

## Unified Propulsion Quiz

May 7, 2004

**Closed Book – no notes other than the equation sheet provided with the exam  
Calculators allowed.**

- **Put your name on each page of the exam.**
- **Read all questions carefully.**
- **Do all work for each problem on the pages provided.**
- **Show intermediate results.**
- **Explain your work --- don't just write equations.**
- **Partial credit will be given (unless otherwise noted), but only when the intermediate results and explanations are clear.**
- **Please be neat. It will be easier to identify correct or partially correct responses when the response is neat.**
- **Show appropriate units with your final answers.**
- **Box your final answers.**

### Exam Scoring

<b>#1 (15%)</b>	
<b>#2 (25%)</b>	
<b>#3 (15%)</b>	
<b>#4 (20%)</b>	
<b>#5 (25%)</b>	
<b>Total</b>	

Below is a photograph of the Joint Strike Fighter (JSF). A lift fan (directly behind the cockpit) is driven by a shaft from the main engine to provide vertical take-off and landing capability. The exhaust nozzle on the main engine can also swivel 90 degrees to point directly down. The inlet for the main engine is split, with half the flow coming in either side of the cockpit, flowing through a y-shaped duct (with the shaft running through it) that wraps around the lift fan and meets in the center of the aircraft. There are also two small roll-control jets that exit the main engine aft of the compressor.



U.S. Air Force photo

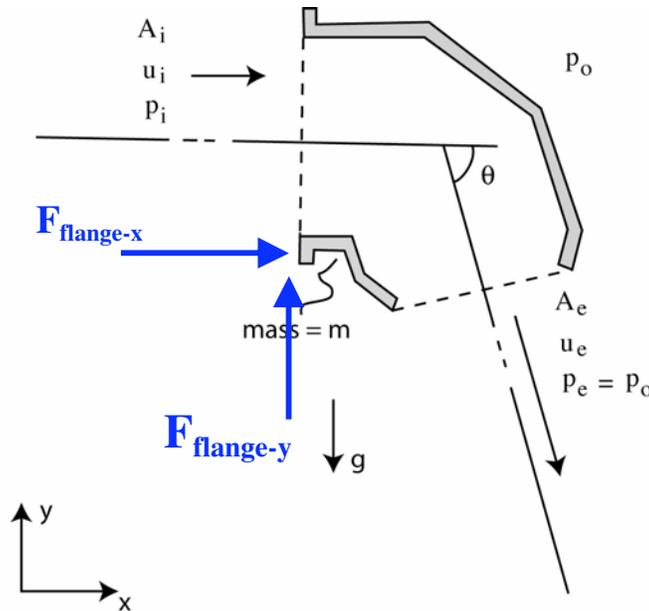
**1. (15 points, partial credit given, L.O. C & D)** What are the principal figures of merit for an engine of this type and why? When answering this question be sure to describe how these figures of merit are related to overall vehicle performance.

The principal figures of merit for gas turbine engine performance are max-thrust, overall propulsive efficiency, impact on vehicle weight and impact on vehicle drag. These parameters directly impact mission performance for example:

$$\text{Range} = \frac{h}{g} \eta_{\text{overall}} \frac{L}{D} \ln \frac{W_{\text{initial}}}{W_{\text{final}}} \quad \text{and for maneuvering:} \quad TV - DV = W \frac{dh}{dt} + \frac{d}{dt} \left( \frac{1}{2} \frac{W}{g} V^2 \right)$$

For this vehicle, the engine configuration alone suggests that maneuvering, especially vertical take-off and landing, is of great importance to the mission. Therefore an additional figure of merit would be the capability to reconfigure or vector the thrust. In addition to the performance figures of merit, it is important to add cost (acquisition cost and non-fuel operating costs).

**2. (25 points, partial credit given, L.O. B)** Consider the main JSF exhaust nozzle when deflected  $\theta$  degrees relative to the engine centerline. Write equations for the x and y components of force on the nozzle flange in terms of the parameters given and the mass flow ( $m_{\dot{}}$ ) through the nozzle.



