

DC Motor / Propeller Characterization

Lab 3 Lecture Notes

24 Feb 06

Nomenclature

r	radial coordinate	B	number of blades
R	prop tip radius	D	prop diameter ($\equiv 2R$)
V	flight velocity	P	prop pitch
Ω	motor and propeller rotation rate	T	prop thrust
v	motor terminal voltage	Q	prop torque
i	motor current	P_{thrust}	thrust power ($\equiv TV$)
η	overall propulsive efficiency	P_{elec}	electrical power ($\equiv i v$)

Motor/Propeller Characterization

The combination of a propeller and an electric motor, shown in Figure 1, can be viewed as a propulsion unit whose thrust $T(V, v)$ and current $i(V, v)$ both depend on the air velocity V into the propeller, and the voltage v applied to the motor terminals. We can then define the thrust power P_{thrust} , the electrical power P_{elec} , and the overall propulsive efficiency.

$$\eta(V, v) = \frac{P_{\text{thrust}}}{P_{\text{elec}}} = \frac{TV}{i v} \quad (1)$$

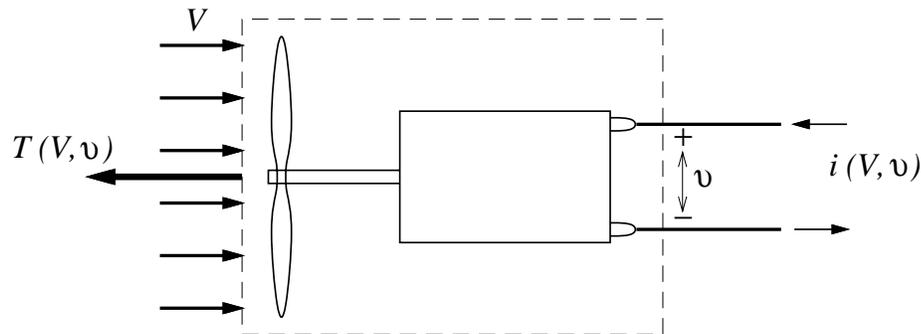


Figure 1: Propulsion unit consisting of a propeller driven by an electric motor. For any given propeller and motor, the thrust T and current i depend only the oncoming air velocity V and applied voltage v .

Calculation of the $T(V, v)$ and $i(V, v)$ functions for any given propeller and motor combination is rather complicated, and is beyond scope here. However, these can be readily measured in a motor/prop test stand in a wind tunnel. Typical behavior of these functions is shown in Figure 2. The rotation speed $\Omega(V, v)$ is also shown for interest.

Each thrust curve in Figure 2 is seen to drop to zero at a particular *aerodynamic pitch speed* $V_o(v)$, which is roughly proportional to the applied voltage v . The reason for this drop to zero is primarily due to the aerodynamic characteristics of the propeller, although the motor characteristics also come into play to a smaller extent.

An electric aircraft in level flight requires $T > 0$, and hence its maximum level-flight speed clearly cannot exceed the pitch speed. Hence, to increase the aircraft's maximum speed, it is necessary to examine what features of the propeller and motor combination might be changed to increase V_o (in addition to the obvious solution of increasing the voltage v).

