a) Given: $\phi_1(x,y)$, $\phi_2(x,y)$, $p_1(x,y)$, $p_2(x,y)$

$\nabla^2\phi_1 = 0$, $\nabla^2\phi_2 = 0$ (satisfy mass conservation)

Define: $\phi_3 = \phi_1 + \phi_2$

Determine p_3

$$\Rightarrow p_3 = p_0 - \frac{1}{2}\rho|\nabla\phi_3|^2 = p_0 - \frac{1}{2}\rho(\nabla\phi_3 \cdot \nabla\phi_3)$$

$$= p_0 - \frac{1}{2}\rho\left[(\nabla\phi_1 + \nabla\phi_2)\cdot(\nabla\phi_1 + \nabla\phi_2)\right]$$

$$\boxed{p_3 = p_0 - \frac{1}{2}\rho\left[|\nabla\phi_1|^2 + |\nabla\phi_2|^2 + 2\nabla\phi_1\cdot\nabla\phi_2\right]}$$

Note: $p_3 \neq p_1 + p_2$!

b) Given $\phi_4 = \partial\phi_1/\partial x$

Is it physically realizable?

Test: $\nabla^2\phi_4 \stackrel{?}{=} 0$

$$\nabla^2(\partial\phi_1/\partial x) \stackrel{?}{=} 0$$

$$\boxed{\frac{\partial}{\partial x}(\nabla^2\phi_1) \stackrel{\checkmark}{=} 0}$$

$\frac{\partial()}{\partial x}$ and $\nabla^2()$ commute.

Since $\nabla^2\phi_1 = 0$ as given

Example: $\phi_1 = \frac{\Lambda}{2\pi}\ln\sqrt{x^2+y^2} = \frac{\Lambda}{2\pi}\ln r$ source

$\phi_4 = \frac{\partial\phi_1}{\partial x} = \frac{\Lambda}{2\pi}\frac{x}{x^2+y^2} = \frac{\Lambda}{2\pi}\frac{\cos\theta}{r}$ doublet !

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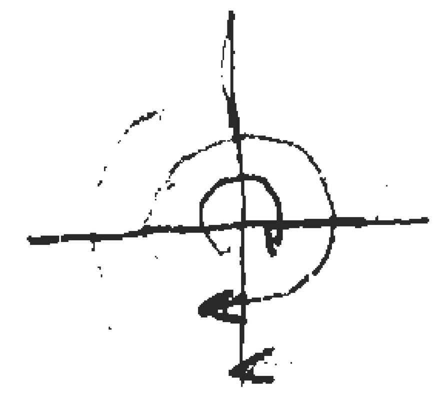
$$u_1 = \frac{y}{x^2+y^2} \qquad v_1 = \frac{-x}{x^2+y^2} \qquad \text{"}\tfrac{1}{r}\text{ vortex"}$$



$$u_2 = V_\infty \qquad v_2 = 0 \qquad \text{uniform flow} \longrightarrow$$

$$\boxed{u_3 = u_1 + u_2 = \frac{y}{x^2+y^2} + V_\infty} \quad \boxed{v_3 = v_1 + v_2 = \frac{-x}{x^2+y^2}}$$