Starting with the integral form of the momentum equation for a control volume of fixed mass:

$$\underbrace{\sum \bar{F} - \bar{F}_o}_{\text{sum of forces on C.V.}} = \underbrace{\int_V \left[ \frac{\partial(\rho \bar{u})}{\partial t} \right] dV}_{\text{change in momentum of mass contained in C.V. over time}} + \underbrace{\int_s \bar{u}(\rho \bar{u}) \cdot \bar{n} ds}_{\text{change in momentum flux across surface of C.V.}}$$

Assume steady flow and no acceleration of the reference frame (2<sup>nd</sup> and 3<sup>rd</sup> terms are zero). Then the x- and y-components are:

$$\sum F_x = \int_s u_x(\rho \bar{u}) \cdot \bar{n} ds \quad \text{and} \quad \sum F_y = \int_s u_y(\rho \bar{u}) \cdot \bar{n} ds$$

$$F_{\text{flange-x}} + (p_i - p_o)A_i = u_i(-u_i)\rho A_i + u_e \cos(\theta)(u_e)\rho A_e$$

$$F_{\text{flange-x}} = m_{\dot{}} [u_e \cos(\theta) - u_i] - (p_i - p_o)A_i$$

$$F_{\text{flange-y}} - mg = 0 + (-u_e)\sin(\theta)(u_e)\rho A_e$$

$$F_{\text{flange-y}} = mg - m_{\dot{}} [u_e \sin(\theta)]$$

**3. (15 points, partial credit given, L.O. B, C,D, G)** If the mass flow in the lift fan is 150kg/s and it produces enough thrust to lift half the weight of the aircraft (aircraft mass=30,000 kg) then approximate the minimum power in the shaft driving the lift fan. (Note that the lift fan is engaged with a clutch – an incredible feat given the power transient!) Assume the changes in kinetic energy of the fan flow are large relative to the change in internal energy.

Starting with the steady flow energy equation (J/kg):

$$q_{1-2} - w_{s_{1-2}} = h_{T_2} - h_{T_1}$$

$$w_s = \left( C_p T_e + \frac{u_e^2}{2} \right) - \left( C_p T_i + \frac{u_i^2}{2} \right) \approx \left( \frac{u_e^2}{2} \right) - \left( \frac{u_i^2}{2} \right)$$

since changes in internal energy are small relative to changes in kinetic energy. This is the work per kilogram. To find the power (work per time) we need to multiply by the mass flow (kg/s). So

$$P_s \approx \dot{m} \left[ \left( \frac{u_e^2}{2} \right) - \left( \frac{u_i^2}{2} \right) \right] \approx \dot{m} \left( \frac{u_e^2}{2} \right) \quad \text{for } u_e \gg u_i$$

Therefore we must to determine  $u_e$ . This comes from the integral momentum equation. Assuming no acceleration of the reference frame and steady flow, and neglecting and pressure forces then the lift fan thrust is

$$\text{Thrust} \approx \dot{m} u_e$$

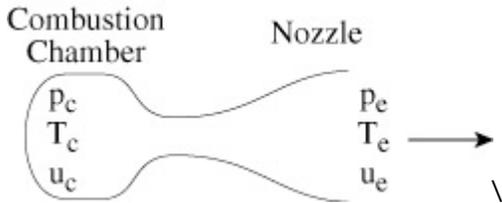
And the problem states that the thrust is equal to one-half the weight = 1/2 (9.8)(30,000) =147kN (assuming the other half comes from the main engine thrust vectored 90 degrees). The problem also gives the mass flow as 150kg/s, so the exit velocity is 147,000kN/150kg/s = 980m/s

$$\text{So } \boxed{\text{Power} = 150 \text{ kg/s } (980\text{m/s})^2/2 = 72\text{MW}}$$

**4. (20 points, partial credit given, L.O. C & E)** You are given the design for a rotor for an existing turbopump for a rocket engine. As part of an effort to improve the vehicle performance you are considering a new material for the rotor. The properties of the new material are such that the rotor would be heavier, but you would be able to spin it faster.

