# F13 – Lecture Notes

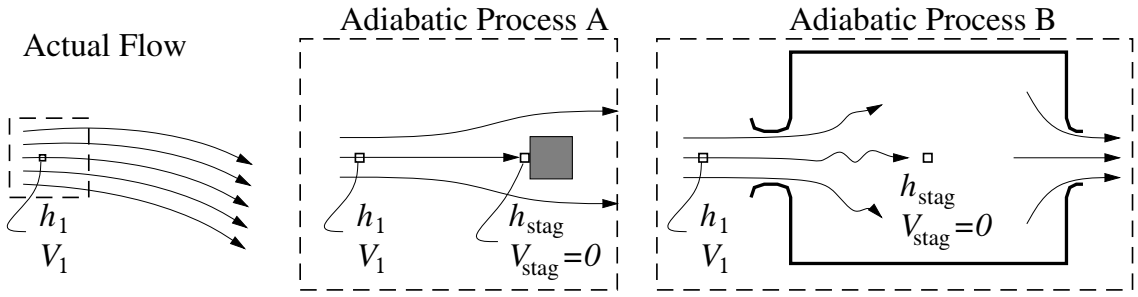
1. Stagnation Quantities
2. Introduction to Shock Waves

Reading: Anderson 7.5, 7.6

## Stagnation Quantities

### Adiabatic stagnation processes

An *adiabatic stagnation process* is one which brings a moving fluid element to rest adiabatically (without heat addition or removal). The figure a fluid element at station 1 in some flow being brought to rest by two hypothetical adiabatic processes. Process A is done by placing a blunt object in the flow, such that the fluid element reaches the stagnation point, where $V = 0$. Process B lets the fluid element flow into a large insulated chamber where it will mix with the stationary fluid there and thus come to rest.




The changes of $h$ and $V$ for either process are governed by the total enthalpy relation

$$h_o \equiv h + \frac{1}{2}V^2 = \text{constant}$$

derived previously. Therefore, we have

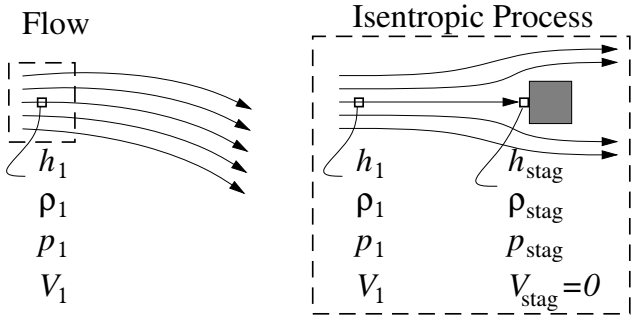
$$h_{o_1} \equiv h_1 + \frac{1}{2}V_1^2 = h_{\text{stag}} + \frac{1}{2}V_{\text{stag}}^2 = h_{\text{stag}}$$

We see that at the end of the stagnation process, $h_{\text{stag}}$ is equal to the total enthalpy $h_{o_1}$ at the beginning. For this reason, the terms *stagnation enthalpy* and *total enthalpy* are largely synonymous, although they are two distinct concepts.

The total enthalpy $h_o$ on the streamline can therefore be measured by setting up an actual stagnation process, typically with a small obstruction like a small-scale version of Process A, and measuring the resulting temperature $T_{\text{stag}}$. One can then calculate $h_o = h_{\text{stag}} = c_p T_{\text{stag}}$.

### Isentropic stagnation processes

An *isentropic stagnation process*, is one which brings a moving fluid element to rest adiabatically and reversibly (without friction). Of the above figures, only Process A is of this type.



In addition to the total enthalpy relation

$$h_{o_1} \;=\; h_{\text{stag}}$$

we now also have the isentropic relations between station 1 and the stagnation point.

