APPLICATIONS OF THE FIRST LAW
TO HEAT ENGINES

I.   Thermodynamic cycles and heat engines *(VW, S & B: Chapter 9)*

This section of the course is devoted to describing the basic fundamentals of how various heat engines work (e.g. a refrigerator, an IC engine, a jet). You will also learn how to model these heat engines as thermodynamic cycles and how to apply the First Law of Thermodynamics to estimate thermal efficiency and work output as a function of pressures and temperatures at various points in the cycle. This is called **ideal cycle analysis**. The estimates you obtain from the analysis represent the best achievable performance that may be obtained from a heat engine. In reality, the performance of these systems will be somewhat less than the estimates obtained from ideal cycle analysis—you will learn how to make more realistic estimates in 16.050.

We will deal with only "air-standard" thermodynamic cycles, where we ignore the changes in properties of the working fluid brought about by the addition of fuel or the presence of combustion products (we do of course account for the heat release that occurs due to the combustion of the fuel-air mixture). In general this is a good assumption since in typical combustion applications the fuel accounts for only about 5% of the mass of the working fluid.

A. The Otto Cycle (VW, S & B: 9.13)

The Otto cycle is an idealization of a set of processes used by spark ignition internal combustion engines (2-stroke or 4-stroke cycles). These engines a) ingest a mixture of fuel and air, b) compress it, c) cause it to react, thus effectively adding heat through converting chemical energy into thermal energy, d) expand the combustion products, and then e) eject the combustion products and replace them with a new charge of fuel and air. The various steps are illustrated on page 9 of these notes. There is a nice animation (and other useful material) at http://www.uq.edu.au/~e4nsrdja/teaching/e4213/intro.htm

We model all of these happenings by a thermodynamic cycle consisting of a set of processes all acting on a fixed mass of air contained in a piston-cylinder arrangement. The exhaust and intake processes are replaced by constant-volume cooling.

1. Representation of the heat engine as a thermodynamic cycle.

   (Ingest mixture of fuel and air)

   1' - 2 Compress mixture quasi-statically and adiabatically
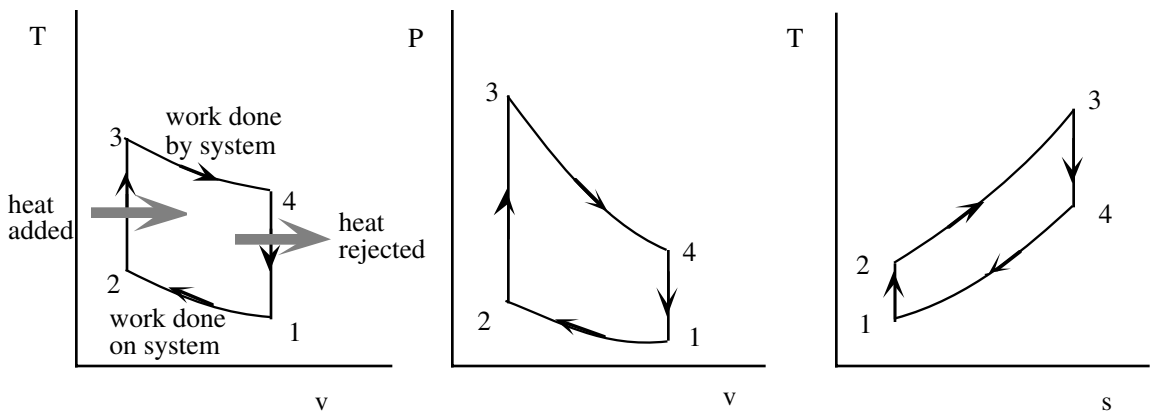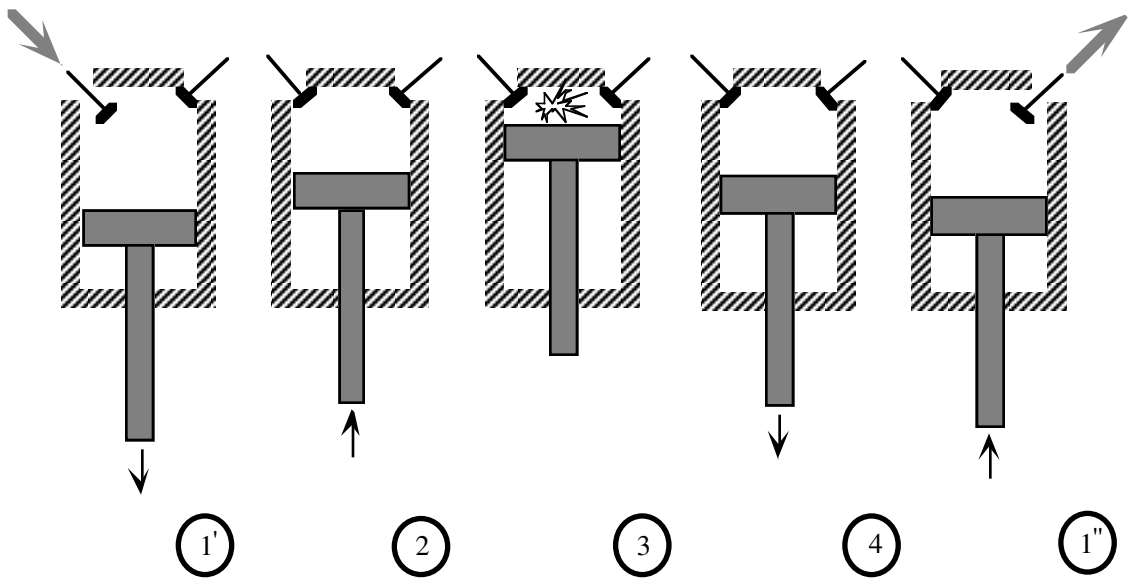
   2 - 3 Ignite and burn mixture at constant volume (heat is added)

