

## F5 – Lecture Notes

1. Effects of Reynolds Number
2. Effects of Mach Number

Reading: Anderson 1.7

Dimensional Analysis has identified the important dimensionless parameters (or  $\Pi$  products), which determine the nature of a given aerodynamic flow situation. The main parameters are the Reynolds number and the Mach number, and their values in any given flow situation will determine the nature of the flow (strongly viscous vs. nearly inviscid, low speed vs. high speed)

### Effects of Reynolds Number

The Reynolds number can be interpreted as a comparison of the pressure forces and viscous shear forces which act on the body, done by forming their ratio.

$$\frac{\text{pressure forces}}{\text{shear forces}} = \frac{p - p_\infty}{\tau} \sim \frac{\rho u^2}{\mu du/dn} \sim \frac{\rho_\infty V_\infty^2}{\mu_\infty V_\infty/c} \sim \frac{\rho_\infty V_\infty c}{\mu_\infty} \equiv Re_\infty$$

Note that  $p - p_\infty$  is considered rather than  $p$  itself, since  $p_\infty$  is just a large constant bias which produces no net force on a body.

The Reynolds number is therefore a measure of the importance of pressure forces relative to viscous shear forces. Hence, if the Reynolds number increases, the viscous effects on the flow get progressively less important.

The viscosity of air and water is quite small when expressed in common units.

	Air @ STP	Water @ 15°C
$\mu$	$1.78 \times 10^{-5} \text{kg/m-s}$	$1.15 \times 10^{-3} \text{kg/m-s}$
$\mu/\rho = \nu$	$1.45 \times 10^{-5} \text{m}^2/\text{s}$	$1.15 \times 10^{-6} \text{m}^2/\text{s}$

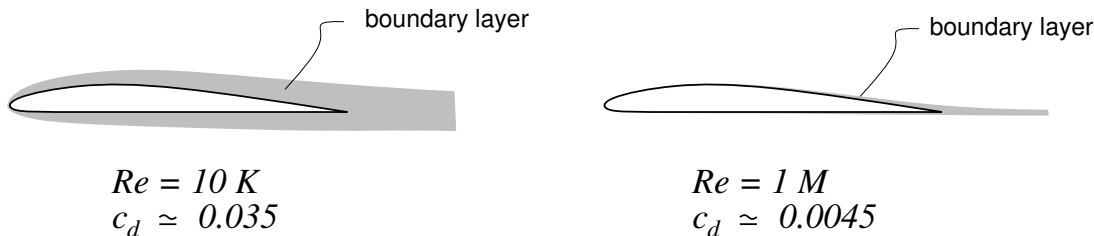
The ratio  $\mu/\rho = \nu$  is called the *kinematic viscosity*. This is more relevant than  $\mu$  itself, since the ratio is what actually appears in the Reynolds number.

$$Re_\infty = \frac{V_\infty c}{\nu_\infty}$$

From the values of  $\nu$ , it is clear that that typical aerodynamic and hydrodynamic flows will have very large Reynolds numbers. This can be seen in the following table, which gives the Reynolds numbers based on the chord length of common winged objects.

Object	$Re$
Butterfly	5 K
Pigeon	50 K
RC glider	100 K
Sailplane	1 M
Business jet	10 M
Boeing 777	50 M

The Reynolds number is large even for insects, which means that the flow can be assumed to be inviscid (i.e.  $\mu = 0$  and  $\tau = 0$ ) almost everywhere. The only place where the viscous shear is significant is in *boundary layers* which form adjacent to solid surfaces and become a *wake* trailing downstream.



