LECTURE 57

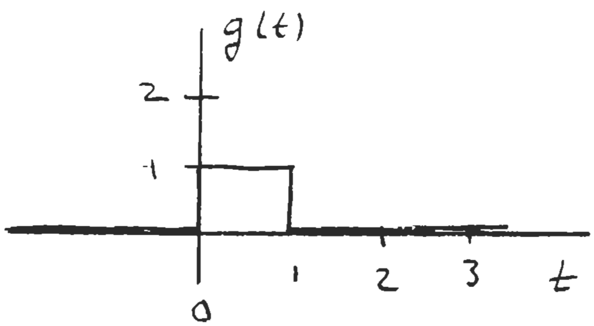
Graphical Interpretation of Convolution

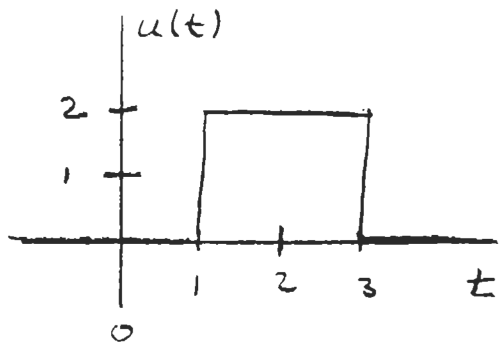
We can interpret convolution graphically:

$$y(t) = g(t) * u(t)$$
$$= \int_{-\infty}^{\infty} g(t-\tau)\, u(\tau)\, d\tau$$

let's draw these functions of τ.

Suppose $g(t), u(t)$ are:

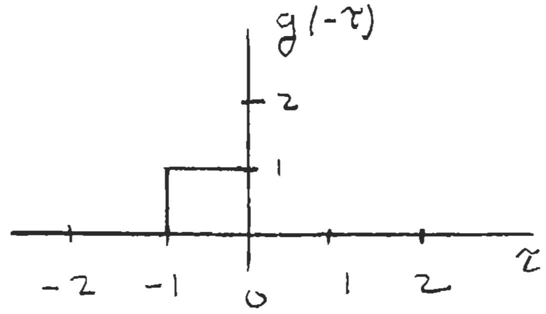




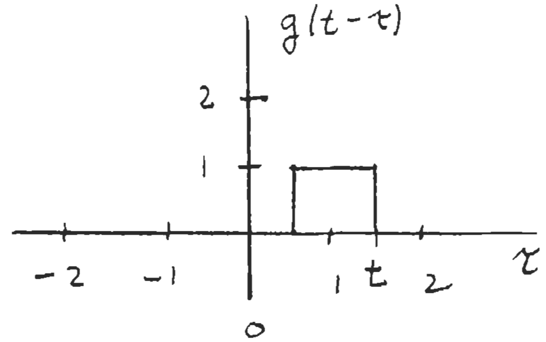
$$g(t) = \begin{cases} 1, & 0 \le t \le 1 \\ 0, & \text{else} \end{cases}$$

$$u(t) = \begin{cases} 2, & 1 \le t \le 3 \\ 0 & \text{else} \end{cases}$$

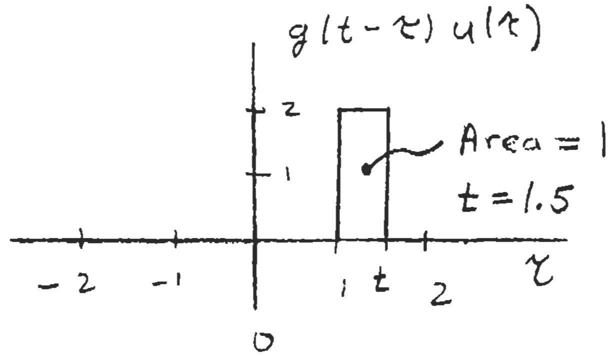
Now plot as function of τ. $u(\tau)$ is unchanged. Let's do $g(t-\tau)$:



