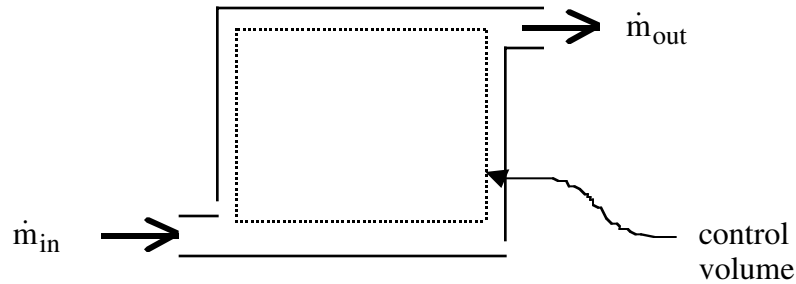


STEADY FLOW ENERGY EQUATION

I. First Law for a Control Volume (VW, S & B: Chapter 6)

A. Frequently (especially for flow processes) it is most useful to express the First Law as a statement about rates of heat and work, for a control volume.

1. Conservation of mass (VW, S & B: 6.1)



$$\frac{dm_{cv}}{dt} = \dot{m}_{in} - \dot{m}_{out} \quad \left(\begin{array}{l} \text{rate of change} \\ \text{of mass in c.v.} \end{array} \right) = \left(\begin{array}{l} \text{mass flow} \\ \text{in} \end{array} \right) - \left(\begin{array}{l} \text{mass flow} \\ \text{out} \end{array} \right)$$

For steady state, $\frac{d}{dt} = 0$ therefore

$$\dot{m}_{in} = \dot{m}_{out} = \dot{m}$$

2. Conservation of Energy (First Law) (VW, S & B: 6.2)

Recall, $dE = \delta Q - \delta W$

For the control volume,

$$\frac{dE_{c.v.}}{dt} = \dot{Q}_{c.v.} - \dot{W}_{c.v.} + \dot{m}_{in}e_{in} - \dot{m}_{out}e_{out}$$

$$\left(\begin{array}{c} \text{rate of change} \\ \text{of energy in c.v.} \end{array}\right) = \left(\begin{array}{c} \text{rate of heat} \\ \text{added to c.v.} \end{array}\right) - \left(\begin{array}{c} \text{rate of work} \\ \text{done} \end{array}\right) + \left(\begin{array}{c} \text{rate of energy} \\ \text{flow in to c.v.} \end{array}\right) - \left(\begin{array}{c} \text{rate of energy} \\ \text{flow out of c.v.} \end{array}\right)$$

where

$$\dot{Q} = \lim_{dt \rightarrow 0} \left(\frac{\delta Q}{dt} \right) \quad \text{rate of energy transfer to the system as heat}$$

$$\dot{W} = \lim_{dt \rightarrow 0} \left(\frac{\delta W}{dt} \right) \quad \text{rate of work done by the system}$$

3. For steady-state (*VW, S & B: 6.3*)

$$\frac{d}{dt} = 0 \quad \text{and} \quad \dot{m}_{in} = \dot{m}_{out} = \dot{m}$$

so

$$\dot{Q}_{c.v.} - \dot{W}_{c.v.} = \dot{m}(e_{out} - e_{in}) \quad (\text{units J/s})$$

or

$$\dot{Q}_{c.v.} - \dot{W}_{c.v.} = \dot{m}[(IE + KE + PE + CE)_{out} - (IE + KE + PE + CE)_{in}]$$

Neglecting potential and chemical energy (PE and CE)

$$\dot{Q}_{c.v.} - \dot{W}_{c.v.} = \dot{m} \left[\left(u + \frac{c^2}{2} \right)_{out} - \left(u + \frac{c^2}{2} \right)_{in} \right]$$

Where c is the speed of the fluid, and $c^2/2$ is the kinetic energy of the fluid per unit mass relative to some coordinate system.

