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**16.03/16.04 Unified Engineering III, IV  
Spring 2004**

**Problem Set 4**

Name: \_\_\_\_\_

Due Date: 3/2/04

	<b>Time Spent (min)</b>
<b>F11/F12</b>	
<b>F13</b>	
<b>F14</b>	
<b>M10</b>	
<b>M11</b>	
<b>M12</b>	
<b>M13</b>	
<b>Study Time</b>	

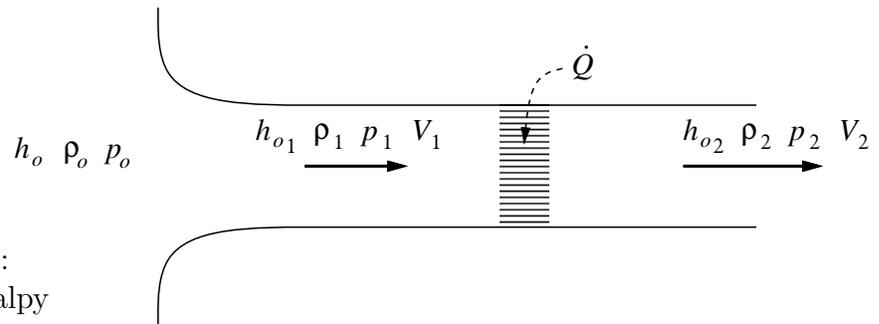
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Announcements: The Materials Quiz on Friday (2/27) will be at 9am

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F11+12. Air is drawn at high speed out of a large reservoir through a duct of constant area  $A$ , which contains a radiator delivering a known  $\dot{Q}$  to the flow (in Watts). The heating and friction of the duct walls are negligible.



The known flow quantities are:

- $h_o = h_{o1}$  reservoir total enthalpy
- $\rho_o$  reservoir total density
- $p_o$  reservoir total pressure
- $p_2$  outlet pressure (drives the flow)

The remaining six unknown quantities inside the duct are:

- $h_{o2}$  outlet total enthalpy
- $V_1$  inlet velocity
- $V_2$  outlet velocity
- $\rho_1$  inlet density
- $\rho_2$  outlet density
- $p_1$  inlet pressure

A total of six equations are needed to solve for the six unknowns. One of these equations is the isentropic relation between the reservoir and station 1,

$$\frac{p_1}{p_o} = \left( \frac{h_{o1} - \frac{1}{2}V_1^2}{h_o} \right)^{\gamma/(\gamma-1)}$$

and two additional ones are the state equations at stations 1 and 2.

$$p_1 = \frac{\gamma - 1}{\gamma} \rho_1 \left( h_{o1} - \frac{1}{2}V_1^2 \right)$$

$$p_2 = \frac{\gamma - 1}{\gamma} \rho_2 \left( h_{o2} - \frac{1}{2}V_2^2 \right)$$

Write down the remaining three equations by constructing a suitable control volume and applying the integral mass, momentum, energy equations. (Do not try to solve the six equations — it gets very messy!)

F13. You stick your hand out the window of a car traveling at 75 mph, palm into the wind. The ambient air conditions are  $\rho = 1.2 \text{ kg/m}^3$ ,  $p = 10^5 \text{ Pa}$ ,  $T = 300 \text{ K}$ . For air,  $c_p = 1004 \text{ J/kg K}$ . Determine the conditions on the stagnation point on your palm.

F14. An aircraft is flying at speed  $V_\infty$ , in an atmosphere with  $p_\infty$ , and  $\rho_\infty$ .

a) What is the flight Mach number  $M_\infty$ ? Give in terms of the quantities above.

b) Determine the stagnation pressure  $p_o$  at the nose of the aircraft in two ways:

i) The exact full compressible equation.

ii) The incompressible Bernoulli equation, pretending  $\rho = \rho_\infty$  is constant.

Plot  $p_o/p_\infty$  versus  $M_\infty$  for the two equations. Also plot the “Bernoulli error”

$$(p_o/p_\infty)_{\text{exact}} - (p_o/p_\infty)_{\text{Bernoulli}}$$

versus  $M_\infty$ . What would you judge to be a reasonable upper Mach limit on the validity of the Bernoulli equation?