   3 - 4 Expand mixture quasi-statically and adiabatically

   4 - 1'' Cool mixture at constant volume

   (then repeat)

   work ~ Force* distance ~ $\int p dv$
   fuel use ~ heat added ~ $T_3 - T_2$
   efficiency ~ work out/fuel use

2. Method for estimating thermal efficiency and work output (application of the First Law of Thermodynamics).

   a) **Net work done by system** = work of expansion + work of compression (-)

      Both expansion and compression are adiabatic so

      $$\Delta w = (u_3 - u_4) - (u_2 - u_1)$$

      Assuming an ideal gas with constant c_v

      $$\Delta w = c_v[(T_3 - T_4) - (T_2 - T_1)]$$

   b) While the above expression is accurate, it is not all that useful. We would like to put the expression in terms of the typical design parameters: the compression ratio ($v_1/v_2 = v_4/v_3$), and the heat added during combustion ($q_{comb.} = c_v(T_3 - T_2)$).

      For a quasi-static, adiabatic process,

      $$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = \frac{T_3}{T_4} \quad \text{so we can write the net work as}$$

      $$\Delta w = c_v T_1\left(\frac{T_4}{T_1} - 1\right)\left(r^{\gamma-1} - 1\right) \quad \text{we also know that}$$

      $$\frac{T_4}{T_1} = \left(\frac{T_4}{T_3}\right)\left(\frac{T_3}{T_2}\right)\left(\frac{T_2}{T_1}\right) = \frac{T_3}{T_2} \quad \text{since} \quad \frac{T_2}{T_1} = \frac{T_3}{T_4} \quad \text{so}$$

      $$\frac{T_4}{T_1} - 1 = \frac{T_3}{T_2} - 1 = \left(\frac{T_3 - T_2}{T_1}\right)\left(\frac{1}{r^{\gamma-1}}\right)$$

      and finally, the desired result in terms of typical design parameters:

      $$\boxed{\Delta w = q_{comb.}\frac{\left(r^{\gamma-1} - 1\right)}{r^{\gamma-1}}}$$

   c) The **thermal efficiency** of the cycle is

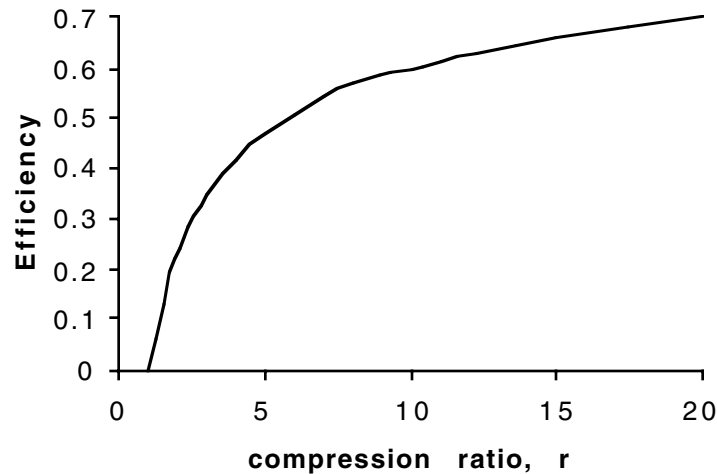 $$\eta = \frac{\text{net work}}{\text{heat input}} = \frac{\Delta w}{q_{comb.}} \quad \text{so}$$

$$\eta_{\text{Otto}} = \frac{\left(r^{\gamma-1} - 1\right)}{r^{\gamma-1}}$$

(Note that we could also rewrite this as:

$\eta_{\text{Otto}} = 1 - \dfrac{T_1}{T_2}$ showing that the efficiency of an Otto cycle depends only

