NAME SOLUTIONS (WAITZ)

Unified Propulsion Quiz

April 25, 2001

One 81/2" x 11" sheet (two sides) of notes
Calculators allowed.

- **Put your name on each page of the exam.**
- **Read all questions carefully.**
- **Do all work for each problem on the pages provided.**
- **Show intermediate results.**
- **Explain your work --- don't just write equations.**
- **Partial credit will be given (unless otherwise noted), but only when the intermediate results and explanations are clear.**
- **Please be neat. It will be easier to identify correct or partially correct responses when the response is neat.**
- **Show appropriate units with your final answers.**
- **Box your final answers.**

Exam Scoring

#1 (10%)	
#2 (30%)	
#3 (15%)	
#4 (15%)	
#5 (30%)	
Total	

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1. (10 points, partial credit given, L.O. A) In a few sentences describe how jet propulsion works.

FORCES ARE RELATED TO CHANGES IN MOMENTUM. IN THE CASE OF A ROCKET, MASS STORED IN THE VEHICLE IS THROWN BACKWARDS PROPELLING THE VEHICLE FORWARDS (LIKE THROWING A HEAVY STONE FROM A BOAT FLOATING ON THE WATER — THE BOAT MOVES IN THE OPPOSITE DIRECTION.) IN THE CASE OF A JET ENGINE MASS (THE AIR) IS TAKEN IN WITH LOW MOMENTUM AND IT IS THROWN BACKWARDS WITH A HIGHER MOMENTUM. THE FORCE ASSOCIATED WITH THE CHANGE IN MOMENTUM OF THE AIR PASSING THROUGH THE ENGINE PROPELS THE VEHICLE FORWARD.

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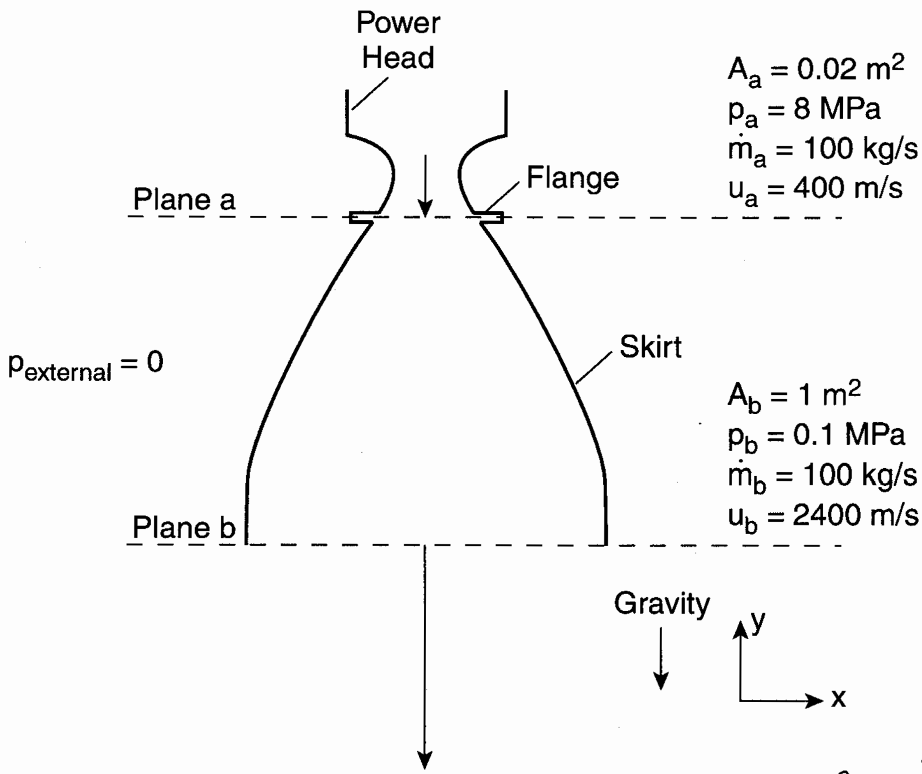
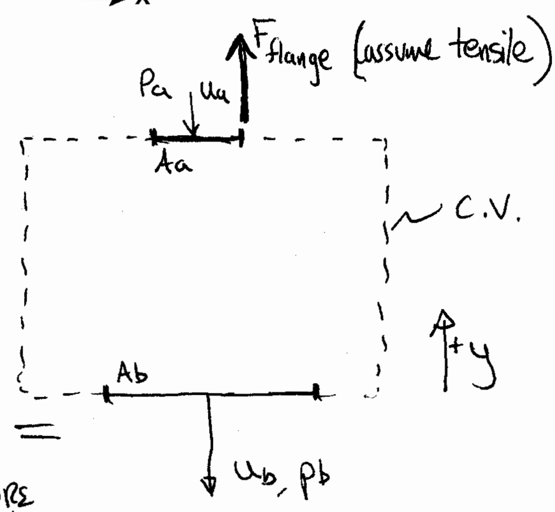
2. (30 points, partial credit given, L.O. B) A rocket motor is shown below. The "skirt" is attached to the rest of the power head of the engine by a flange at plane "a-a". The mass of the skirt is 500 kg, it is operating in a 1 g environment (where $g = 10 m/s^2$) in space (i.e. the external pressure is zero), and it is oriented as shown. <u>What is the force on the flange? Is it in tension or compression?</u>