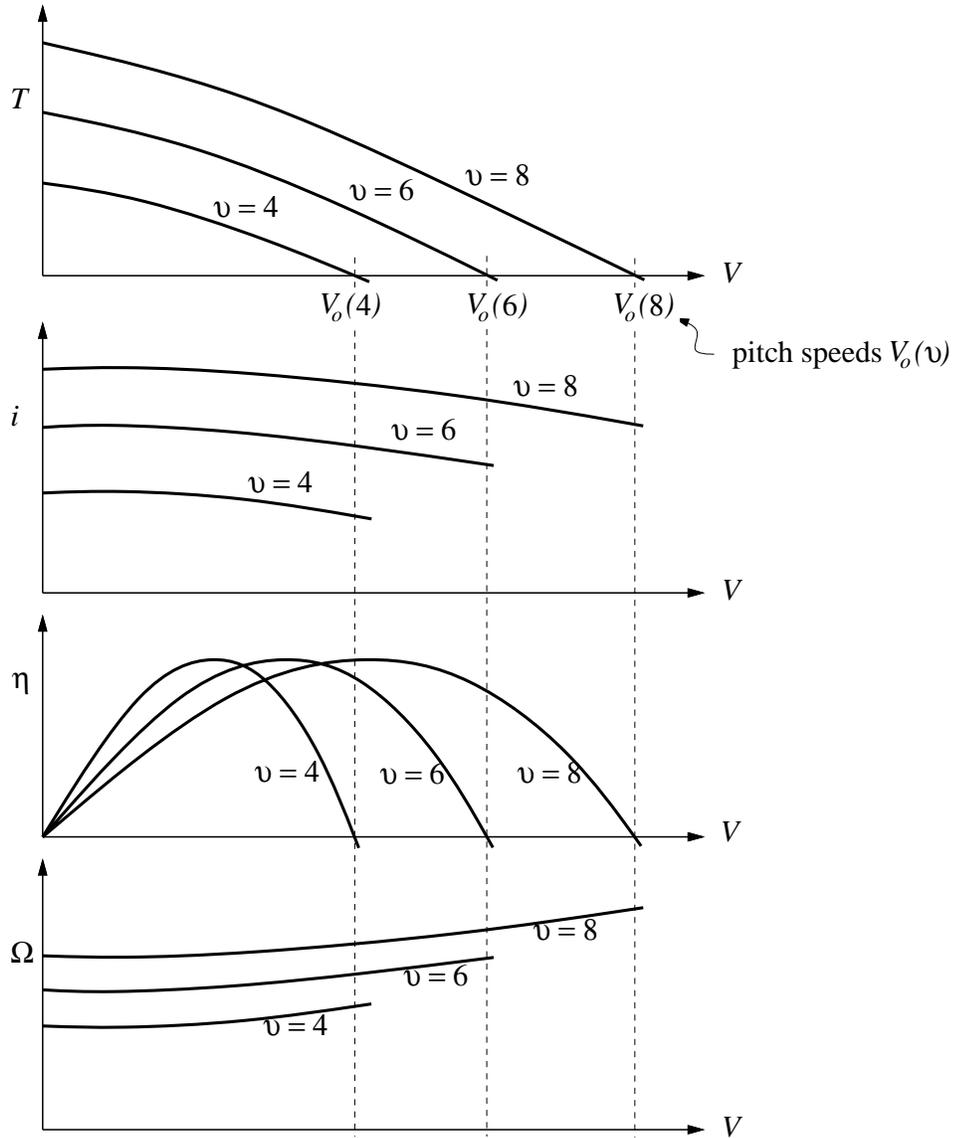


Figure 2: Typical thrust T , current i , overall efficiency η , and rotation speed Ω of a propeller/motor combination, versus airspeed V and three applied voltages v .

Propeller Thrust Characteristics

Figure 3 shows the blade airfoil at some radius r , which sees a total oncoming flow velocity $W = \sqrt{V^2 + \Omega^2 r^2}$, which is the hypotenuse of the *velocity triangle* composed of V and Ωr .

The local resultant force/span R' has the usual lift and drag components L' and D' . This R' can also be considered to be composed of the thrust and torque components T' and Q'/r , which are also shown in Figure 3. The overall thrust T and torque Q of the propeller is then the result of integrating these T' and Q' along the B blades.