The larger the Reynolds number is, the thinner the boundary layers are relative to the size of the body, and the more the flow behaves as though it was inviscid. This has a number of consequences of great practical significance:

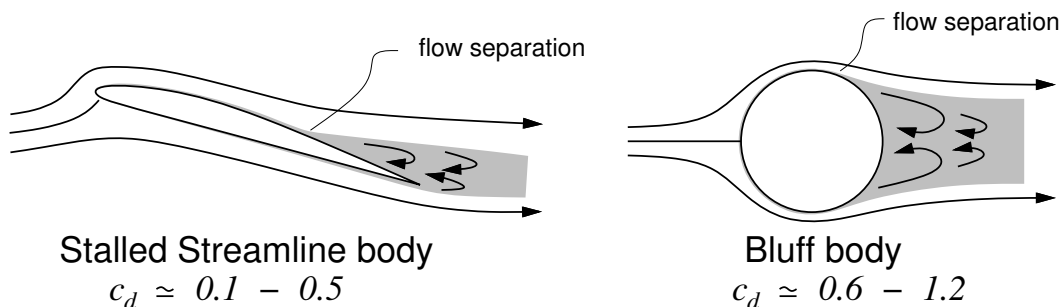
1. Neglecting the viscous shear stress greatly simplifies the equations of fluid motion, allowing them to be manipulated and solved much more easily. Simple solutions then give better insight and understanding of aerodynamic behavior of bodies such as airfoils, wings, propellers, turbine blades, etc.
2. If the Reynolds number is large enough (i.e. viscosity is small enough), then its effects on some aerodynamic forces and moments can be neglected. For example, if  $Re > 10^6$  or so, then it has almost no influence on the aerodynamic lift and moment, which means that the general dependencies for  $c_l$  and  $c_m$  simplify as follows.

$$\begin{aligned} c_l &= c_l(\alpha, M_\infty, Re_\infty) & \rightarrow & & c_l &= c_l(\alpha, M_\infty) \\ c_m &= c_m(\alpha, M_\infty, Re_\infty) & & & c_m &= c_m(\alpha, M_\infty) \end{aligned} \quad (Re_\infty \text{ large})$$

Such a simplification is not appropriate for  $c_d(\alpha, M_\infty, Re_\infty)$ , however.

### Streamlined vs. Bluff Bodies

The above assumption that viscosity has negligible effects on lift and moment is valid only for *streamline bodies* which gradually taper to a point or sharp edge, and are roughly aligned with the freestream flow direction. Examples are slender bodies, airfoils, and wings, operating at small angles of attack. For *bluff bodies* which have blunt rear faces, or for streamline bodies at large angles of attack, viscosity always plays an important role for all the forces and moments. Such flows exhibit *flow separation*, which is the sudden thickening or breakaway of the boundary layer from the surface, resulting in a thick trailing wake.



Bodies with separation exhibit much larger drag forces than comparably-sized streamline bodies without separation. When an airfoil stalls, its drag will typically increase by a factor

of 10 or more. Most of this drag increase is due to a sharp rise of *pressure drag*, defined as the integrated pressure  $p - p_\infty$  normal to the body. The *friction drag*, which is the integrated tangential shear stress  $\tau$ , is not significantly different between streamline and bluff bodies of similar size.

### Drag reference length

The reference length used to define  $c_d$  is arbitrary. Traditionally, for streamline bodies such as an airfoil we use the streamwise length  $c$ . For bluff bodies we use the transverse height, typically denoted by  $h$ . For a circular cylinder they are of course the same, and equal to the diameter  $d$ .

### Effects of Mach Number

The Mach number is defined as the ratio of the freestream speed to the speed of sound.