<u>INTEGRAL MOMENTUM THEOREM:</u>

$$\Sigma F_y = \iint_S \rho u_y \bar{u}\cdot\bar{n}\, ds$$

$$F_{flange} - m_{skirt} g - p_a A_a + p_b A_b =$$

(ASSUME TO BE IN DIRECTION CORRESPONDING TO A TENSILE LOAD) (+ y direction)

WEIGHT OF SKIRT (− y direction)

PRESSURE AT a ACTS IN − y direction

PRESSURE AT b ACTS IN +y direction

$$= \underbrace{u_a}_{\substack{u_y \\ -y\ dir}} \underbrace{(-u_a)}_{\bar{u}\cdot\bar{n}} \rho A_a - \underbrace{u_b}_{\substack{u_y \\ -y\ dir}} \underbrace{(u_b)}_{\bar{u}\cdot\bar{n}} \rho A_b$$

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(BLANK PAGE FOR EXTRA WORK)

$$F_{flange} = \dot{m}u_a - \dot{m}u_b + P_aA_a - P_bA_b + m_{skirt}\,g$$

(check: with no flow F_{flange} is (+) → tensile due to weight of skirt ✓)

$$= 100(400-2400) + 8\times10^6(0.02) - 0.1\times10^6(1) + 500(10)$$

$$= -200\times10^3 + 160\times10^3 - 100\times10^3 + 5\times10^3 \quad (N)$$

$$F_{flange} = -135\times10^3\ N \quad (\text{WHERE + IS TENSION})$$

THEREFORE, THE FLANGE IS IN <u>COMPRESSION</u>.

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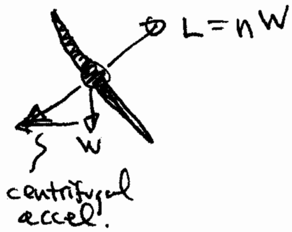
3. (15 points, partial credit given, L.O. D) Describe what happens to an airplane which is initially in a steady, constant altitude turn if it rolls to level and begins to fly straight at constant speed with no change in engine thrust setting. Why does this happen?

THE AIRPLANE'S ALTITUDE INCREASES.

LIFT FOR TURNING FLIGHT IS GREATER THAN IN STEADY LEVEL FLIGHT SINCE IT MUST BALANCE BOTH WEIGHT AND CENTRIFUGAL ACCELERATION SO AT CONSTANT L/D THEN DRAG IS INCREASED FOR TURNING FLIGHT RELATIVE TO STEADY LEVEL FLIGHT.