Consider two wings with the same span  $b = 2 \text{ m}$ , and average chord  $c_{\text{avg}} = 0.25 \text{ m}$ , but different taper ratios:

Constant chord ( $c_r = 0.250 \text{ m}$ ,  $c_t = 0.250 \text{ m}$ ,  $r = 1.0$ )

Straight taper ( $c_r = 0.333 \text{ m}$ ,  $c_t = 0.167 \text{ m}$ ,  $r = 0.5$ )

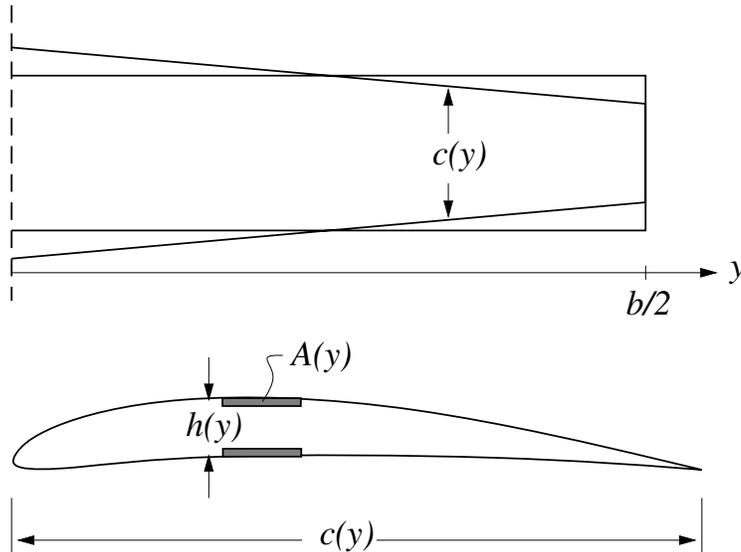
Constant airfoil thickness/chord ratio  $\tau = 0.08$  everywhere.

Assume local loading  $q(y)$  is proportional to chord  $c(y)$ .

Total load on half-wing is  $F = 10 \text{ N}$  (15 oz gross weight at 5 g's).

For each wing ...

- Determine  $q(y)$ ,  $S(y)$ ,  $M(y)$
- Assuming load is carried by top and bottom spars with separation equal to local  $h(y) = \tau c(y)$ , determine sparcap load  $\pm P(y)$ .
- Assuming max permissible sparcap stress (1 ksi = 7 MPa for balsa), calculate minimum cap area  $A(y)$ . Assuming balsa density of  $\rho = 0.125 \text{ g/cm}^3$ , estimate sparcap mass.
- Compute beam curvature at wing center  $\kappa = M(0)/EI(0)$ , and estimate tip deflection  $\delta = w(b/2)$  assuming  $w''(y) = \kappa$  is constant along span. Balsa modulus:  $E = 200 \text{ ksi} = 1.36 \text{ GPa}$ .
- Discuss structural and aerodynamic merits of straight vs tapered wing. Plotting of the various distributions is suggested to help with interpretation.



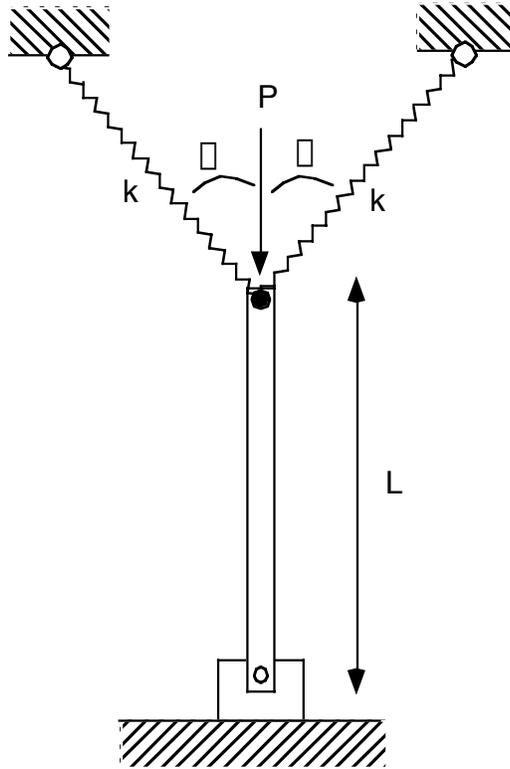
**Problem M11**

A solid steel shaft of length 3 m is required to transmit a torque of 200 kNm. The maximum allowable shear stress that the steel can support,  $\tau_y$ , is 200 MPa (shear yield stress).