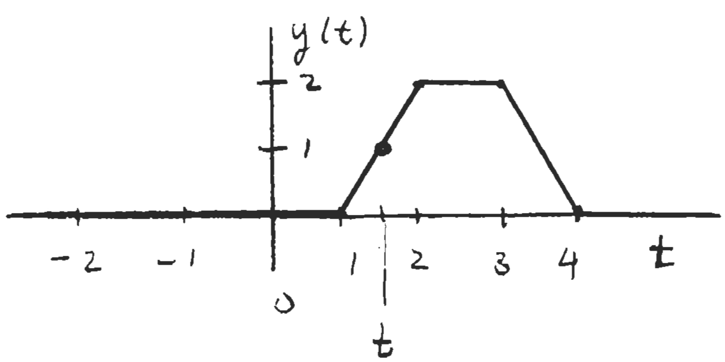
"flip" —
Reflect $g(\tau)$ about $\tau = 0$.



"slide" —
Slide $g(-\tau)$ by amount t (In this case, $t = 1.5$)

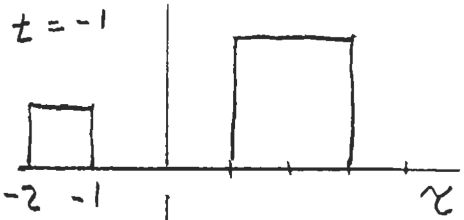
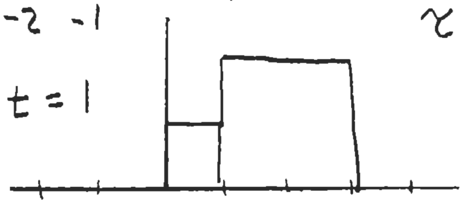
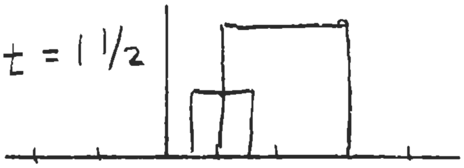
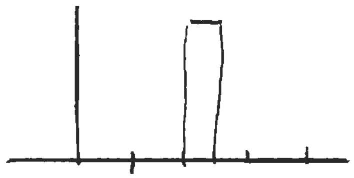
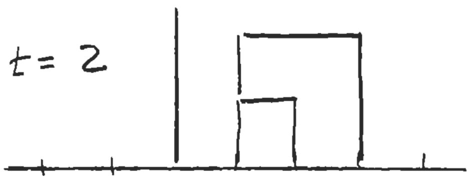
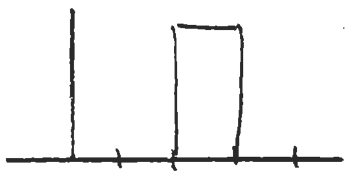
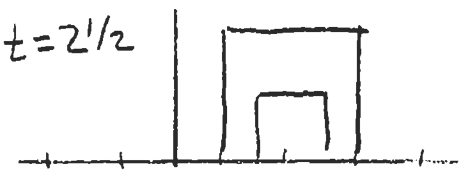
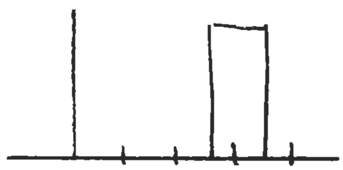
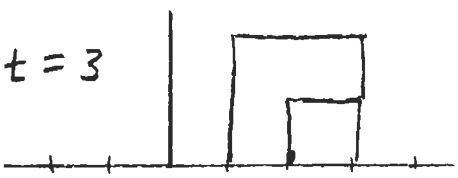
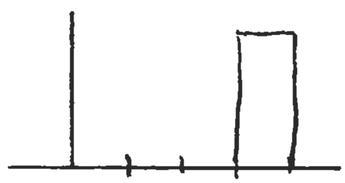
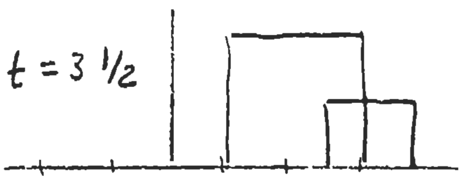
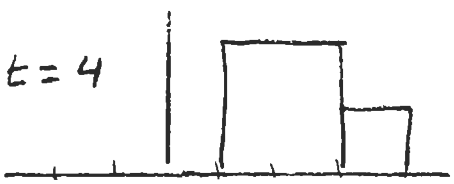


