

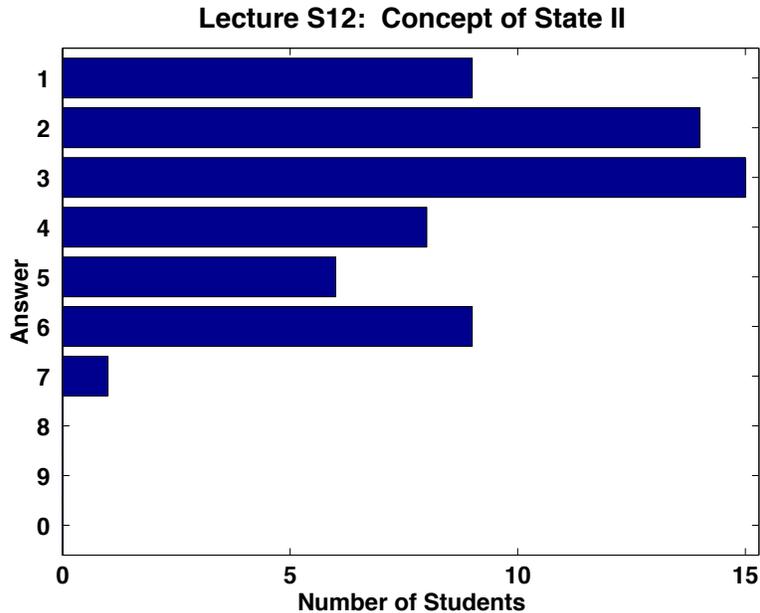
Concept of State II

Concept Test

You are to model the dynamics of a roller coaster, in order to predict the transit time of each car around the track. The first step in the modeling is to determine the number of states in the system.

Enter the number of states, or 0 (zero) if the number of states is greater than 9.

Concept of State II Solution



The best answer is that there are two state variables: the position of the coaster along the track, and the velocity of the coaster.

There was a wide range of answers in the class. The best way to narrow down the answer is to think about the absolute minimum number of variables needed to specify the initial condition.

Eigenvalues and Eigenvectors

Concept Test

For the matrix A given by

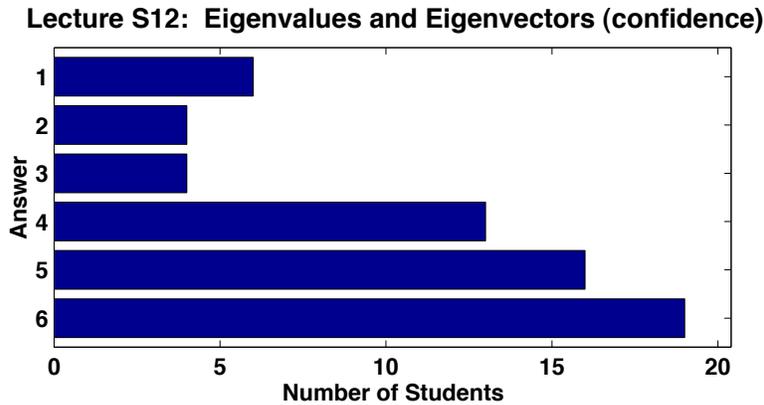
$$A = \begin{bmatrix} -1/2 & -2 \\ 1/2 & -3 \end{bmatrix}$$

find the eigenvalues and eigenvectors.

My confidence in my answer is

1. 0%
2. 20%
3. 40%
4. 60%
5. 80%
6. 100%

Eigenvalues and Eigenvectors Solution



The eigenvalues are the roots of the equation

$$\begin{aligned}\phi(s) &= \det(sI - A) = 0 \\ &= \begin{vmatrix} s + 1/2 & 2 \\ -1/2 & s + 3 \end{vmatrix} \\ &= (s + 1/2)(s + 3) - (a/s)(-2) = s^2 + (7/2)s + 5/2\end{aligned}$$

The roots of this quadratic are

$$\begin{aligned}s_1 &= -1 \\ s_2 &= -2.5\end{aligned}$$

The corresponding eigenvectors are found by solving

$$(sI - A)\underline{v} = \underline{0}$$

It is easily verified that

$$\underline{v}_1 = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

$$\underline{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

These eigenvectors can be found by Gaussian elimination or elimination of variables.

Steps in Solving a Linear Dynamic Network Using Eigenvalue Method

1. Identify the states of the circuit (capacitor voltages and inductor currents), \underline{x} .
2. Use the node method (or loop method) to solve for unknown node voltages (or loop currents).
3. Voltage across inductors is difference in node potentials. (Current in capacitors is difference in loop currents.)
4. Find current in capacitors using KCL. (Find voltage across inductor using KVL.)
5. Using the constitutive relations for the capacitors and inductors, find the state equation of the form

$$\dot{\underline{x}} = A\underline{x}$$

6. Find the eigenvalues s_i and the eigenvectors \underline{v}_i of the matrix A .

7. The general solution is

$$\underline{x}(t) = \sum_i a_i \underline{v}_i e^{s_i t}$$

8. Solve for the a_i that give the correct initial conditions. This may be calculated as

$$\underline{a} = \left[\begin{array}{cccc} \underline{v}_1 & \underline{v}_2 & \cdots & \underline{v}_n \end{array} \right]^{-1} \underline{x}(0)$$