$$T = B \int_0^R T' dr \quad (2)$$

$$Q = B \int_0^R (Q'/r) r dr \quad (3)$$

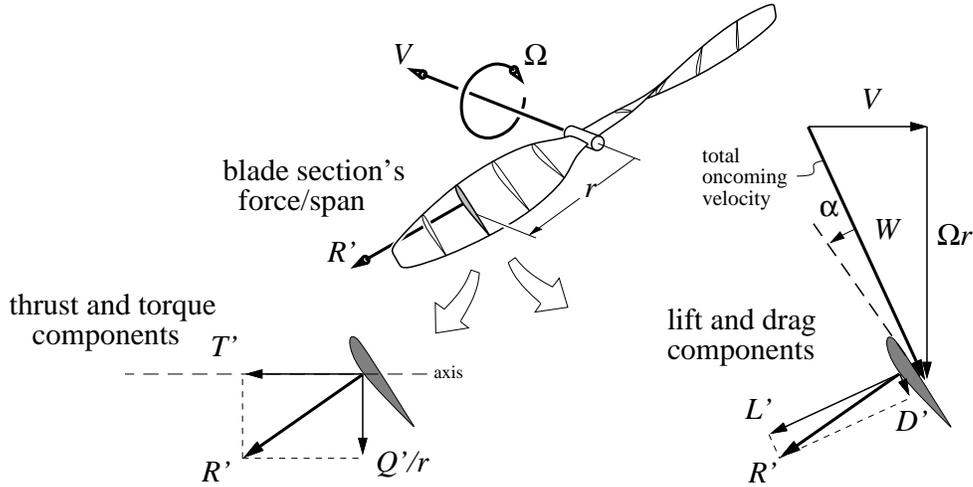


Figure 3: Propeller blade resultant force/span R' at some radius r , due to angle of attack α and the total oncoming flow velocity W with components V and Ωr . Additional small induced velocity normal to W not shown.

The thrust behavior shown in Figure 2 can now be explained by considering what happens to a typical velocity triangle as the velocity is changed. Figure 4 shows this situation for three different airspeeds. As V is increased towards V_o , the local angle of attack α and thrust T' go towards zero. The overall thrust then also falls to zero. One complication is that Ω

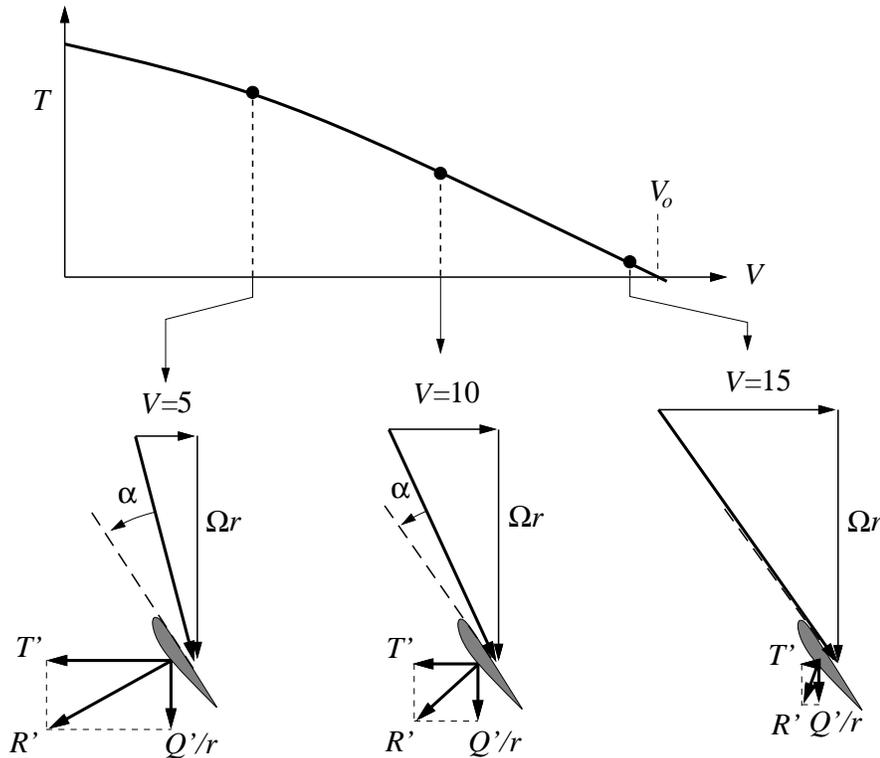


Figure 4: Velocity triangle seen by propeller blade section for three different flight velocities V . As V increases towards V_o , the local α and thrust/span T' decrease.