INITIALLY

$$T_iV_i - D_iV_i = \cancelto{0}{W\frac{dh}{dt}} + \cancelto{0}{\frac{d}{dt}\left(\frac{1}{2}\frac{W}{g}V^2\right)}$$

THEN T STAYS SAME, V STAYS SAME & ROLL TO LEVEL, SO DRAG DECREASES

$$\underbrace{TV}_{\text{SAME}} - \underbrace{DV}_{\text{DECREASES}} = W\frac{dh}{dt} + \cancelto{0}{\frac{d}{dt}\left(\frac{1}{2}\frac{W}{g}V^2\right)}$$

$\therefore \frac{dh}{dt}$ IS + $\longrightarrow$ A/C CLIMBS

BECAUSE $P_{AVAIL} > P_{REQ'D}$

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4. (15 points, partial credit given, L.O. F) Consider a single stage sounding rocket launched vertically with g = 10 m/s^2, no drag and an ideally expanded nozzle. If the specific impulse is 300 s, the total vehicle mass is 15000 kg, and the propellant mass flow rate is 100 kg/s, what is the maximum burn time before the vehicle experiences an acceleration of 5g ~~is reached~~?

$T = m_v a$, FOR CONSTANT THRUST AS VEHICLE GETS LIGHTER, ACCELERATION INCREASES.

PROCEDURE:

- a IS GIVEN (5g)
- FIND T FROM $T = \dot{m} u_e + \cancel{(p_e - p_o)A_e}$ — ideally expanded
- SOLVE FOR m_v FOR $a = 5g$
- THEN $\Delta m_v / \dot{m} = t_b$

$T = \dot{m} u_e = m_v(t)\, a \qquad u_e = I_{sp} g$

$$\frac{\dot{m} I_{sp} g}{5g} = m_v(t) = \frac{100 \cdot 300}{5} = 6000\ \text{kg}$$

(Mass of vehicle at time that 5g accel is achieved)

m_v initial $= 15000$ kg $\qquad m_{v_{5g}} = 6000$ kg

$\Delta m_v = 9000$ kg

$\dot{m} = 100$ kg/s $\qquad \therefore \boxed{t_b = 90 \text{ seconds}}$

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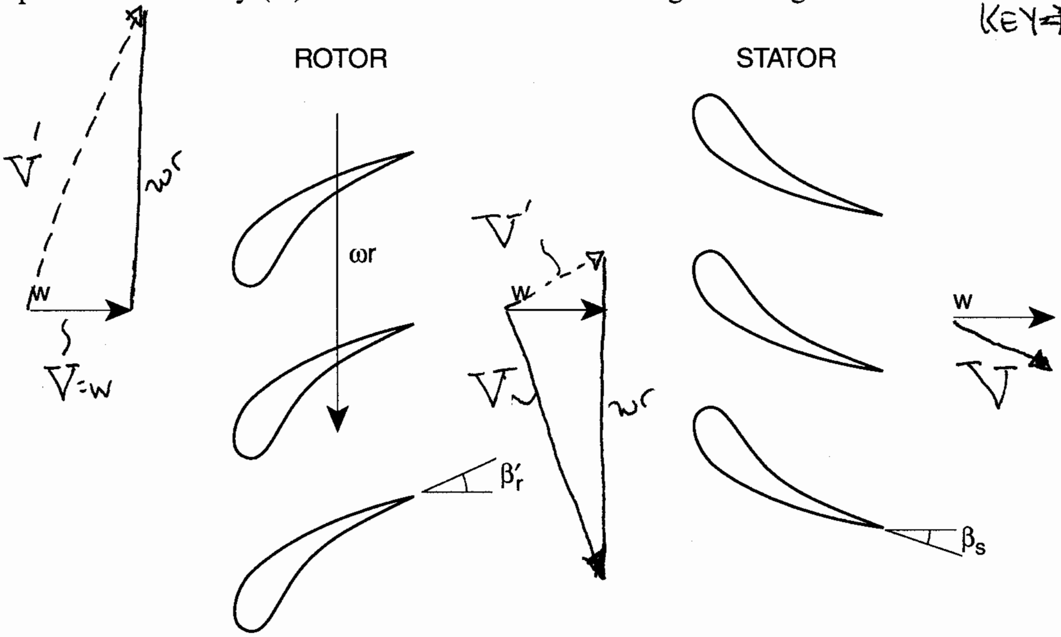
5. (30 points, partial credit given, L.O. B, I & J) Two rows turbomachine blades are shown below. At inlet to the rotor, the flow is purely axial. Assume that the axial component of velocity (w) then remains constant through the stage.

