Prove that in this case the principal directions of stress and strain coincide. Compute the maximum shear strains.

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Question 2 [20 points]

A bar of length L, area A, mass density ρ , elastic modulus E and CTE α rotates around $\mathbf{e_2}$ with an angular velocity ω , as shown in Figure ??. The free end x = L is constrained from extensional motion by a ring housing (i.e. the displacement u(L) = 0at the extremity A). After testing a first design, it was found that friction between the bar end and the housing produced unacceptable heating of the material surfaces. A solution that eliminates frictional forces at the interface completely is sought by modifying the temperature along the whole length of the bar uniformly by a value $\Delta \theta$.





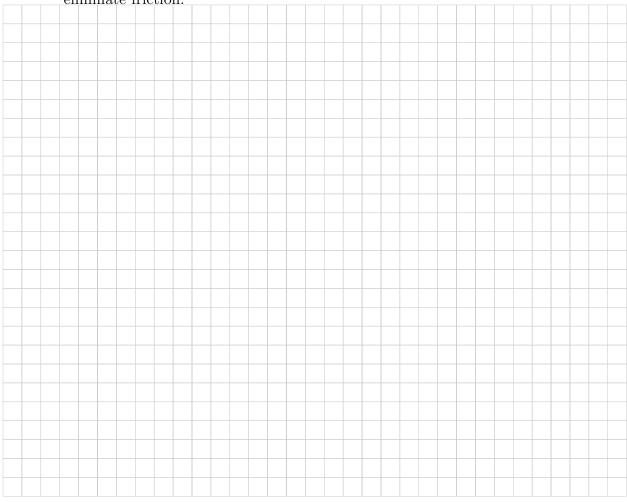
2.1 (5 points) Specialize the governing equations of rod theory to this problem. What principles do they represent?



2.2 (5 points) State the boundary conditions of this problem. Is the problem statically determinate or indeterminate? Justify your answer.

2.3 (5 points) Reduce the system of equations above and integrate the resulting equation(s). Apply the boundary conditions to obtain the distribution of displacements u(x) and internal axial load N(x) as a function of the external actions ω and $\Delta \theta$.

2.4 (5 points) Use the solution above to obtain an expression for the value of the temperature change $\Delta \theta$ which would make the normal N(L), and therefore the friction forces vanish. Interpret the result in terms of the sign of N(L) when there is no temperature change and the sign of the required temperature change to eliminate friction.



- 2.5 There are many additional interesting questions about this problem, but you are not asked to answer them:
 - Is the displacement affected by the temperature applied in this case (why?)
 - For the $\Delta \theta$ obtained, is there any penalty to be paid, e.g. in terms of the increase of N(0)? How much is it?
 - Use realistic values of the problem parameters to get a sense of how practicable this approach of controlling mechanical response with temperature changes due to cooling or heating would be. For the requisite temperature change (cooling or heating?), one could do a thermodynamic calculation (wait for the Spring), to get a sense of the power consumption of such an approach.

Solution: DO NOT ANSWER THIS

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