F10 – Lecture Notes

1. Aircraft Performance Analysis

Aircraft Performance Analysis

Drag breakdown

The drag on a subsonic aircraft can be broken down as follows.

$$D = D_o + D_p + D_i$$

where
D_o = "parasite" drag of fuselage + tail + landing gear + ...
D_p = wing profile drag
D_i = induced drag

We now use the wing airfoil drag polar $c_d(c_\ell; Re)$ to give the wing profile drag, and use lifting line to give the induced drag. The nondimensional total drag coefficient is then

$$\frac{D}{\frac{1}{2}\rho V^2 S} \equiv C_D = C_{Do}\frac{S_o}{S} + c_d(C_L; Re) + \frac{C_L^2}{\pi e AR} \tag{1}$$

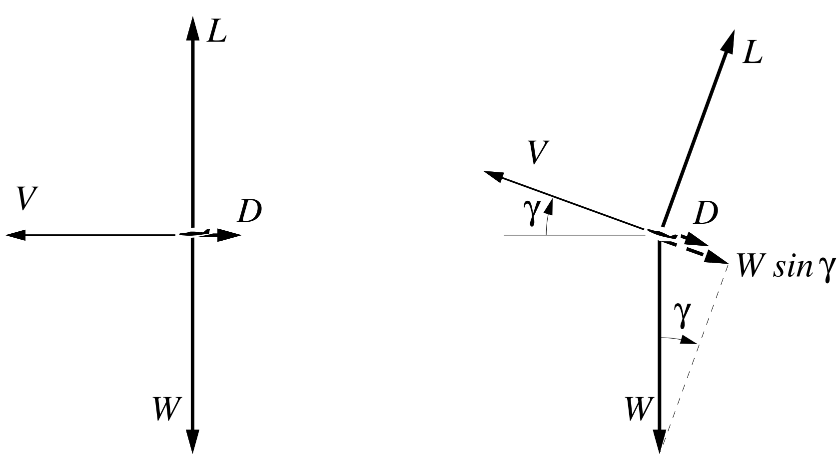
where the "∞" subscript on the flight speed V has been dropped. The parasite drag coefficient C_{Do} is based on its own reference area S_o, which may or may not be the same as the wing's reference area S. The factor S_o/S allows any convenient S_o to be used.

Flight power

The mechanical power P needed for constant-velocity flight is given by

$$\eta_p P = V(D + W \sin\gamma) \tag{2}$$

where W is the weight, γ is the *climb angle*, and η_p is the propulsive efficiency. Typically P is defined as the motor shaft power, in which case η_p is the propeller efficiency.



In level flight, $\gamma = 0$, and the power is

$$\eta_p P = V D = \frac{1}{2}\rho V^3 S C_D \tag{3}$$

The flight speed V is given by the Lift = Weight condition, together with the definition of the lift coefficient C_L.

$$L \;=\; \frac{1}{2}\rho V^2 S\, C_L \;=\; W$$

$$V \;=\; \left(\frac{2\,W/S}{\rho\, C_L}\right)^{1/2}$$

The ratio W/S is called the *wing loading*, and has the units of force/area, or pressure. The level-flight power equation (3) then takes the following form.

$$P \;=\; \frac{1}{\eta_{\rm p}} \left(\frac{2\,W/S}{\rho}\right)^{1/2} W\,\frac{C_D}{C_L^{3/2}} \qquad (4)$$

Power dependencies

Equation (4) indicates how the level flight power depends on the quantities of interest. The following dependencies are particularly worthy to note:

$$P \;\sim\; \frac{1}{\eta_{\rm p}}$$

$$P \;\sim\; \frac{1}{S^{1/2}}$$

$$P \;\sim\; W^{3/2}$$

$$P \;\sim\; \frac{C_D}{C_L^{3/2}}$$

Note the very strong effect of overall weight W. This indicates that to reduce flight power, considerable effort should be directed towards weight reduction if that's possible.