is not fixed here, but tends to increase as V is increased and the motor-loading prop torque Q is reduced. However, the Ω curves in Figure 2 show that the increase in Ω is not large percentagewise, so the velocity triangles in Figure 4 are essentially correct.

Propeller Geometry and Operation

Important geometric parameters of a propeller are its tip radius R and its *pitch* P , illustrated in Figure 5. The pitch has the same meaning as the thread pitch on a wood screw — it’s the distance that the propeller would travel in one revolution if advanced into a solid material. Although conceptually useful, this definition of pitch is somewhat ambiguous. Nevertheless,

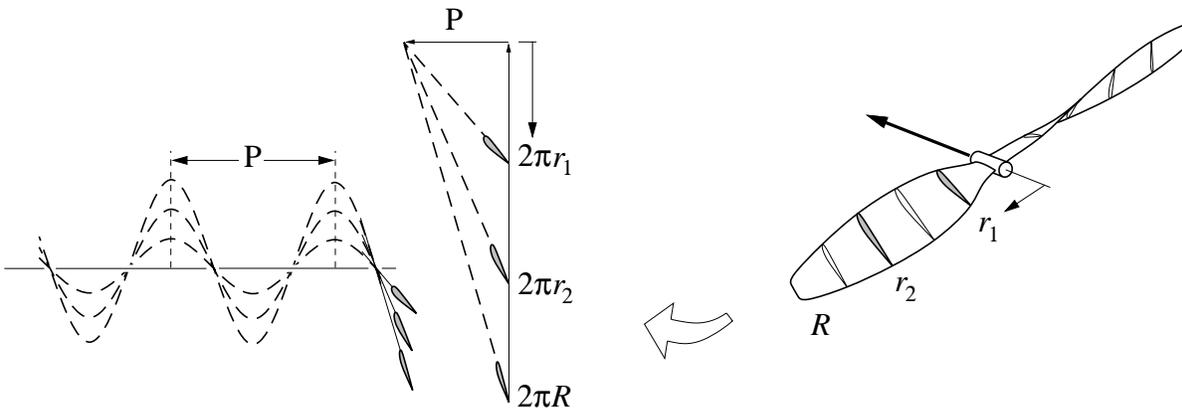


Figure 5: Propeller pitch P , equal at the three radii shown.

almost every commercial propeller has a quoted pitch value (e.g. a 7×4 propeller corresponds to $D = 7$ in, $P = 4$ in), even though how this pitch is defined is never stated. Despite this ambiguity, the quoted pitch is in fact useful for categorizing and comparing different propellers.

High Pitch versus Low Pitch Propellers

The $V = 15$ velocity triangle in Figure 4, which is close to the pitch speed case $V = V_o$, is congruent to the pitch triangle in Figure 5 at the same radius. Both cases have $\alpha \simeq 0$ and hence $T \simeq 0$. Hence, the pitch is the distance the propeller travels in one revolution time $t_{\text{rev}} = 2\pi/\Omega$ at speed V_o . The relation is then

$$V_o \simeq \frac{P}{t_{\text{rev}}} = \frac{P \Omega}{2\pi} \quad (4)$$

which is the origin of the term “pitch speed”. This clearly suggests that one way to increase V_o , in the interest of increasing the maximum speed of the aircraft, is to increase the pitch of the propeller. However, increasing the pitch has some drawbacks at low speeds, and a tradeoff is required.