If we divide through by the mass flow and set the inlet of the control volume as station 1, and the outlet as station 2, then

$$q_{1-2} - w_{1-2} = u_2 - u_1 + \frac{c_2^2}{2} - \frac{c_1^2}{2}$$

It is also more convenient to divide the work into two terms: 1) the flow work done by the system which is $p_2v_2 - p_1v_1$, and 2) any additional work which we will term external work or shaft work, w_s . Then we have

$$q_{1-2} - w_{s1-2} = (u_2 + p_2 v_2) - (u_1 + p_1 v_1) + \frac{c_2^2}{2} - \frac{c_1^2}{2}$$

or

$$q_{1-2} - w_{s1-2} = h_2 - h_1 + \frac{c_2^2}{2} - \frac{c_1^2}{2}$$

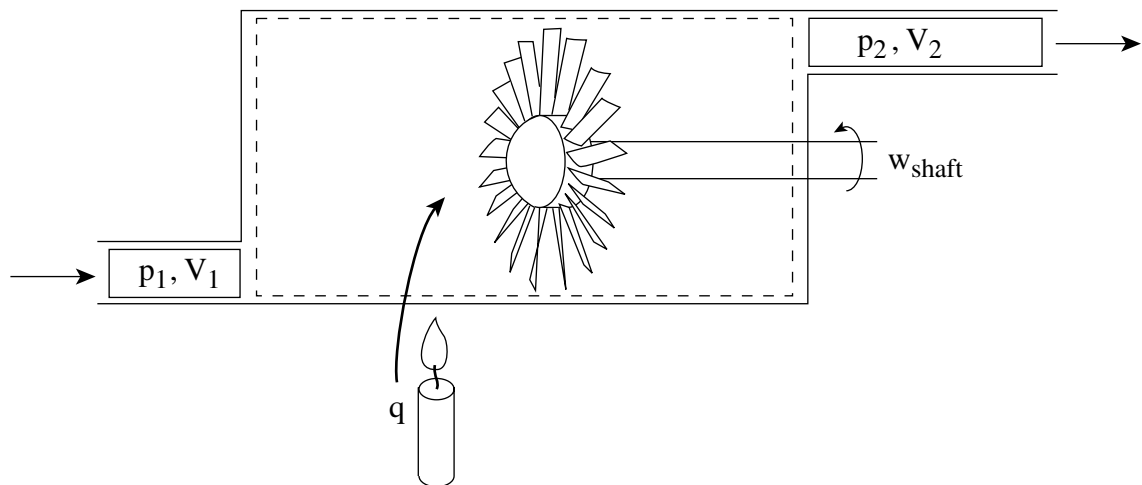
We will call this the **steady flow energy equation**.

For an **ideal gas** $dh = c_p dT$ so

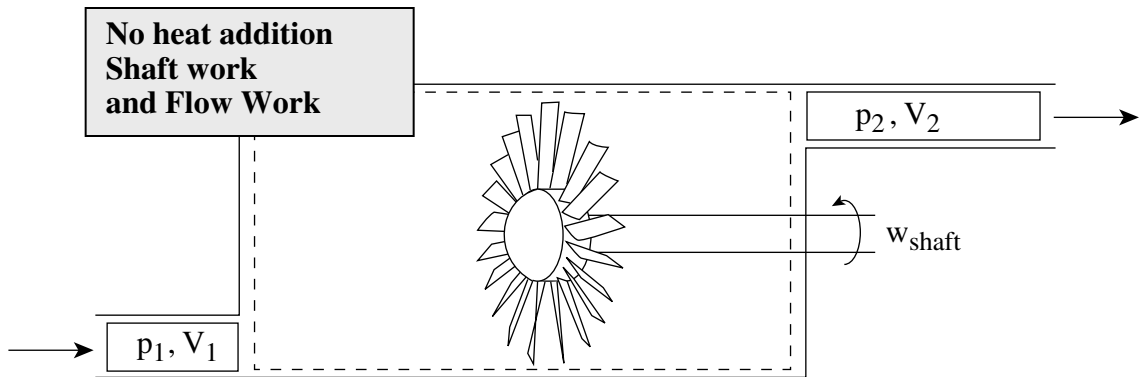
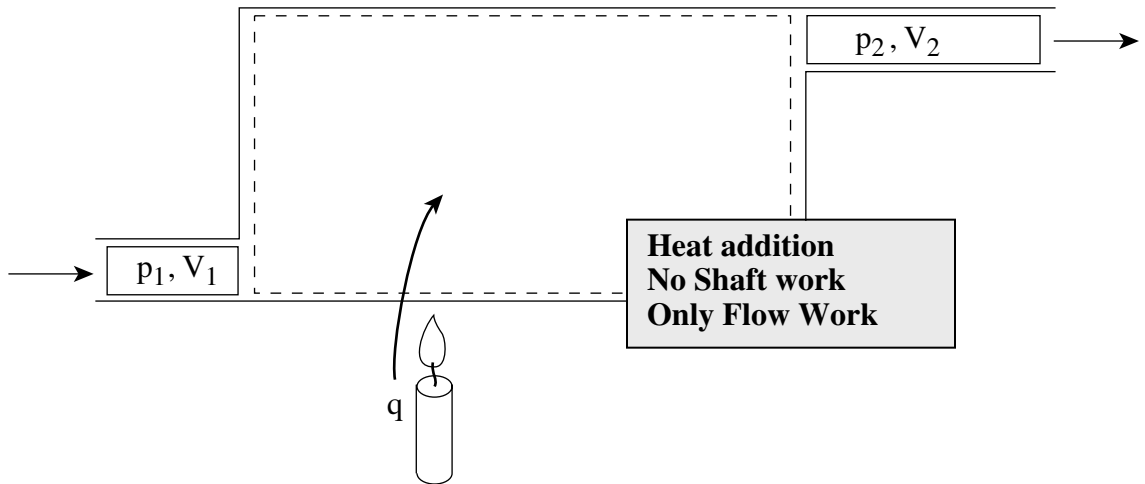
$$q_{1-2} - w_{s1-2} = \left(c_p T_2 + \frac{c_2^2}{2} \right) - \left(c_p T_1 + \frac{c_1^2}{2} \right)$$

