Fluids Lab 1 – Lecture Notes

1. Bernoulli Equation
2. Pitot-Static Tube
3. Airspeed Measurement
4. Pressure Nondimensionalization

Reading: Anderson 3.2, 1.5
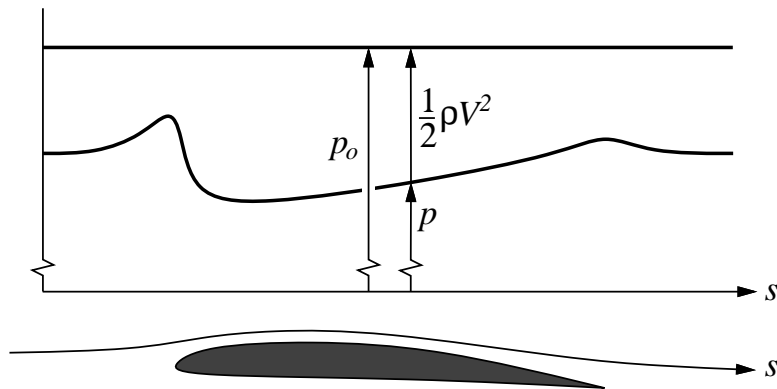
Bernoulli Equation

Definition

For every point s along a streamline in constant-density frictionless flow, the local speed $V(s) = |\vec{V}|$ and local pressure $p(s)$ are related by the *Bernoulli Equation*

$$p + \frac{1}{2}\rho V^2 = p_o \tag{1}$$

This p_o is a constant for all points along the streamline, even though p and V may vary. This is illustrated in the plot of $p(s)$ and $p_o(s)$ along a streamline near a wing, for instance.

Standard terminology is as follows.

$$
\begin{aligned}
p &= \text{static pressure} \\
\frac{1}{2}\rho V^2 &= \text{dynamic pressure} \\
p_o &= \text{stagnation pressure, or total pressure}
\end{aligned}
$$

Also, a commonly-used shorthand for the dynamic pressure is

$$q \equiv \frac{1}{2}\rho V^2$$

so we can also write the Bernoulli equation in the following compact form.

$$p + q = p_o$$

<u>Uniform Upstream Flow Case</u>

Many practical flow situations have uniform flow somewhere upstream, with $\vec{V}(x, y, z) = \vec{V}_\infty$ and $p(x, y, z) = p_\infty$ at every upstream point. This uniform flow can be either at rest with $\vec{V}_\infty \simeq 0$ (as in a reservoir), or be moving with uniform velocity $\vec{V}_\infty \neq 0$ (as in a upstream wind tunnel section), as shown in the figure.

Reservoir/Jet   Wind Tunnel
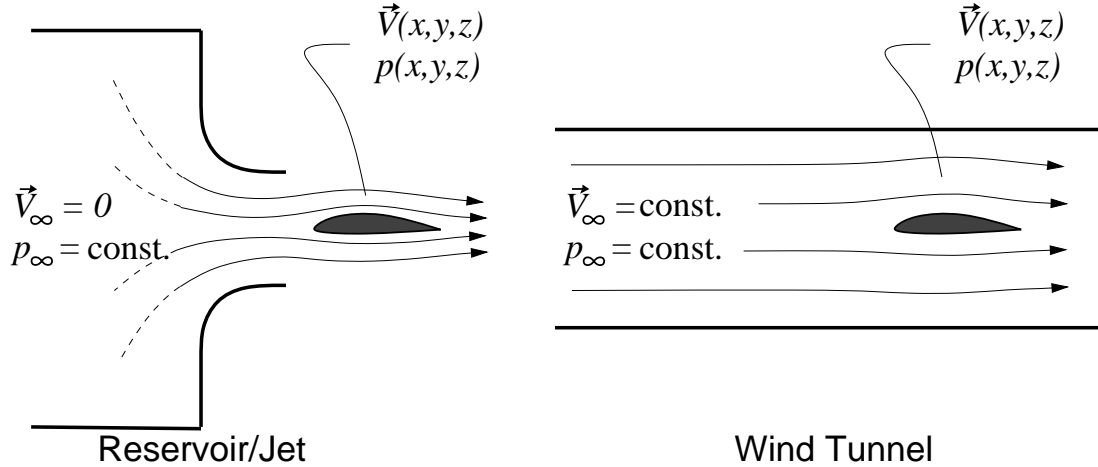
In these situations, p_o is the same for all streamlines, and can be evaluated using the upstream conditions in equation (1).

$$p_o \ = \ p_{o_\infty} \ = \ p_\infty + \frac{1}{2}\rho V_\infty^2$$

This gives an alternative form of Bernoulli Equation which in these flow situations is valid for all downstream points.

$$p \ + \ \frac{1}{2}\rho V^2 \ = \ p_{o_\infty} \qquad \text{(uniform upstream flow)} \qquad (2)$$

If the upstream quantities are known, then p_{o_∞} is also known, and this equation then uniquely relates the pressure field $p(x, y, z)$ and the speed field $V(x, y, z)$.