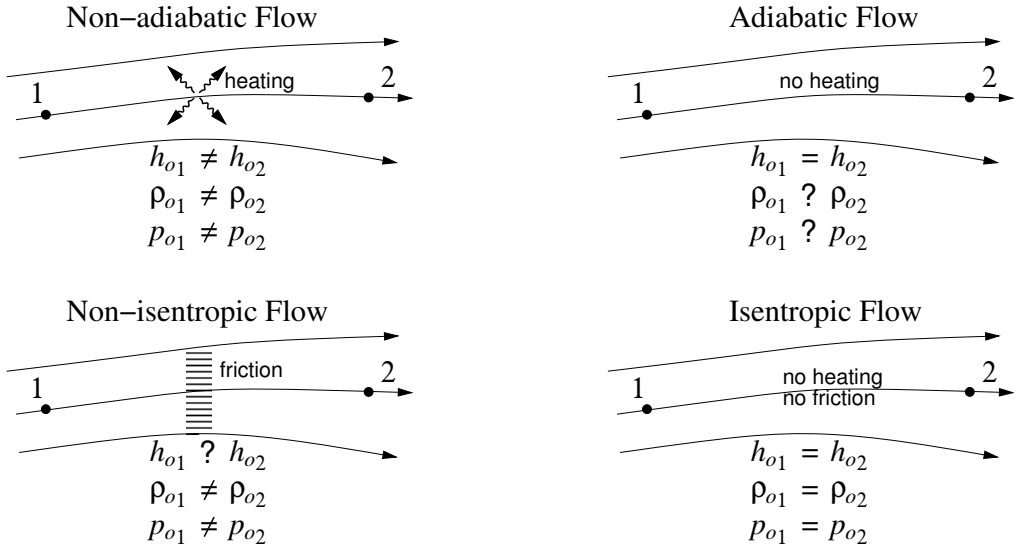
$$\frac{\rho_{\text{stag}}}{\rho_1} \;=\; \left(\frac{T_{\text{stag}}}{T_1}\right)^{1/(\gamma-1)} \;=\; \left(\frac{h_{\text{stag}}}{h_1}\right)^{1/(\gamma-1)}$$
$$\frac{p_{\text{stag}}}{p_1} \;=\; \left(\frac{T_{\text{stag}}}{T_1}\right)^{\gamma/(\gamma-1)} \;=\; \left(\frac{h_{\text{stag}}}{h_1}\right)^{\gamma/(\gamma-1)}$$

Substituting $h_{\text{stag}} = h_{o_1}$, and $h_1 = h_{o_1} - \frac{1}{2}V_1^2$, we can now define the total density and total pressure at station 1 in terms of station 1 quantities.

$$\rho_{\text{stag}} \;\equiv\; \rho_{o_1} \;=\; \rho_1 \left(1 - \frac{V_1^2}{2h_{o_1}}\right)^{-1/(\gamma-1)}$$
$$p_{\text{stag}} \;\equiv\; p_{o_1} \;=\; p_1 \left(1 - \frac{V_1^2}{2h_{o_1}}\right)^{-\gamma/(\gamma-1)}$$

**Relations along streamline**

Any point along a streamline can be subjected to a hypothetical adiabatic or isentropic stagnation process in order to define the local total quantities $h_o$, $\rho_o$, and $p_o$. Whether any two such points on a streamline have the same total quantities depends on whether a non-adiabatic or non-isentropic process occurred on the streamline between them. The figure shows four possible situations, resulting in equalities or inequalities between the two points on the streamline. The "?" relation in the non-isentropic and adiabatic cases indicates that the relation is unknown without additional information about the friction or heating, respectively.




For the adiabatic case, a unique total enthalpy $h_o$ can be assigned to the whole streamline. Then for any point on the streamline we have

$$h \;=\; h_o \;-\; \frac{1}{2}V^2 \tag{1}$$

For the more restrictive isentropic case, a unique total density $\rho_o$ and total pressure $p_o$ can also be assigned to the whole streamline, which gives

$$\rho = \rho_o \left(1 - \frac{V^2}{2h_o}\right)^{1/(\gamma-1)} \tag{2}$$

$$p = p_o \left(1 - \frac{V^2}{2h_o}\right)^{\gamma/(\gamma-1)} \tag{3}$$

These equations are in effect a compressible-flow replacement for the incompressible Bernoulli equation.