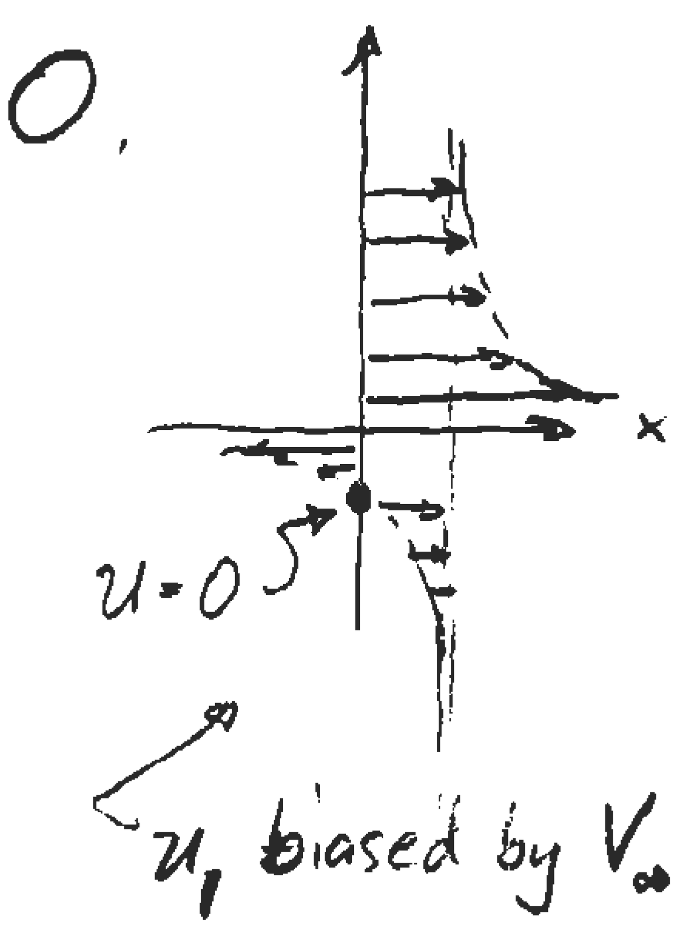
$$p_3 = p_0 - \tfrac{1}{2}\rho\left(u_3^2 + v_3^2\right) = p_0 - \tfrac{1}{2}\rho\left[\frac{y^2}{(x^2+y^2)^2} + \frac{2yV_\infty}{x^2+y^2} + V_\infty^2 + \frac{x^2}{(x^2+y^2)^2}\right]$$

$$\boxed{p_3 = p_0 - \tfrac{1}{2}\rho\left[\frac{1+2yV_\infty}{x^2+y^2} + V_\infty^2\right]}$$

Maximum pressure is where $u_3^2 + v_3^2$ is minimum.

Note that on y-axis where $x=0$, we have $v_3 = 0$.
Also, at $y = -1/V_\infty$ we also have $u_3 = 0$

$$\Rightarrow \text{max } p_3 \text{ at } x, y = 0, \frac{-1}{V_\infty}$$



Alternative mathematical approach (hard way)

set $\frac{\partial p_3}{\partial x} = 0$ and $\frac{\partial p_3}{\partial y} = 0$

$$\frac{\partial p_3}{\partial x} = -\tfrac{1}{2}\rho\,\frac{1+2yV_\infty}{(x^2+y^2)^2}(-2x) = 0 \Rightarrow (1+2yV_\infty)\,x = 0 \qquad (1)$$

$$\frac{\partial p_3}{\partial y} = -\tfrac{1}{2}\rho\,\frac{1+2yV_\infty}{(x^2+y^2)^2}(-2y) - \tfrac{1}{2}\rho\,\frac{2V_\infty}{x^2+y^2} = 0 \Rightarrow (1+2yV_\infty)\,y - (x^2+y^2)V_\infty = 0$$

$$\text{or} \quad (1+yV_\infty)\,y - x^2V_\infty = 0 \qquad (2)$$

Two possibilities from equation (1)