- (a) If the shaft has a solid circular cross section, determine the minimum diameter of the shaft to transmit the torque without yielding.
- (b) If the shaft is instead hollow with a ratio of external to internal diameter of 5/4, what is now the required diameter of the shaft, what is the weight saving over case (a)?.
- (c) What is the ratio of (i) the angle of twist between the ends of the shafts and (ii) the torsional stiffness in the two cases?.
- (d) What is the maximum extensional stress acting in the shaft in (b), in what direction(s) does it act?.

### Problem M12

The diagram below shows a rigid bar, length  $L$ , pinned at one end, which is initially held vertical. The bar is stabilized by the pair of linear springs, stiffness  $k$ , orientated at the angle  $\alpha$  to the vertical. Find the critical value of the load  $P$  which will cause collapse of this assembly. Hint: Consider the equilibrium of the bar at a small displacement from the equilibrium position shown.



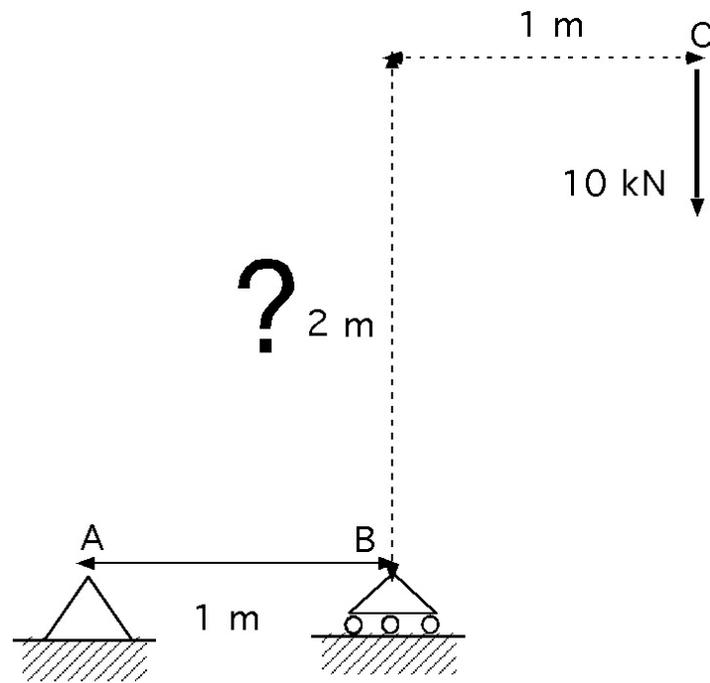
### Problem M13

In problem M1 you designed a truss, however you took no account of the possibility of buckling. Redo the question, with the same set of materials. Reexamine your design strategy (including materials selection). Assume that all the materials are available in the form of bars with a solid circular cross-section.

### Problem M1

This question is designed to provide you with a chance to revise material we met last term, and an opportunity to start thinking about structural design. You are asked to design a minimum mass truss structure that will be attached at points A, and B, 1 m apart on a horizontal floor in order to support a vertical load of 10 kN, without exceeding the strength (assume that the tensile and compressive strengths are the same) of the bars, at a distance 2 m above the floor and 1m to the right of the right hand support point (B).. The truss will be made of constant cross-section members of whatever material you choose to select. All of the bars will have the same cross-section. The following materials are available for selection. Select a material, and then choose an appropriate truss configuration and then estimate the mass of the truss, such that it will meet the design requirement. Explain your thought processes at each step.

Material	Density, $\rho$ (Mg/m <sup>3</sup> )	Modulus, E, (GPa)	CTE, $\alpha$ , $\times 10^{-6} \text{ K}^{-1}$	Price, p, (\$/Mg)	Tensile Strength, $\sigma_f$ , (MPa)
Mild Steel	7.9	203	12	300	220
Aluminum alloy (2000 series)	2.8	71	24	1500	350
Titanium alloy Ti-6Al4V	4.5	120	9.0	8000	850
Carbon fiber composite*	1.5	70	3.0	100000	700
Wood (e.g spruce)*	0.6	12	4.0	300	30
Silicon Carbide (SiC)	3.0	410	4.0	50000	300



Note. Although the design objective is to minimize the mass of the structure, the credit for the question will be based on demonstrating a logical approach to selecting a material and a truss configuration, and then obtaining an estimate for the mass of the truss. Do not spend more than an hour on this question, and do not analyze multiple truss configurations. A useful exercise is to estimate what you think the mass of the truss will be before you do any analysis - developing an intuition for the correct size for structures is a useful skill to cultivate.