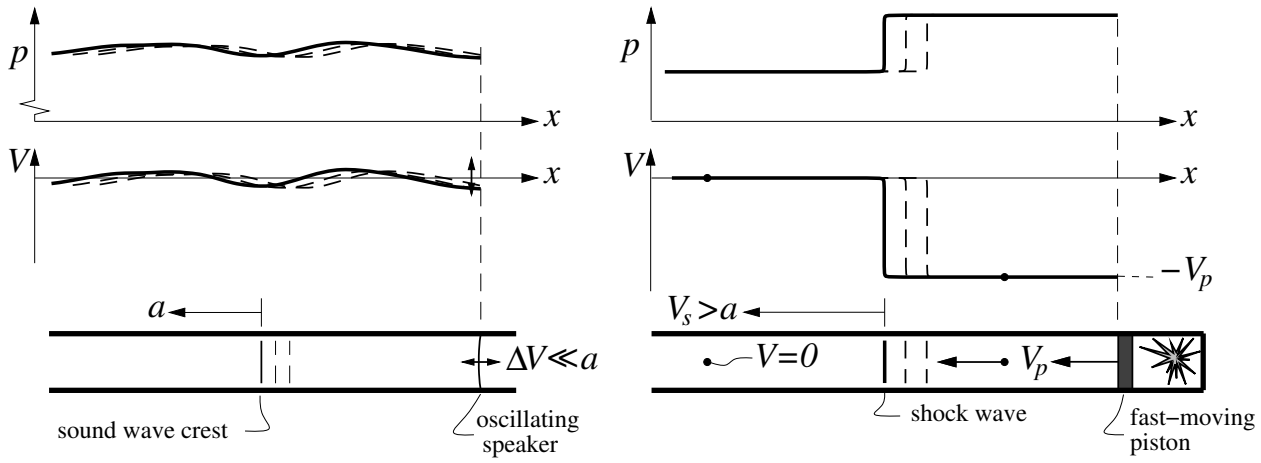
It's useful to note that only two of the three above equations are independent. Any one of them could be removed and replaced by the state equation.