<u>Restrictions</u>

The Bernoulli Equation (2) is subject to several restrictions. As stated above, one restriction is that the upstream flow is uniform, so that p_∞ and V_∞ are well-defined constants. Another restriction is that the streamline containing the point where equation (2) is applied has not been influenced by friction, or fluid viscosity, anywhere upstream. Fortunately, these viscosity-influenced points are restricted to thin regions called *boundary layers*, which are adjacent to solid surfaces as shown in the figure. In these shaded regions the stagnation
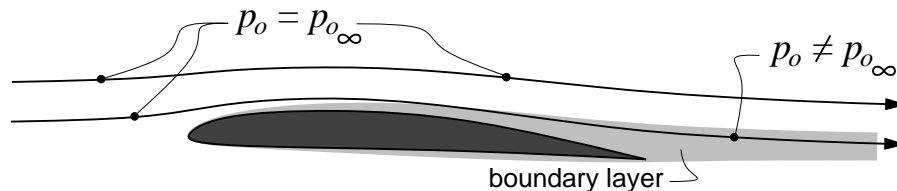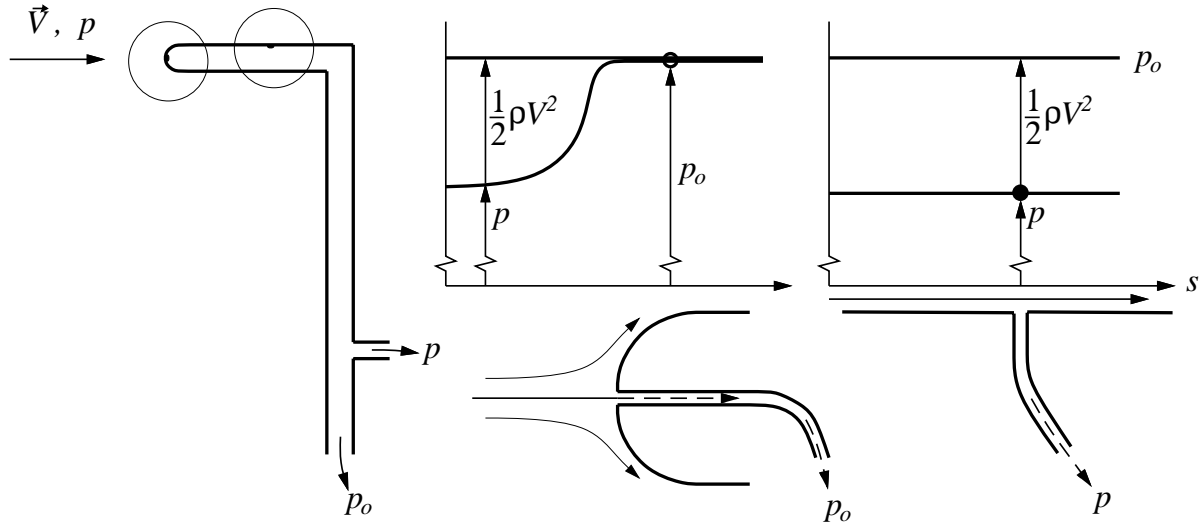

pressure p_o has some unknown value which differs from the constant freestream value,

$$p_o \ \neq \ p_{o_\infty} \qquad \text{(inside boundary layers)}$$

so that the general Bernoulli Equation (1) is not usable to relate p and V.

Pitot-Static Tube

A *Pitot-Static* tube is a device which allows measurement of both the static and stagnation pressures p and p_o at any point in the flowfield.

The p_o port is in the center of the front blunt end, which is faced into the oncoming flow. The flow stagnates (V drops to zero) at the port location, so that locally

$$p \;=\; p_o \qquad\qquad \text{(at stagnation port, where } V = 0\text{)}$$

which is then transmitted by the port tube to the pressure-sensing device. The static port for p is on the side of the stream-aligned tube, so there is no local change in V, and so the local p is transmitted into the port tube.