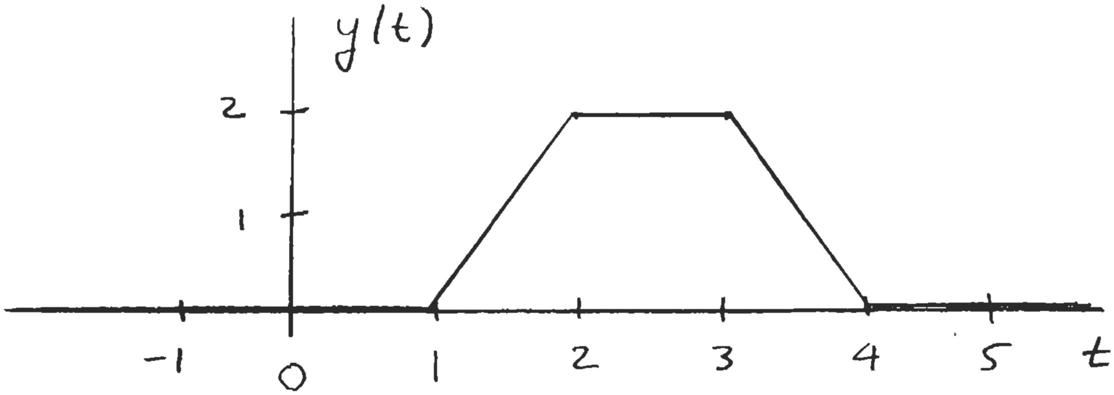
Multiply —
Multiply $g(t-\tau)$ by $u(\tau)$, point by point



Integrate —
Find area of $g(t-\tau)u(\tau)$, plot point.

This is the "flip and slide" method — good for keeping track of limits; insight

	$u(\tau),\ g(t-\tau)$	$g(t-\tau)u(\tau)$	Area
$t=-1$			0
$t=1$			0
$t=1\frac{1}{2}$			1
$t=2$			2
$t=2\frac{1}{2}$			2
$t=3$			2
$t=3\frac{1}{2}$			1
$t=4$			0



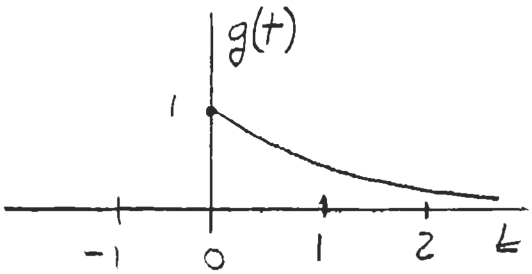
Whew!

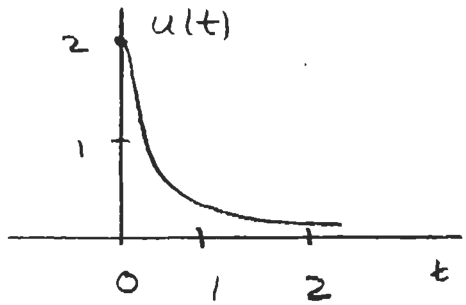
Note –

1. Can easily see limits of integration
2. Gain some insight into where $y(t)$ is small or big. To be big, $g(t-\tau)$ and $u(\tau)$ must have significant overlap where they are big.
3. $y(t)$ is smoother than $u(t)$ or $g(t)$.