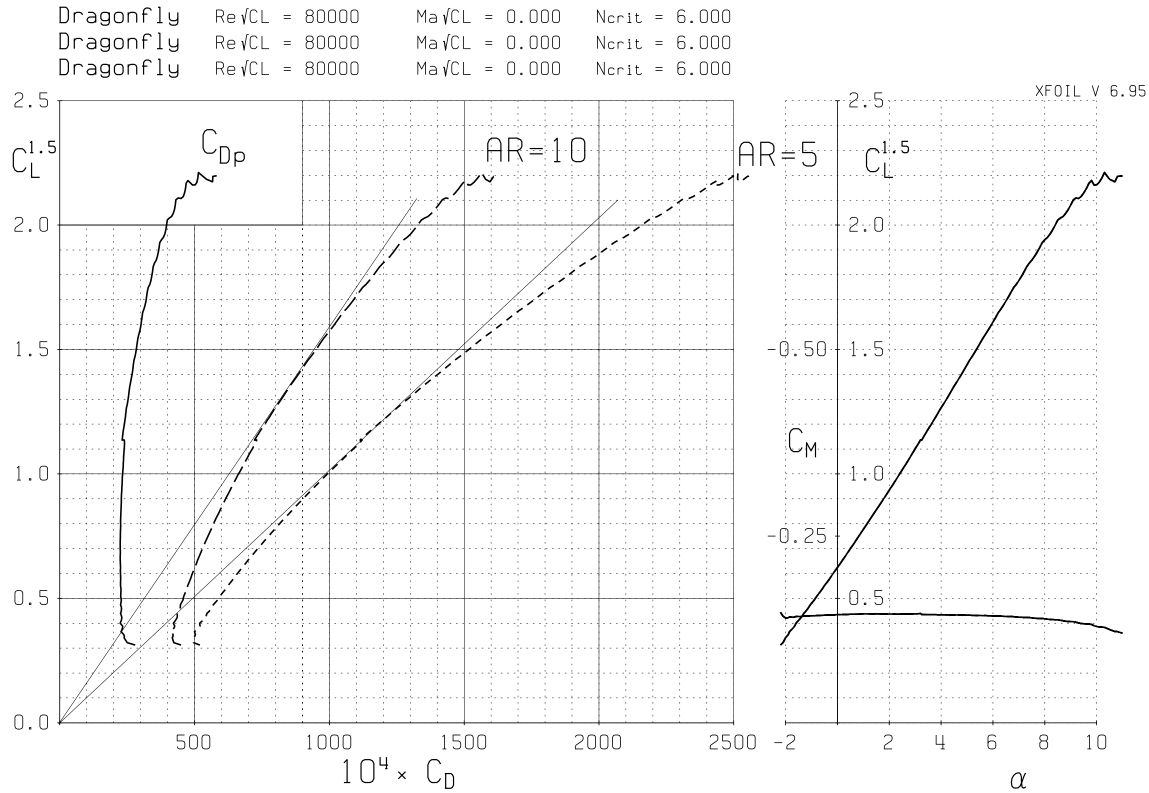
An important consideration is that the proportionality with each parameter assumes that all the other parameters are held fixed, which is difficult if not impossible to do in practice. For example, increasing the wing area S is likely to also increase the weight W, so the net influence on the flight power requires a closer analysis of the area-weight relation.

The flight power is seen to vary as the inverse of the ratio $C_L^{3/2}/C_D$, must be maximized to obtain minimum-power flight, or maximum-duration flight. But again, increasing the maximum achievable $C_L^{3/2}/C_D$ will typically require changes in the other aircraft parameters. For example, increasing the aspect ratio $A\!R$ will typically increase the maximum $C_L^{3/2}/C_D$, but it will also increase the weight W which offsets the benefit.

Drag and power polars

One convenient way to examine the aircraft's power-requirement characteristics is in a *power polar*. This is a variation on the more common drag polar, with the vertical C_L axis being replaced by $C_L^{3/2}$. An example power polar is shown below, showing three curves:

Only C_{Dp}, which assumes $C_{Do} = 0$ and $A\!R = \infty$.
Total C_D, assuming $C_{Do} S_o/S = 0.01$, and $e\,A\!R = 10$.
Total C_D, assuming $C_{Do} S_o/S = 0.01$, and $e\,A\!R = 5$.



The slope of the line tangent to each polar curve indicates the maximum $C_L^{3/2}/C_D$ value, and the point of contact gives the C_L at which this condition occurs. For the two example polars, we have:

For $AR = 10$: $(C_L^{3/2}/C_D)_{\max} = 16.0$ at $C_L \simeq 0.90$
For $AR = 5$: $(C_L^{3/2}/C_D)_{\max} = 10.0$ at $C_L \simeq 0.70$

The $AR = 10$ aircraft can therefore be expected to have a power requirement which is lower by a factor of $10.0/16.0 = 0.625$. The expected duration is longer by the reciprocal factor $16.0/10.0 = 1.6$. But this assumes that the larger AR has no other adverse effect on the other important parameters affecting P, which is very unlikely.

It's useful to compare the effects of increasing the parasite drag C_{Do} versus increasing the induced drag C_{Di} via aspect ratio. The optimum C_L (i.e. optimum speed) varies in opposite direction. In general, larger C_{Do} favors lower speed, while smaller AR favors higher speed.