The tradeoffs become more clear if we examine how the propeller/motor operating parameters shown in Figure 2 depend on the propeller pitch, or more precisely, on the propeller pitch/diameter ratio, or P/D . Figure 6 compares these parameters for a “low pitch” and a “high pitch” propeller, for the same motor and same applied voltage.

The low pitch propeller is seen to be superior at low velocities, where it has relatively large thrust and a larger efficiency. Such a propeller is most effective for a rapid takeoff and efficient low-speed cruise. But its relatively low pitch speed V_o prevents the aircraft from reaching high speeds.

The high pitch propeller is inferior at low speeds, but it has a markedly larger pitch speed

V_o , and hence maintains its thrust and good efficiency to larger airspeeds. Such a propeller is most effective for reaching large maximum flight speeds.

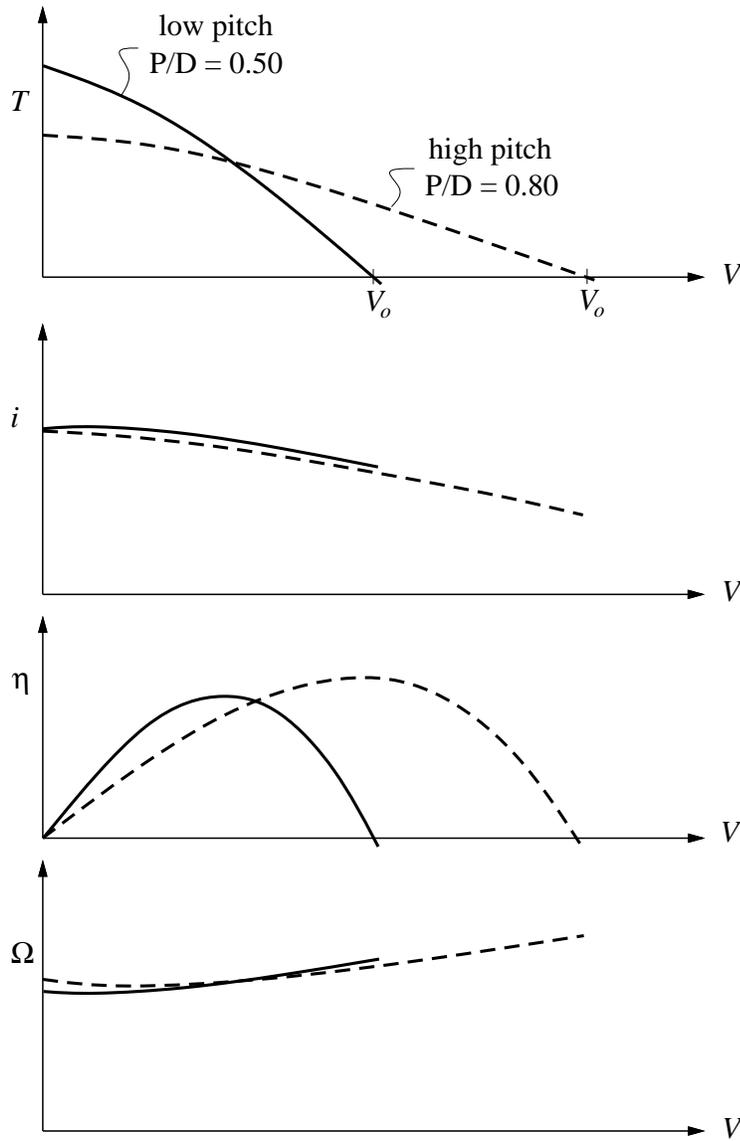


Figure 6: Effects of pitch/diameter ratio on propeller performance parameters versus airspeed V . Same motor and same applied voltage are used for both propellers.