What impacts might this have on vehicle performance and why? Please *present the equations and concepts* you would use to connect a change in turbopump rotor speed, to changes in principal figures of merit for a rocket motor, to changes in vehicle performance. Discuss the trade-offs for this new material— which parameters would increase, which would stay the same, which would decrease?

Start with the Euler turbine equation:  $\text{Power} = \omega(\text{mass flow})(r_{\text{out}}v_{\text{out}} - r_{\text{in}}v_{\text{in}})$ . Spinning the rotor faster would enable increased  $\omega$  and probably increased  $v_{\text{out}}$  (since the tangential speed of the flow at the exit of the centrifugal turbopump would be closely related to the tangential speed of the rotor). So if the power increases then from the steady flow energy equation the change in stagnation enthalpy across the pump will increase.  $\text{Power} = \dot{m}(h_{T_{\text{out}}} - h_{T_{\text{in}}})$  assuming adiabatic flow.



This will enable the propellants to be delivered to the combustion chamber at higher pressure and temperature. So when they react the pressure and temperature will be higher. This will increase the exit velocity of the rocket (again from the steady flow energy equation for an adiabatic expansion with no shaft work):

$$u_e = \sqrt{2C_p T_c \left[ 1 - \left( \frac{P_e}{P_c} \right)^{\frac{\gamma-1}{\gamma}} \right]}$$

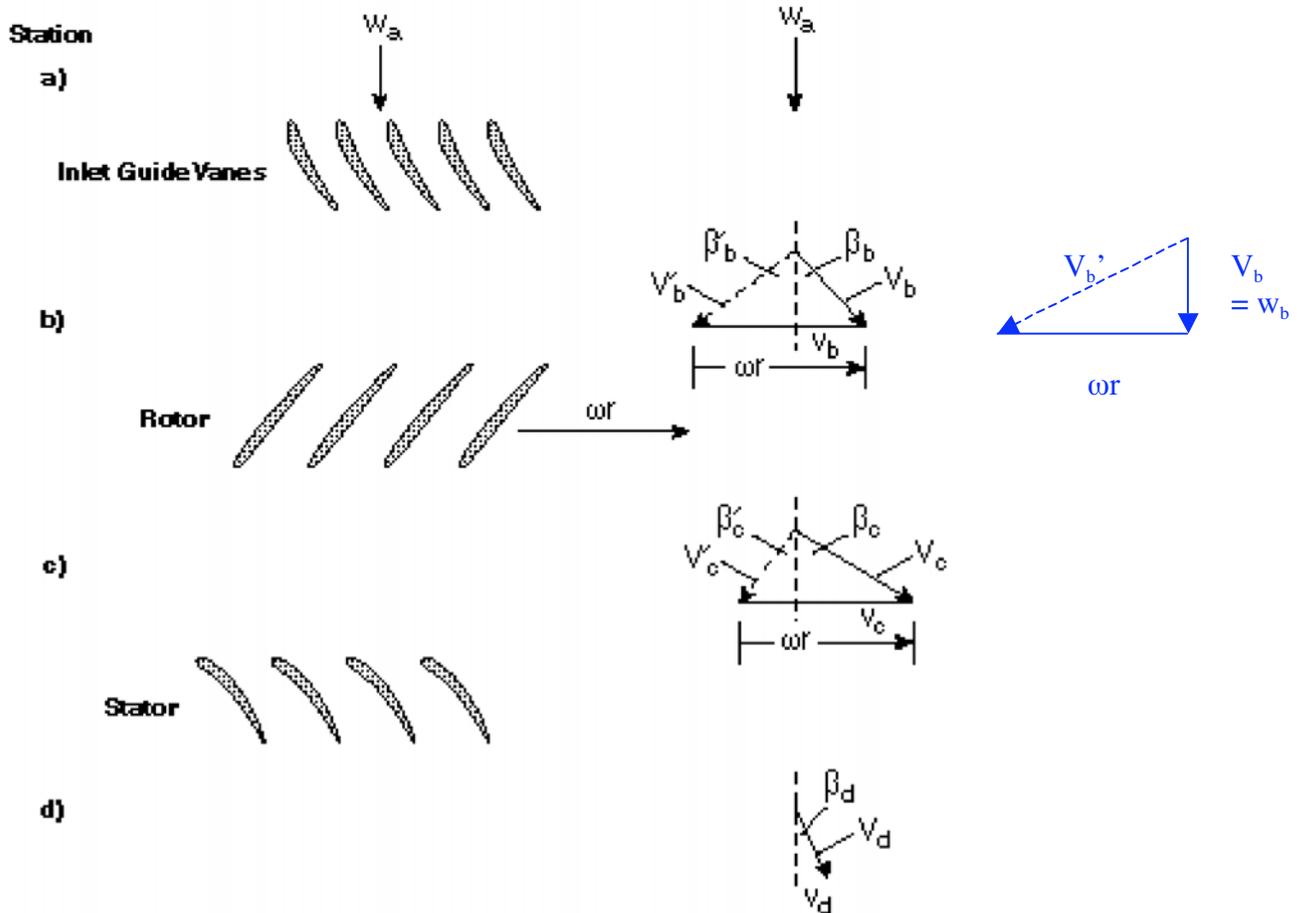
The higher exit velocity will increase the specific impulse ( $I_{sp} = u_e/g$ ). There will also be a higher mass flow through the nozzle since the upstream stagnation temperature and pressure are higher (although we haven't handled this very well in terms of the impact of