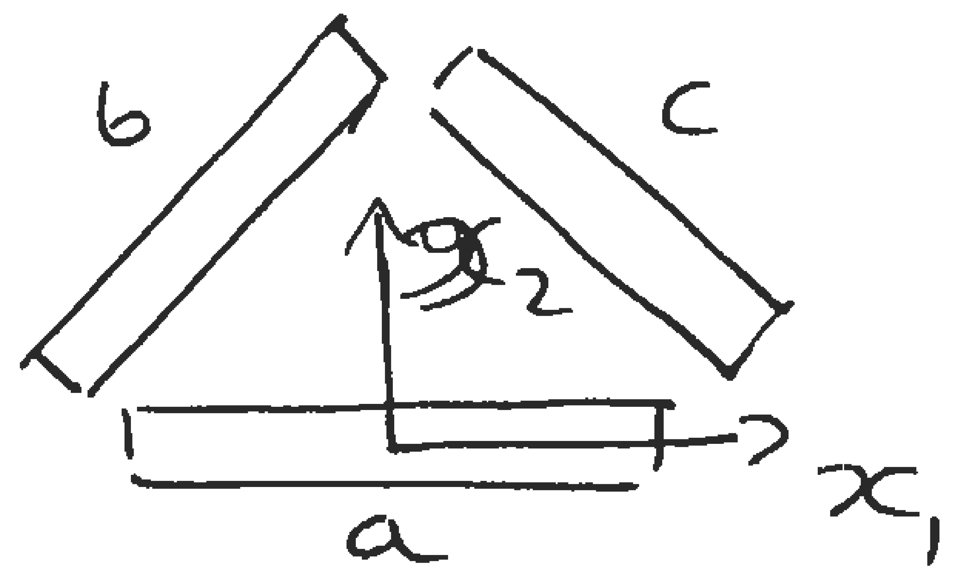
a) $1+2yV_\infty = 0$, $x \neq 0$ $\rightarrow$ $y = \frac{-1}{2V_\infty}$

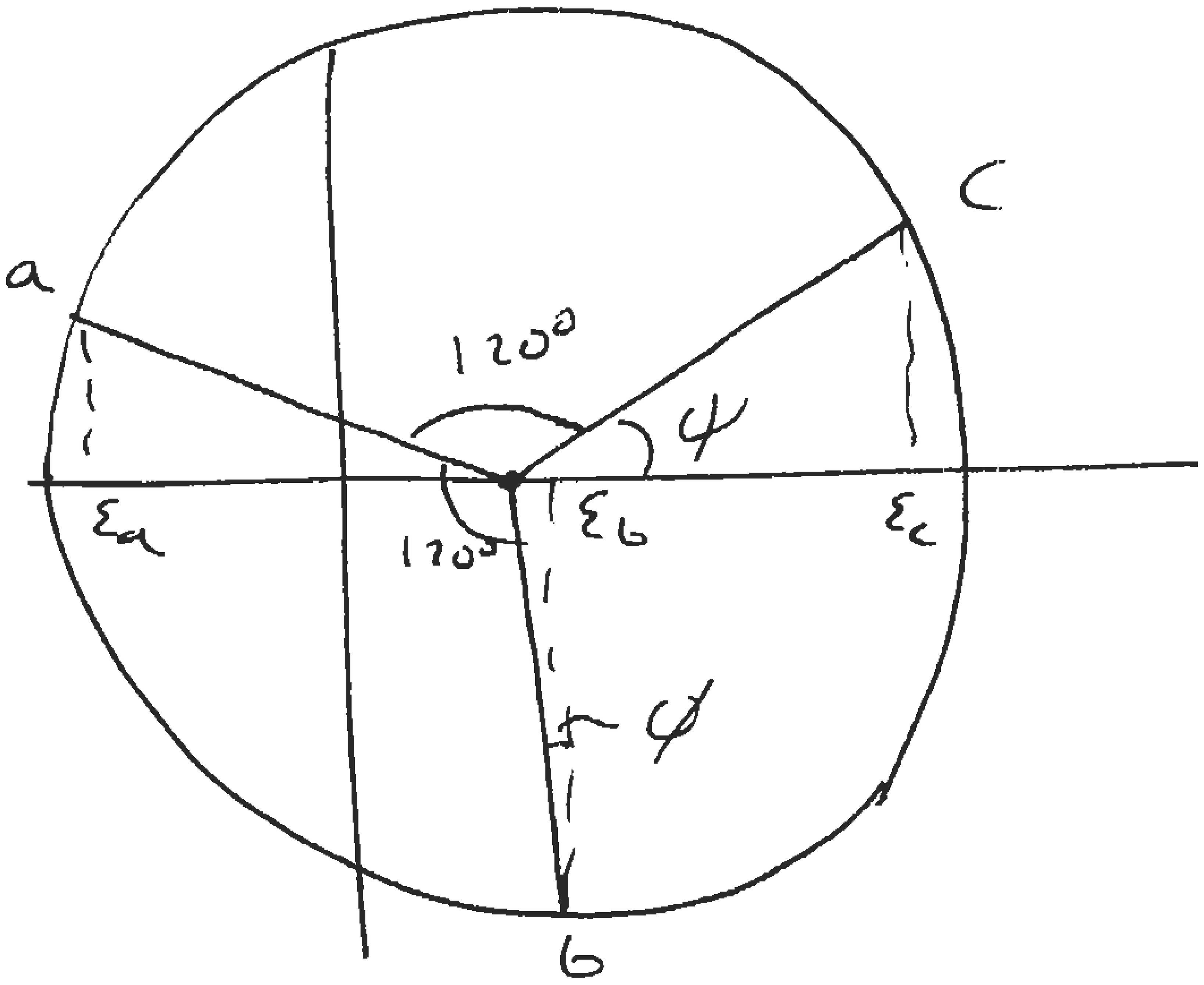
Plug into equation (2) $\rightarrow$ $\frac{1}{2}\cdot\left(\frac{-1}{2V}\right) - x^2V_\infty = 0 \rightarrow x^2 = \frac{-1}{4V_\infty}$ no real solution