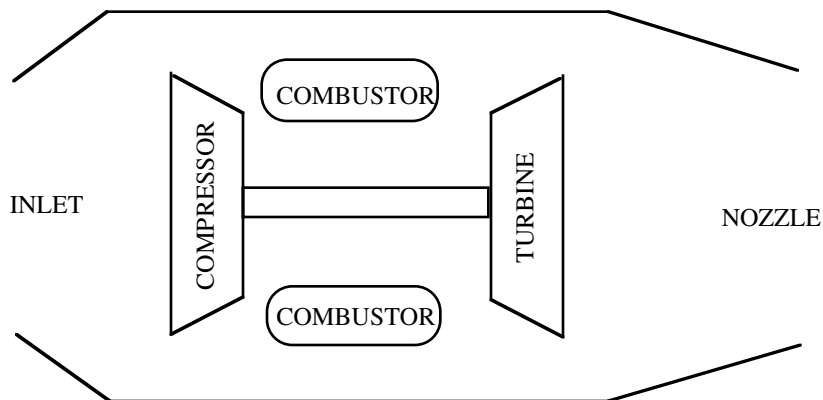on the temperature ratio of the compression process.)
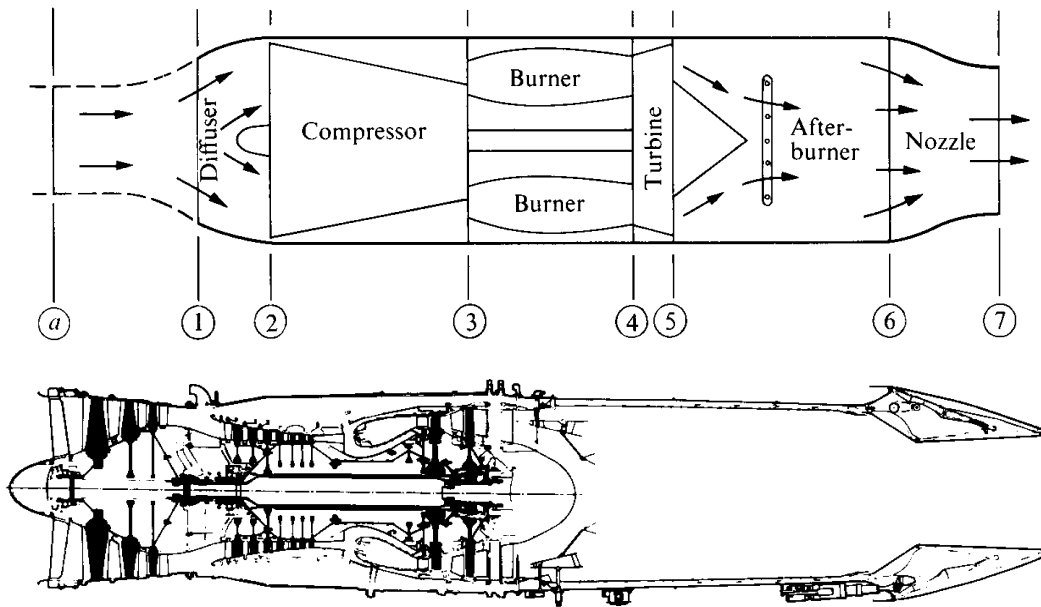
Otto Cycle Efficiency

B. Brayton Cycle (VW, S & B: 9.8-9.9, 9.12)

The Brayton cycle is an idealization of a set of thermodynamic processes used in gas turbine engines, whether for jet propulsion or for generation of electrical power.

1. Components of a gas turbine engine

Schematics of typical military gas turbine engines. Top: turbojet with afterburning, bottom: GE F404 low bypass ratio turbofan with afterburning (Hill and Peterson, 1992).

2. The thermodynamic cycle

The cycle consists of four processes: a) quasi-static adiabatic compression in the inlet and compressor, b) constant pressure heat addition in the combustor, c) quasi-static adiabatic expansion in the turbine and exhaust nozzle, and finally d) constant pressure cooling to get the working fluid back to the initial condition.