Component Efficiencies

So far we have treated the motor/propeller combination as a single propulsive device with one overall efficiency η . This is sufficient for matching this device with a battery and an airframe, via the performance curves in Figures 2 and 6. But to make sure that this device is working as well as possible, it is necessary to consider how well the motor and the propeller are working individually. For this purpose we define a shaft power,

$$P_{\text{shaft}} = Q \Omega \quad (5)$$

which is the output of the motor and is the input into the propeller. This then allows

decomposing η into separate motor and propeller efficiencies.

$$\eta_m = P_{\text{shaft}}/P_{\text{elec}} \quad (6)$$

$$\eta_p = P_{\text{thrust}}/P_{\text{shaft}} \quad (7)$$

$$\eta = \eta_m \eta_p \quad (8)$$

Based on circuit motor models, η_m depends on rotation speed and voltage, and based on aerodynamic prop models, η_p depends on rotation speed and velocity.

$$\eta_m = \eta_m(\Omega, v) \quad , \quad \eta_p = \eta_p(\Omega, V)$$

But since we can measure or otherwise determine the $\Omega(V, v)$ relation for a particular motor/prop combination, both efficiencies can be determined as functions of V and v .

$$\eta_m = \eta_m(V, v) \quad , \quad \eta_p = \eta_p(V, v)$$

Figure 7 shows two cases of how these efficiencies might behave with velocity.

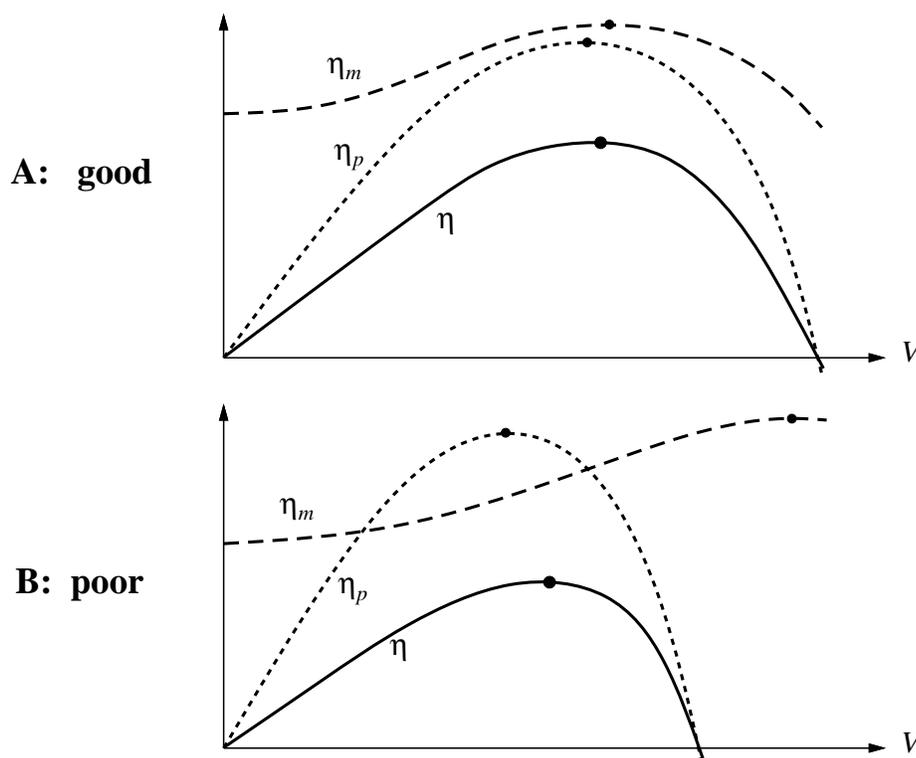


Figure 7: Good and poor impedance matches between motor and propeller.

In Case A, the peaks in η_m and η_p occur at roughly the same velocity V . In this situation, the motor and prop are said to have a *good impedance match*. In Case B, the peaks in η_m and η_p occur at significantly different velocities. This means that the maximum overall η will occur somewhere between these, and will be smaller than in Case A.

In motor/propeller combinations, poor impedance matches are most often the result of an oversized propeller. The motor then cannot reach its most efficient RPM for the required power, which produces excess current flow and large resistive losses.