b) $1+2yV_\infty \neq 0$, $\boxed{x = 0}$

Plug into equation (2) $\rightarrow$ $(1+yV_\infty)\,y = 0$ $\nearrow$ $y \overset{?}{=} 0$ Nope. Inconsistent with (1).
$\searrow$ $1+yV_\infty = 0 \rightarrow \boxed{y = \frac{-1}{V_\infty}}$ ✓

M17





Mohr's circles read

a) 60° rosette - plots as 120° on Mohr's circle

ε_a aligned with x_1 $\therefore$ reads $\varepsilon_{11} = -200\mu\varepsilon$ $\Leftarrow$

ε_b 60° ~~clock~~ counter clockwise

angle $\phi = 120^\circ - 106.85^\circ = 13.15^\circ$

$\therefore \varepsilon_b = \underset{C}{100} + \underset{R}{361} \sin 13.5^\circ = 182.1\,\mu\varepsilon$. $\Leftarrow$

angle $\psi = 180 - 120 - 16.85 = 43.2^\circ$

$\varepsilon_c = 100 + 361 \cos 43.2^\circ = 363.4\,\mu\varepsilon$ $\Leftarrow$

b) $\varepsilon_{mn} = \begin{pmatrix} -200 & -200 & 0 \\ -200 & +400 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

ε_{I} eigenvalues solution to $0 = |M - \lambda I|$

$$(-200 - \lambda)(+400 - \lambda) - (-200)^2 = 0$$

$$\lambda^2 - 200\lambda - 120000 = 0$$

$$\lambda = \frac{+200 \pm \sqrt{200^2 + 4 \times 120000}}{2}$$

$$= 100 \pm 361$$

$= -261\,\mu\varepsilon$ and $+461\,\mu\varepsilon$!!

c) for $\varepsilon_{33} = 300\,\mu\varepsilon$ and $\varepsilon_{23} = \varepsilon_{13} = 0$

ε_{33} is a principal strain (no associated shear

$\therefore\ \varepsilon_{I} = 461\,\mu\varepsilon$ + $\varepsilon_{II} = -261\,\mu\varepsilon$ as before

$\varepsilon_{III} = 300\,\mu\varepsilon$.

M18 i) Youngs modulus is controlled by bonding and crystal structure (atomic packing)

ii) The glass transition temperature is the temperature at which the polymer changes from being an elastic solid to a viscoelastic one (at higher temps) which is due to the breaking (melting) of the van-der-Waals bonds between the polymer molecules

iii) Metals have higher densities because they have higher atomic masses and close packed crystal structures

iv) Interatomic bond energy $U(\underbrace{r}_{\text{separation}})$

Interatomic force $= \dfrac{dU(r)}{dr}$