4. Flow work and shaft work

Enthalpy is most useful for separating flow work from shaft work. In the figure shown below. Heat is added, a compressor is doing work on the system, the flow entering the system does work on the system (work = $-p_1 V_1$), and work is done by the system through pushing out the flow (work = $+p_2 V_2$). The first law relates the change in energy between states 1 and 2 to the difference between the heat added and the work done by the system. Frequently, however, we are interested only in the shaft work that crosses the system boundary, not the volumetric or flow work. In this case it is most convenient to work with enthalpy.

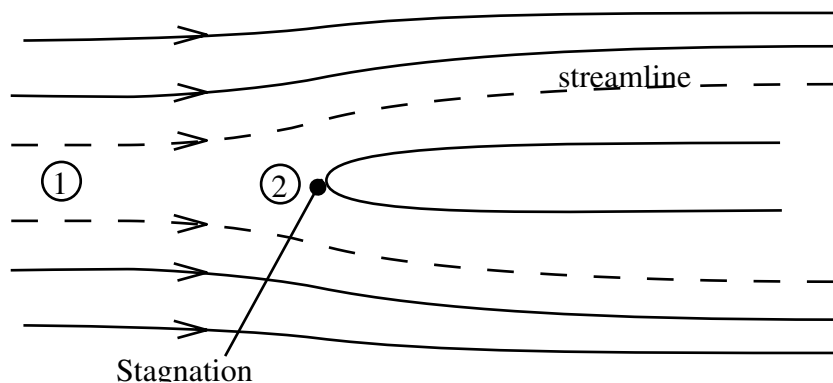


Note that both of the following cases are also frequently encountered:



B. Stagnation Temperature and Stagnation Enthalpy

Suppose that our steady flow control volume is a set of streamlines describing the flow up to the nose of a blunt object.



The streamlines are stationary in space, so there is no external work done on the fluid as it flows. If there is also no heat transferred to the flow (adiabatic), then the steady flow energy equation becomes

$$c_p T_2 + \frac{c_2^2}{2} = c_p T_1 + \frac{c_1^2}{2}$$

1. The quantity that is conserved is called the stagnation temperature.

$$T_T = T + \frac{c^2}{2c_p} \quad \left(\text{or } \frac{T_T}{T} = 1 + \frac{\gamma-1}{2} M^2 \quad \text{using } a = \sqrt{\gamma RT} \right)$$

The stagnation temperature is the temperature that the fluid would reach if it were brought to zero speed by a steady adiabatic process with no external work. Note that for any steady, adiabatic flow with no external work, the stagnation temperature is constant.

(The Mach number, M , is the ratio of the flow speed, c , to the speed of sound, a . You will learn more about these quantities in fluids, but it is interesting to see that M^2 measures the ratio of the kinetic energy of the gas to its thermal energy.)