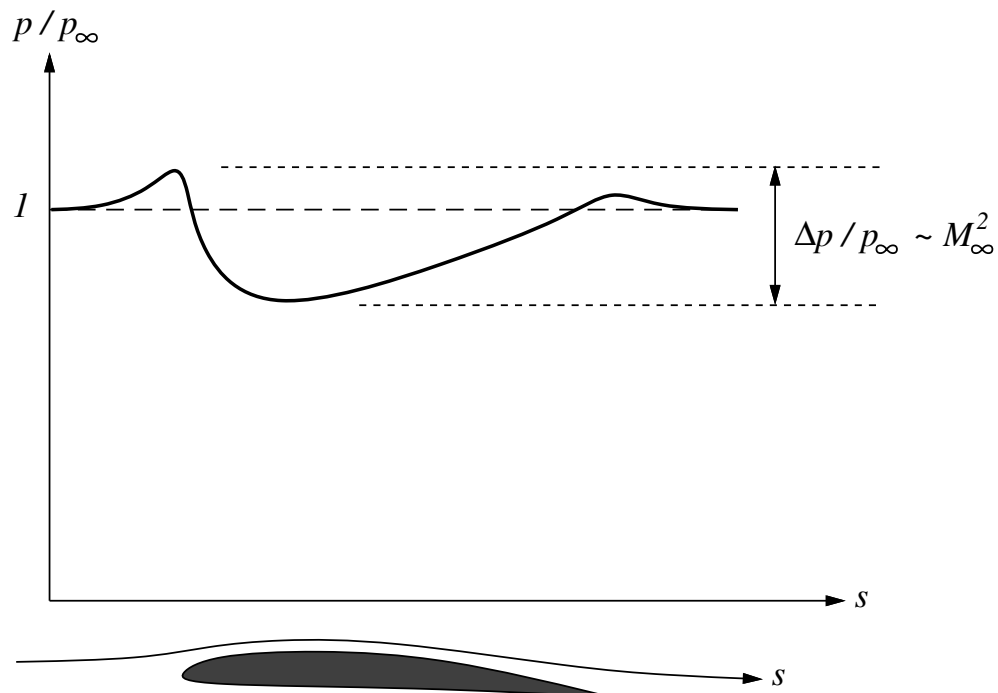
$$\frac{V_\infty}{a_\infty} \equiv M_\infty$$

The speed of sound  $a_\infty$  is the velocity at which pressure disturbances travel in the fluid, although what this implies for the flow is not immediately apparent.

Another interpretation of the Mach number arises when we estimate the *fractional* variation of pressure in the flowfield.

$$\frac{\Delta p}{p_\infty} \sim \frac{q_\infty}{p_\infty} = \frac{\frac{1}{2}\rho_\infty V_\infty^2}{p_\infty} = \frac{\gamma \rho_\infty V_\infty^2}{2 \gamma p_\infty} = \frac{\gamma V_\infty^2}{2 a_\infty^2} = \frac{\gamma}{2} M_\infty^2$$

where  $\gamma = c_p/c_v$  is the thermodynamic ratio of specific heats. So the Mach number squared is a measure of the *fractional* pressure variations in the flowfield, as shown in the figure.



## Low-Speed flows

A *low-speed flow* is defined as one for which

$$M_\infty^2 \ll 1.$$

From the above derivation this implies that fractional pressure changes are small, and from thermodynamics we then conclude that that density and temperature changes must also be small.

$$\frac{\Delta p}{p} \ll 1 \quad , \quad \frac{\Delta \rho}{\rho} \ll 1 \quad , \quad \frac{\Delta T}{T} \ll 1$$

So an important feature of a low-speed flow is *nearly constant fluid properties* throughout the flow field. All hydrodynamic flows can be classified as low-speed flows, since liquids feature an essentially constant density.

One practical feature of low-speed flows is that they are extremely insensitive to Mach number variations, as long as  $M_\infty^2 \ll 1$ . So a flow with  $M_\infty = 0.01$  is almost indistinguishible from a flow with  $M_\infty = 0.1$  when compared using dimensionless variables. So for low speed flows we can remove the influence of  $M_\infty$  from the dependency list.

$$\begin{array}{lcl} c_\ell = c_\ell(\alpha, M_\infty) & & c_\ell = c_\ell(\alpha) \\ c_m = c_m(\alpha, M_\infty) & \rightarrow & c_m = c_m(\alpha) \\ c_d = c_d(\alpha, M_\infty, Re_\infty) & & c_d = c_d(\alpha, Re_\infty) \end{array} \quad (M_\infty^2 \ll 1)$$

This is a considerable simplification.