mass flow changes on pump performance). This will reduce the burn time for a given propellant mass. And the weight of the rocket motor will increase since the rotor will be heavier. Thus, the impacts on the principal figures of merit are that Isp will increase, rocket motor weight will increase, and the overall thrust will increase since the mass flow and exit velocity will increase. These changes would be reflected in the vehicle performance (for a simple single stage sounding rocket for example) as shown in the Rocket Equation:

$$u = g \left[ I_{sp} \ln \left( \frac{m_{v_o}}{m_v} \right) - t \right] \quad h_b = g \left[ - t_b I_{sp} \frac{\ln \left( \frac{m_{v_o}}{m_{v_{final}}} \right)}{\left( \frac{m_{v_o}}{m_{v_{final}}} \right) - 1} + t_b I_{sp} - \frac{1}{2} t_b^2 \right]$$

Isp would increase, the vehicle initial to final mass ratio would decrease, and burn time would decrease. The net impact on performance of these trade-offs would have to be evaluated given the detailed specifications.

5. (25 points, partial credit given, L.O. G) Three rows of turbomachine blades are shown below. At inlet to the first blade row, the flow is purely axial. Assume that the axial component of velocity ( $w$ ) then remains constant through the stage.



a) (5 pts) Describe in words the energy exchange processes in each of the three blade rows shown in the figure.

IGV: A nozzle—it accelerates the flow without changing the energy (or more appropriately the total enthalpy). Converts internal energy to swirling kinetic energy.

ROTOR: Increases the swirling kinetic energy, increasing the overall energy (total enthalpy) of the flow.

STATOR: A diffuser—it slows the flow down without changing the energy (total enthalpy). Converts swirling kinetic energy to internal energy.

b) (5 pts) Which of the three blade rows experiences torque? Why?

All three blade rows experience a torque since the tangential velocity in the stationary frame changes across each of the blade rows (however, it is only the moving blade that does work changing the total enthalpy of the flow).

$$\text{Torque} = (\text{mass flow})(r_{\text{out}}v_{\text{out}} - r_{\text{in}}v_{\text{in}})$$

c) (15pts) If the first blade row (the inlet guide vanes, IGV) were to be removed, with all else staying the same, would the magnitude of work for the device increase, decrease or stay the same? Why? Substantiate your answer by drawing the appropriate velocity triangles to the right of the figure on the preceding page.

The work would increase since the change in stationary frame tangential velocity across the moving blade row in the direction of rotor motion would increase.

$$\text{Power} = \omega(\text{mass flow})(r_{\text{out}}v_{\text{out}} - r_{\text{in}}v_{\text{in}}), \quad v_{\text{out}} \text{ remains the same, } v_{\text{in}} \text{ is reduced to zero.}$$

See drawing on previous page.