2. It is also convenient to define the **stagnation enthalpy**, h_T

$$h_T = c_p T + \frac{c^2}{2}$$

So we can write the **Steady Flow Energy Equation** in a convenient form as

$$q_{1-2} - w_{s1-2} = h_{T2} - h_{T1}$$

3. Note that for a quasi-static adiabatic process

$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2} \right)^{\frac{\gamma-1}{\gamma}} \quad \text{so we can write} \quad \frac{T_T}{T} = \left(\frac{p_T}{p} \right)^{\frac{\gamma-1}{\gamma}}$$

and define the relationship between **stagnation pressure** and **static pressure** as

$$\frac{p_T}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/\gamma - 1}$$

where, the stagnation pressure is the pressure that the fluid would reach if it were brought to zero speed, via a steady, adiabatic, quasi-static process with no external work.

4. Frame dependence of stagnation quantities

An area of common confusion is the frame dependence of stagnation quantities. The stagnation temperature and stagnation pressure are the conditions the fluid would reach if it were brought to zero speed *relative to some reference frame*, via a steady adiabatic process with no external work (add quasi-static for stagnation pressure).

What if the body or reference frame is moving? We know from looking at re-entry vehicles, that the skin temperature is much hotter than the atmospheric temperature. If the atmosphere is assumed still, and we stagnate a fluid particle on the nose of a high speed vehicle (carrying it along with the vehicle and thus essentially giving it kinetic energy) its stagnation temperature is given by

$$T_T = T + \frac{c^2}{2c_p}$$

where c is the vehicle speed. The temperature the skin reaches (to first approximation) is the stagnation temperature. The atmospheric temperature, T , is not frame dependent.

The confusion comes about because T is usually referred to as the **static** temperature and in common language this has a similar meaning as “stagnation”. In fluid mechanics and thermodynamics *static* is commonly used to label the thermodynamic properties of the gas p , T , etc.—these are not frame dependent. *Stagnation* quantities are those the flow would arrive at if brought to zero speed relative to some reference frame, via a steady adiabatic process with no external work (add quasi-static for stagnation pressure). Stagnation quantities depend on the frame of reference.

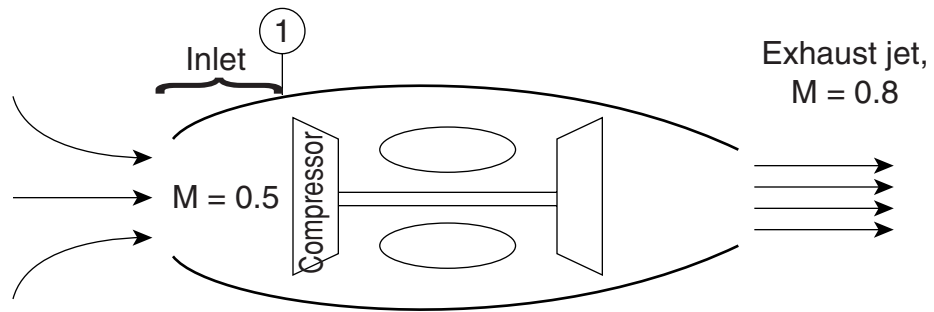
Thus for a re-entry vehicle, the stagnation temperature (vehicle frame) is hotter than the atmospheric (static) temperature. And in a still atmosphere, the static temperature is the same as the stagnation temperature (atmospheric frame).

5. Example:

For the case shown below, a jet engine is sitting motionless on the ground prior to take-off. Air is entrained into the engine by the compressor. The inlet can be assumed to be frictionless and adiabatic.

Atmosphere:

T_{atm}
 p_{atm}
 $M = 0$



Considering the state of the gas within the inlet, prior to passage into the compressor, as state (1), and working in the *reference frame of the motionless airplane*:

a) Is T_{T1} greater than, less than, or equal to T_{atm} ?

The stagnation temperature of the atmosphere, $T_{T\text{atm}}$, is equal to T_{atm} since it is moving the same speed as the reference frame (the motionless airplane). The steady flow energy equation tells us that if there is no heat or shaft work (the case for our adiabatic inlet) the stagnation enthalpy (and thus stagnation temperature for constant C_p) remains unchanged. Thus $T_{T1} = T_{T\text{atm}} = T_{\text{atm}}$

b) Is T_1 greater than, less than, or equal to T_{atm} ?