Airspeed Measurement

The main purpose of a pitot-static tube is to measure airspeed. Using the difference between the measured p_o and p, the speed at the location of the pitot-static tube can be computed using the Bernoulli Equation.

$$V \;=\; \sqrt{\frac{2(p_o - p)}{\rho}}$$

To allow safe operation of an aircraft, it is necessary to measure the flight speed. This is simply V_∞, or the speed of the airmass relative to the moving aircraft. This can be computed from measured p_{o_∞} and p_∞:

$$V_\infty \;=\; \sqrt{\frac{2(p_{o_\infty} - p_\infty)}{\rho}}$$

On an aircraft, measurement of p_{o_∞} poses little difficulty. We know that

$$p_o(x, y, z) \;=\; p_{o_\infty}$$

for any point x, y, z which lies outside the thin boundary layer on the aircraft skin. So the pitot tube can be put into the flow almost anywhere on the aircraft to measure the local p_o and hence p_{o_∞}. In contrast, measuring the static pressure p_∞ is far more difficult, since it is not possible to put a fragile pitot-static probe "at infinity", or even "far away" from the aircraft's pressure field.

3

The method actually used is to place several static ports on the aircraft skin at strategic locations, such that the various different local p measurements can be combined to provide a sufficiently accurate estimate of the true freestream p_∞.

Pressure Nondimensionalization

Pressure Coefficient

Forces exerted by the fluid on an object typically depend only on *pressure differences* within the fluid rather than the absolute pressure itself. For example, the lift on a wing depends on the pressure difference between the top and bottom surface. At low airspeeds (low Mach numbers to be more precise), the absolute pressure is very much greater than the pressure differences of interest. Hence, it is convenient to *bias* and *normalize* the pressure p by using p_∞ and q_∞, which defines the dimensionless *pressure coefficient* variable.

$$C_p = \frac{p - p_\infty}{q_\infty} \tag{3}$$
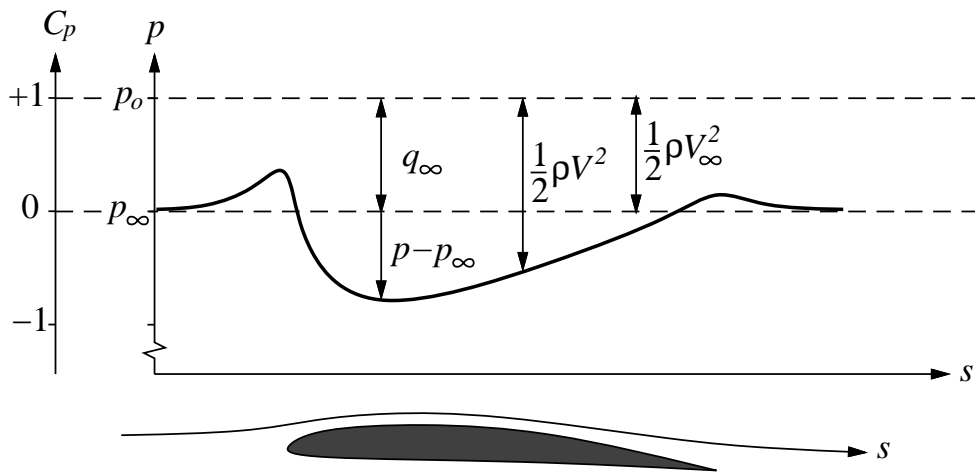
The figure shows how the p axis is shifted and rescaled into the C_p axis. Using the Bernoulli Equation we can express p as

$$p = p_\infty + q_\infty - \frac{1}{2}\rho V^2$$

which can be used to eliminate p from the C_p definition (3). This then gives C_p in terms of the speeds alone.

$$C_p = 1 - \frac{\frac{1}{2}\rho V^2}{q_\infty} = 1 - \frac{V^2}{V_\infty^2} \tag{4}$$

So C_p is not only a normalized pressure variable, but a normalized speed^2 variable as well.

Surface Pressure Distributions

The most convenient way to present pressure distributions on a body such as an airfoil is with C_p. The figure shows the C_p distributions on a NACA 4412 airfoil for three angles of attack, 1°, 3°, and 5°. The traditional presentation is with negative C_p upwards, so that the typically negative C_p of the top surface is on the top of the plot, and vice versa.

4

XFOIL
V 6.95

NACA 4412
Ma = 0.000

α	C_L	C_M
5.000	1.1111	-0.120
3.000	0.8714	-0.116
1.000	0.6306	-0.113

Top surface Cp

Bottom surface Cp

C_p

-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0