$$p = \frac{\gamma - 1}{\gamma} \rho h$$

## Introduction to Shock Waves

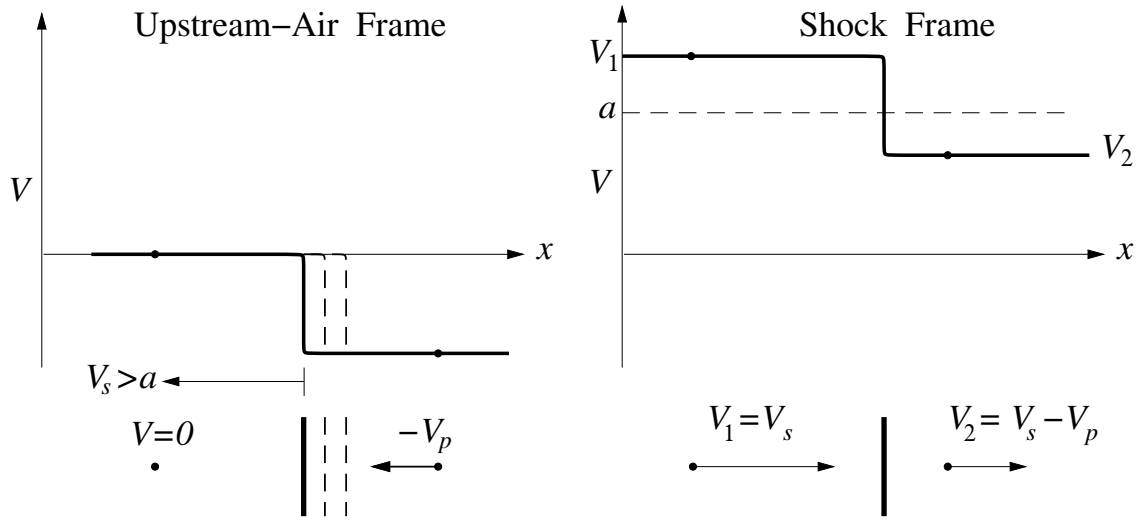
### Wave features

Compressibility of a fluid allows the existence of *waves*, which are variations in $\rho$, $p$, and $h$ (or temperature $T$), which self-propagate through the fluid at some speed. Ordinary sound consists of very small variations which move at the speed of sound $a$, while a *shock wave* has a finite variation in flow quantities and moves at a larger speed $V_s > a$. The figure illustrates the difference in the two types of waves. The shock wave has a flow velocity behind it equal to the piston speed $V_p$, but the shock itself advances into the still air at a much higher speed $V_s > a$. The air properties $\rho$, $p$, and $h$ are all increased behind the shock.



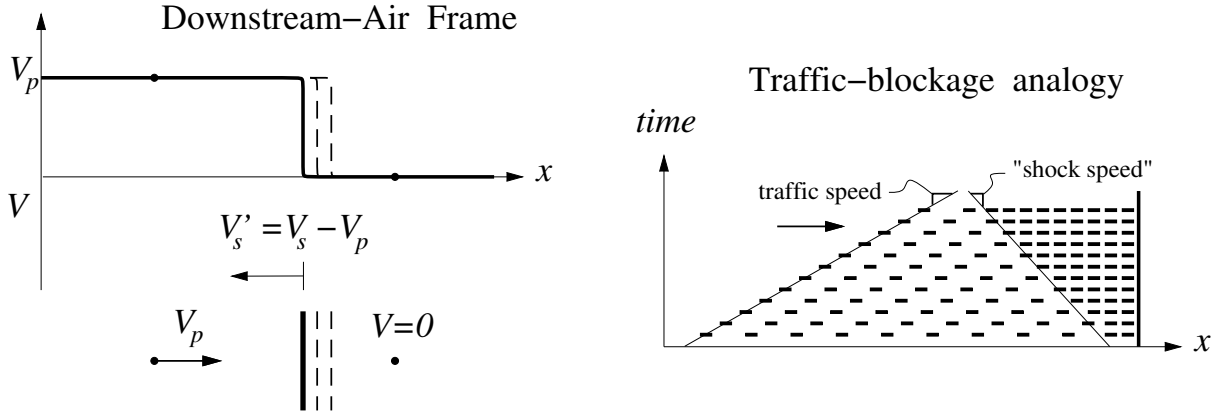

### Shock Frame

We now examine the piston shock flow in the frame of the shock, by shifting all the velocities by $+V_s$. In this frame the flow is steady, and is the most convenient frame for analyzing the shock. The upstream and downstream quantities are usually denoted by the subscripts $()_1$ and $()_2$, respectively. The static air properties $\rho$, $p$, and $h$ are of course unchanged by this frame change.




**Downstream-Air Frame**

An intuitive understanding of a shock wave is perhaps best obtained by looking at the situation yet again, in the downstream-air frame. The shock now propagates against the oncoming upstream flow. This situation is closely analogous to how a traffic blockage propagates backward against the oncoming traffic.




**Dissipation in Shock**

The flow passing through a shock wave undergoes an adiabatic process, since there is no heat being supplied (there's nothing there to provide heat!). But because a shock wave is typically very thin — less than 1 micron at sea level — there are strong viscous forces acting on the fluid passing through it, so the process is irreversible. Therefore, the stagnation quantities have the following relations across a shock wave:

$$\begin{aligned} h_{o_1} &= h_{o_2} \\ \rho_{o_1} &> \rho_{o_2} \\ p_{o_1} &> p_{o_2} \end{aligned}$$

A more detailed analysis will quantify the inequalities.