KEY ⇒ $(\ )'$ & DASHED = RELATIVE



a) Draw the velocity triangles. Is this a compressor or a turbine?

$u_{tangential}$ into rotor $= 0$ $\quad$ u_{tang} out of rotor > 0 $\Longrightarrow$ Compressor

b) Sketch the direction of the force that the flow produces on the stationary blades.

takes flow ↘ turns to →

c) Write an expression for the torque applied to the flow by the stationary blades? How much power is added to the flow by the stationary blades?

$$T = \dot{m} r \left(u_{tang.\,out} - u_{tang.\,in}\right)$$

$$P = T\omega = 0 \qquad (\omega = 0 \text{ for stationary blades})$$

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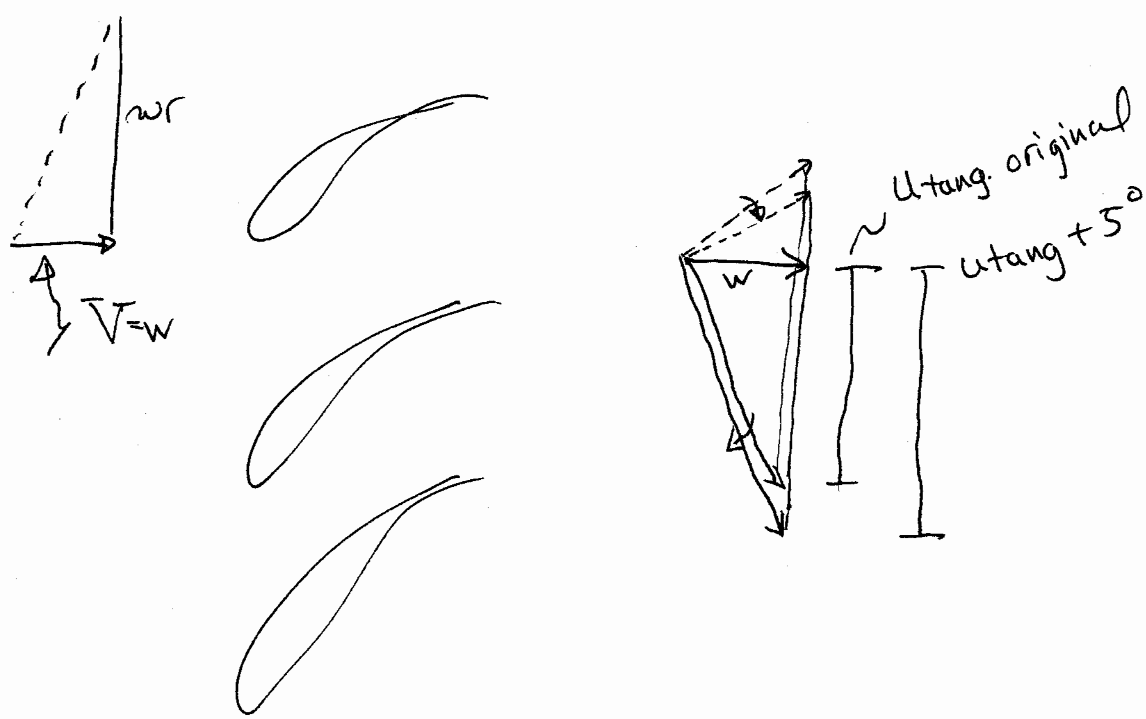
d) In the frame of the moving blades, is the flow accelerated or decelerated?

DECELERATED (BY INSPECTION OF RELATIVE VELOCITY VECTOR LENGTHS AT INLET AND OUTLET OF ROTOR)

e) If you were to increase the camber of the rotating blades by 5 degrees (holding everything else including the inlet angle of the rotating blades) the same, would the work per unit mass flow increase or decrease? (Show this graphically with velocity triangles.)

$u_{\text{TANGENTIAL IN}}$ STAYS THE SAME $= 0$



SO THE WORK PER UNIT MASS FLOW INCREASES: $P = \omega r \cdot \dot{m} \left(u_{\text{TANG OUT}} - u_{\text{TANG. IN}} \right)$