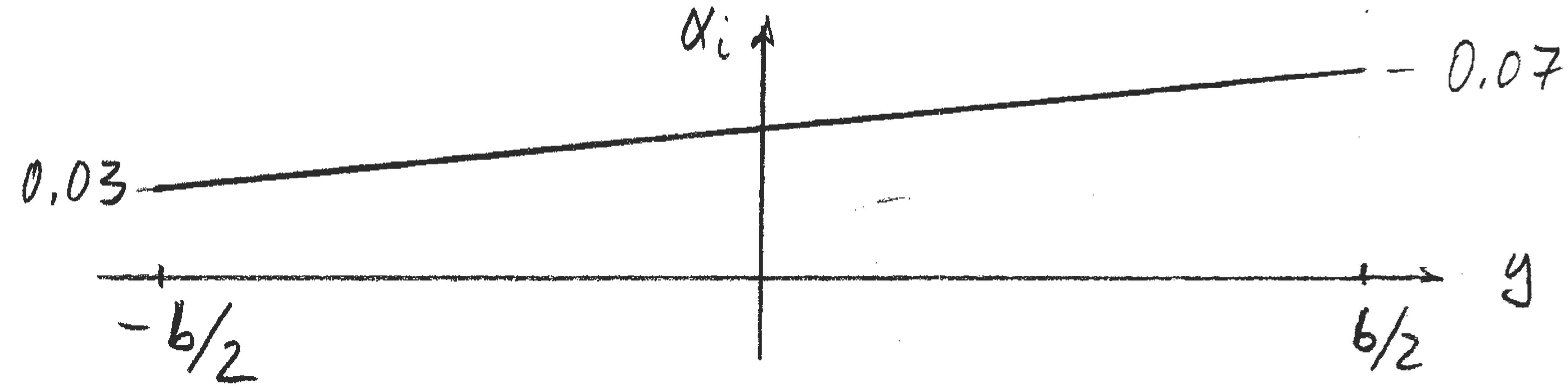
UE Fluids | F9 + F10 Solution | Spring '04

a) $\alpha_i = \sum n A_n \frac{\sin n\theta}{\sin\theta} = A_1 + A_2 \frac{\sin 2\theta}{\sin\theta} = A_1 + A_2\, 2\cos\theta$

but since $\cos\theta = \frac{2y}{b}$, $\alpha_i = 0.05 + 0.02 \cdot \left(\frac{2y}{b}\right)$



b) $M_{roll} = \int_{-b/2}^{b/2} \rho V_\infty \Gamma y \, dy$

Using $\Gamma = 2b_\infty V_\infty (A_1 \sin\theta + A_2 \sin 2\theta)$

$y = \frac{b}{2}\cos\theta$

$dy = -\frac{b}{2}\sin\theta \, d\theta$

$$M_{roll} = \int_\pi^0 \rho V_\infty 2bV_\infty (A_1 \sin\theta + A_2 \sin 2\theta) \frac{b}{2}\cos\theta \left(-\frac{b}{2}\sin\theta\, d\theta\right)$$

$$= \frac{1}{2}\rho V_\infty^2 b^3 \int_0^\pi (A_1 \sin\theta + A_2 \sin 2\theta) \frac{1}{2}\sin 2\theta \, d\theta$$

since $\cos\theta \sin\theta = \frac{1}{2}\sin 2\theta$

$$= \frac{1}{2}\rho V_\infty^2 b^3 \left[\frac{1}{2}A_1 \underbrace{\int_0^\pi \sin\theta \sin 2\theta \, d\theta}_{0} + \frac{1}{2}A_2 \underbrace{\int_0^\pi \sin 2\theta \sin 2\theta \, d\theta}_{\frac{\pi}{2}}\right]$$

$$M_{roll} = \frac{\pi}{8}\rho V_\infty^2 b^3 \cdot A_2$$

only A_2 term contributes to rolling moment