If $T_{T1} = T_{\text{atm}}$ then $T_1 < T_{\text{atm}}$ since the flow is moving at station 1 and therefore some of the total energy is composed of kinetic energy (at the expense of internal energy, thus lowering T_1)

c) Is p_{T1} greater than, less than, or equal to p_{atm} ?

Equal, by the same argument as a).

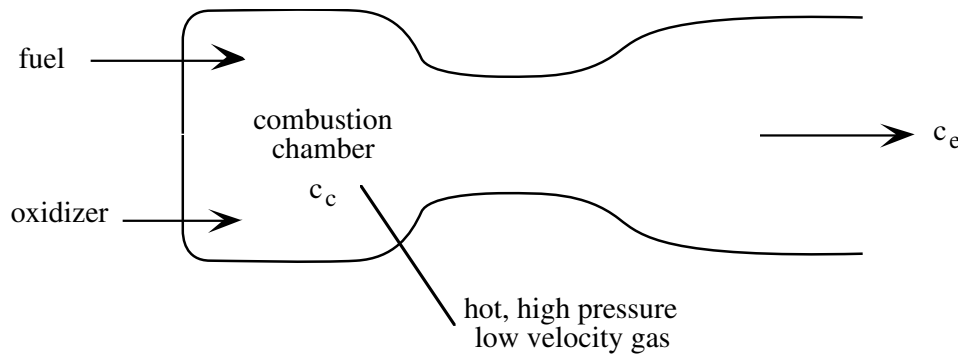
d) Is p_1 greater than, less than, or equal to p_{atm} ?

Less than, by the same argument as b).

C. Example Applications of the Steady Flow Energy Equation (VW, S & B: 6.4)

1. Flow through a rocket nozzle

A liquid bi-propellant rocket consists of a thrust chamber and nozzle and some means for forcing the liquid propellants into the chamber where they react, converting chemical energy to thermal energy.



Once the rocket is operating we can assume that all of the flow processes are steady, so it is appropriate to use the steady flow energy equation. Also, for now we will assume that the gas behaves as an ideal gas, though in general this is a poor approximation. There is no external work, and we assume that the flow is adiabatic.

Then we can write the First Law as

$$q_{1-2} - w_{s1-2} = h_{T2} - h_{T1} \quad \text{which becomes} \quad h_{T2} = h_{T1}$$

or

$$c_p T_c + \frac{c_c^2}{2} = c_p T_e + \frac{c_e^2}{2}$$

therefore

$$c_e = \sqrt{2c_p(T_c - T_e)}$$

If we assume quasi-static, adiabatic expansion then

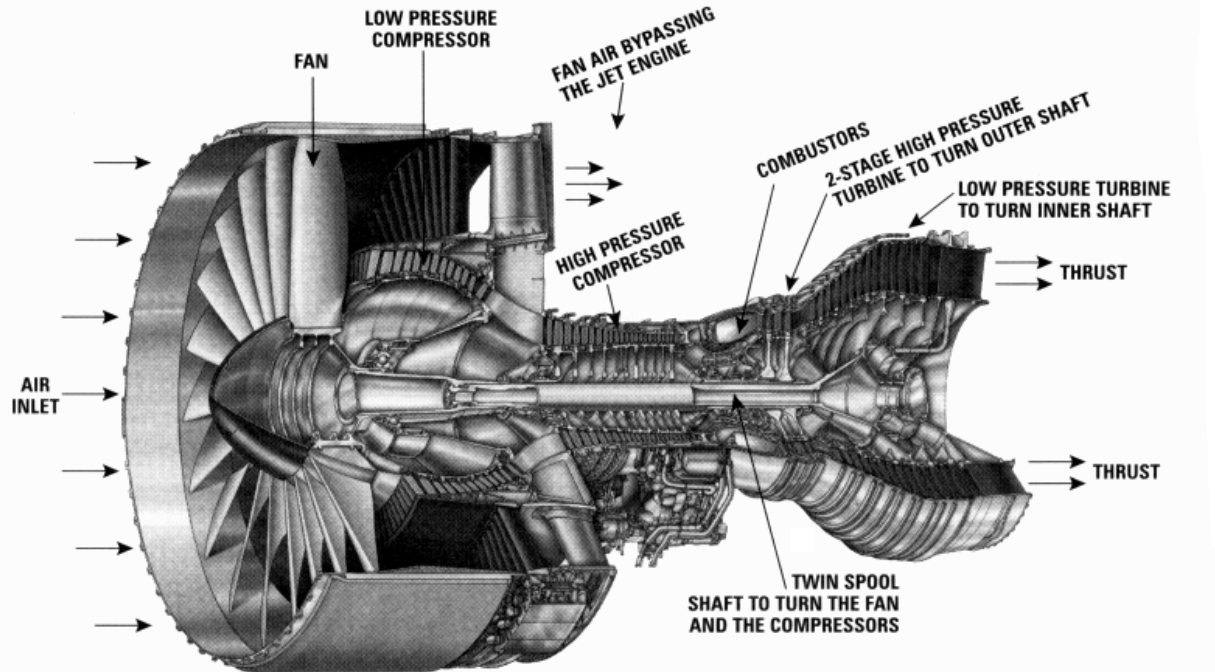
$$\frac{T_e}{T_c} = \left(\frac{p_e}{p_c}\right)^{\frac{\gamma-1}{\gamma}} \quad \text{so}$$

