

$$C_p = 1 - \frac{V^2}{V_\infty^2} = 1 - \left(\frac{u}{V_\infty}\right)^2 - \left(\frac{v}{V_\infty}\right)^2$$

a) Source: $u = V_\infty + \frac{\Lambda}{2\pi} \frac{x}{x^2+y^2}$
 $v = \frac{\Lambda}{2\pi} \frac{y}{x^2+y^2}$

Along $-x, y=0$: $C_p = 1 - \frac{u^2}{V_\infty^2} = 1 - \left(\frac{V_\infty}{V_\infty} - \frac{\Lambda}{2\pi V_\infty} \frac{1}{x}\right)^2 = \left[\frac{\Lambda}{\pi V_\infty} \frac{1}{x} - \left(\frac{\Lambda}{2\pi V_\infty}\right)^2 \frac{1}{x^2}\right]$
 Along $y, x=0$: $C_p = 1 - \left(\frac{V_\infty}{V_\infty}\right)^2 - \left(\frac{\Lambda}{2\pi V_\infty}\right)^2 \frac{1}{y^2} = -\left(\frac{\Lambda}{2\pi V_\infty}\right)^2 \frac{1}{y^2}$

dominant (pointing to $\frac{1}{x}$ term)
much smaller for large x (pointing to $\frac{1}{x^2}$ term)

b) Vortex: $u = V_\infty + \frac{\Gamma}{2\pi} \frac{y}{x^2+y^2}$
 $v = \frac{\Gamma}{2\pi} \frac{-x}{x^2+y^2}$

Along $-x, y=0$: $C_p = 1 - \left(\frac{V_\infty}{V_\infty}\right)^2 - \left(\frac{\Gamma}{2\pi V_\infty}\right)^2 \frac{1}{x^2} = -\left(\frac{\Gamma}{2\pi V_\infty}\right)^2 \frac{1}{x^2}$
 Along $y, x=0$: $C_p = 1 - \left(\frac{u}{V_\infty}\right)^2 = 1 - \left(\frac{V_\infty}{V_\infty} + \frac{\Gamma}{2\pi V_\infty} \frac{1}{y}\right)^2 = \left[-\frac{\Gamma}{\pi V_\infty} \frac{1}{y} - \left(\frac{\Gamma}{2\pi V_\infty}\right)^2 \frac{1}{y^2}\right]$

dominant (pointing to $\frac{1}{y}$ term)

c) Doublet: $\phi = V_\infty x + \frac{K}{2\pi} \frac{x}{x^2+y^2}$ $\left\{ \begin{array}{l} u = \frac{\partial \phi}{\partial x} = V_\infty + \frac{K}{2\pi} \frac{y^2-x^2}{(x^2+y^2)^2} \\ v = \frac{\partial \phi}{\partial y} = \frac{K}{2\pi} \frac{-2xy}{(x^2+y^2)^2} \end{array} \right.$

Along $-x, y=0$: $C_p = 1 - \left(\frac{u}{V_\infty}\right)^2 = 1 - \left(\frac{V_\infty}{V_\infty} - \frac{K}{2\pi V_\infty} \frac{1}{x^2}\right)^2 = \frac{K}{\pi V_\infty} \frac{1}{x^2} - \left(\frac{K}{2\pi V_\infty}\right)^2 \frac{1}{x^4}$
 Along $y, x=0$: $C_p = 1 - \left(\frac{u}{V_\infty}\right)^2 = 1 - \left(\frac{V_\infty}{V_\infty} + \frac{K}{2\pi V_\infty} \frac{1}{y^2}\right)^2 = -\frac{K}{\pi V_\infty} \frac{1}{y^2} - \left(\frac{K}{2\pi V_\infty}\right)^2 \frac{1}{y^4}$

The C_p fields decrease with distance as follows:



Far away the $\frac{1}{x}$ and $\frac{1}{y}$ terms dominate ($\frac{1}{x^2}$ and $\frac{1}{y^2}$ die off much faster)

A lifting airfoil has a nonzero Γ , so it looks mostly like a vortex far away. Largest C_p is above & below.