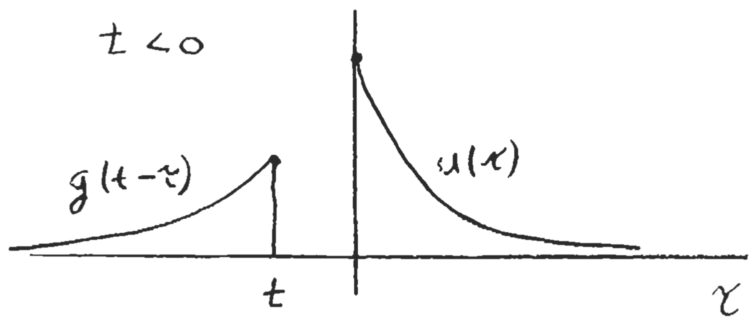
Example $\quad g(t) = e^{-t}\,\sigma(t)$

$$u(t) = 2e^{-2t}\,\sigma(t)$$



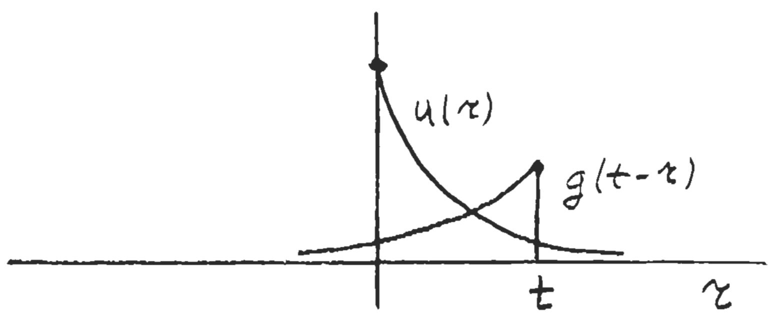


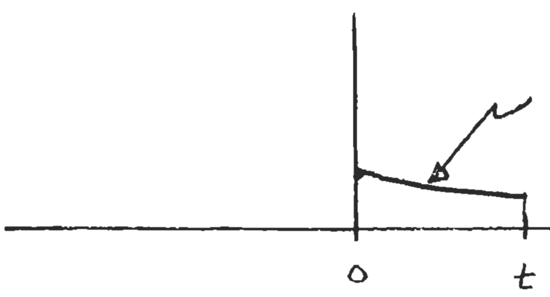
Do flip & slide:



No overlap!

$\Rightarrow y(t) = 0, \quad t < 0$





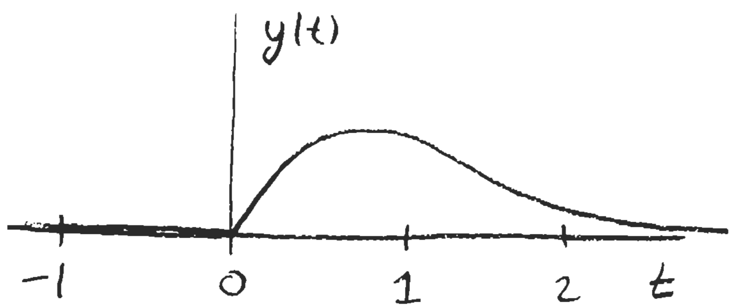
$$g(t-\tau)\,u(\tau) = e^{-(t-\tau)} \cdot 2e^{-2\tau}$$

$$= 2e^{-t}e^{-\tau}$$

$$y(t) = \int_0^t 2e^{-t} e^{-\tau} \, d\tau$$

$$= 2e^{-t}\left(1 - e^{-t}\right)$$

$$= 2e^{-t} - 2e^{-2t}, \qquad t \geq 0$$



<u>Note</u>: Still have to compute integral, <u>but</u> it does help understand the limits of integration, what answer should look like.