$$c_e = \sqrt{2c_p T_c \left[1 - \left(\frac{p_e}{p_c}\right)^{\frac{\gamma-1}{\gamma}} \right]}$$

Where T_c and p_c are conditions in the combustion chamber (set by propellants), and p_e is the external static pressure.

2. Power to drive a gas turbine compressor

Consider for example the PW4084 pictured below. The engine is designed to produce about 84,000 lbs of thrust at takeoff.



(drawing courtesy of Pratt and Whitney)

The engine is a two-spool design. The fan and low pressure compressor are driven by the low pressure turbine. The high pressure compressor is driven by the high pressure turbine.

$$\pi_f = \text{total pressure ratio across the fan} \approx 1.4$$

$$\pi_c = \text{total pressure ratio across the fan + compressor} \approx 45$$

$$\dot{m}_f = 610 \text{ kg/s} \quad \dot{m}_{\text{core}} = 120 \text{ kg/s}$$

$$T_{\text{inlet}} = 300\text{K}$$

Heat transfer from the gas streams is negligible so we write the First Law (steady flow energy equation) as:

$$\dot{Q} - \dot{W}_s = \dot{m}(h_{T2} - h_{T1})$$

For this problem we must consider two streams, the fan stream and the core stream, so

$$\begin{aligned}
 -\dot{W}_s &= \dot{m}_f \Delta h_{Tf} + \dot{m}_c \Delta h_{Tc} \\
 &= \dot{m}_f c_p \Delta T_{Tf} + \dot{m}_c c_p \Delta T_{Tc}
 \end{aligned}$$

We obtain the temperature change by assuming that the compression process is quasi-static and adiabatic. So

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}}$$

$$\text{then } \left(\frac{T_{T_2}}{T_{T_1}} \right)_{\text{fan}} = (\pi_f)^{\frac{\gamma-1}{\gamma}} = 1.1 \quad \Rightarrow \Delta T_{T_{\text{fan}}} = 30\text{K}$$

$$\left(\frac{T_{T_2}}{T_{T_1}} \right)_{\text{core}} = (\pi_{\text{core}})^{\frac{\gamma-1}{\gamma}} = 3.0 \quad \Rightarrow \Delta T_{T_{\text{core}}} = 600\text{K}$$

$$\begin{aligned}
 -\dot{W}_s &= 610 \frac{\text{kg}}{\text{s}} \cdot 30\text{K} \cdot 1008 \frac{\text{J}}{\text{kg K}} + 120 \frac{\text{kg}}{\text{s}} \cdot 600\text{K} \cdot 1008 \frac{\text{J}}{\text{kg K}} \\
 &= 91 \times 10^6 \text{ Joules/sec}
 \end{aligned}$$

$$\dot{W}_s = -91 \text{ Megawatts} \quad \left(\begin{array}{l} \text{negative sign implies} \\ \text{work done on the fluid} \end{array} \right)$$

Note that 1 Hp = 745 watts

If a car engine $\approx 110\text{Hp} = 8.2 \times 10^4$ watts,

then the power needed to drive compressor is equivalent to 1110 automobile engines.

All of this power is generated by the low pressure and high pressure turbines.