1 - 2 Adiabatic, quasi-static compression in inlet and compressor

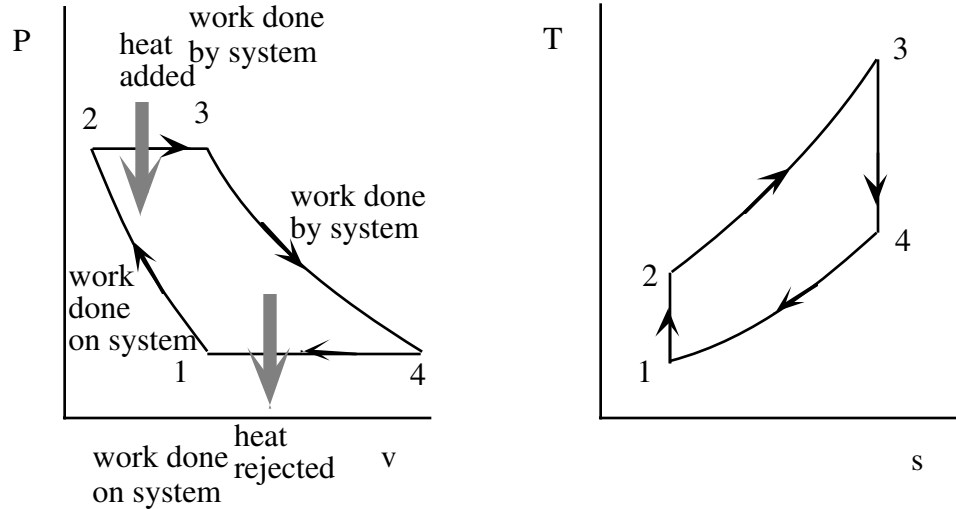2 - 3 Combust fuel at constant pressure (*i.e.* add heat)

3 - 4 Adiabatic, quasi-static expansion in turbine
       a. take work out and use it to drive the compressor
       b. take remaining work out and use it to accelerate fluid for jet propulsion, or to turn a generator for electrical power generation.

4 - 1 Cool the air at constant pressure

Then repeat

3. Estimating the performance of the engine

Our objective with the Brayton cycle is the same as for the Otto cycle. First to derive expressions for the net work and the thermal efficiency of the cycle, and then to manipulate these expressions to put them in terms of typical design parameters so that they will be more useful.

First, remember from the First Law we can show that for any cyclic process heat and work transfers are numerically equal

$\Delta u = q - w$

$u_{final} = u_{initial}$ therefore $\Delta u = 0$ and $q = w$ or $\oint \delta q = \oint \delta w$

This fact is often useful for solving thermodynamic cycles. For instance in this example, we would like to find the net work of the cycle and we could calculate this by taking the difference of the work done all the way around the cycle. Or, since $\Delta q = \Delta w$, we could just as well consider the difference between the heat added to the cycle in process 2-3, and the heat rejected by the cycle in process 4-1.

a) heat added between 2-3 (combustor):

First Law in terms of enthalpy for an ideal gas undergoing a quasi-static process:

$\delta q = dh - vdp$ or $\delta q = c_p dT - vdp$

at constant pressure

$q_{added} = c_p \Delta T$ or $q_{added} = c_p (T_3 - T_2)$

b) heat rejected between 4-1:

similarly

$$q_{added} = c_p \, \Delta T \quad \text{or} \qquad q_{rejected} = c_p \, (T_4 - T_1)$$

c) work done and thermal efficiency:

$$\Delta w = \Delta q = q_{added} - q_{rejected} = c_p [(T_3 - T_2) - (T_4 - T_1)]$$

$$\eta_{Brayton} = (q_{added} - q_{rejected}) / \, q_{added} = [(T_3 - T_2) - (T_4 - T_1)]/(T_3 - T_2)$$

Again, while these expressions are accurate, they are not all that useful. We need to manipulate them to put them in terms of typical design parameters for gas turbine engines. For gas turbine engines the most useful design parameters to use for these equations are often the inlet temperature (T_1), the <u>compressor pressure ratio</u> (p_2/p_1), and the maximum cycle temperature, the <u>turbine inlet temperature</u> (T_3).

d) Rewriting equations in terms of design parameters:

$$\Delta w = c_p T_1 \left(\frac{T_3}{T_1} - \frac{T_2}{T_1} - \frac{T_4}{T_1} + 1 \right)$$

$$\frac{p_2}{p_1} = \frac{p_3}{p_4} = \left(\frac{T_2}{T_1} \right)^{\gamma/\gamma - 1} = \left(\frac{T_3}{T_4} \right)^{\gamma/\gamma - 1} \quad \text{therefore} \quad \frac{T_2}{T_1} = \frac{T_3}{T_4} \quad \text{and} \quad \frac{T_4}{T_1} = \frac{T_3}{T_2}$$

so

$$\Delta w = c_p T_1 \left[\frac{T_3}{T_1} - \left(\frac{p_2}{p_1} \right)^{\gamma - 1/\gamma} - \frac{T_3}{T_2} + 1 \right] \quad \text{with} \quad T_2 = T_1 \left(\frac{p_2}{p_1} \right)^{\gamma - 1/\gamma}$$

becomes

$$\Delta w = c_p T_1 \left[\frac{T_3}{T_1} - \left(\frac{p_2}{p_1} \right)^{\gamma - 1/\gamma} - \frac{T_3}{T_1} \left(\frac{p_2}{p_1} \right)^{-(\gamma - 1)/\gamma} + 1 \right]$$
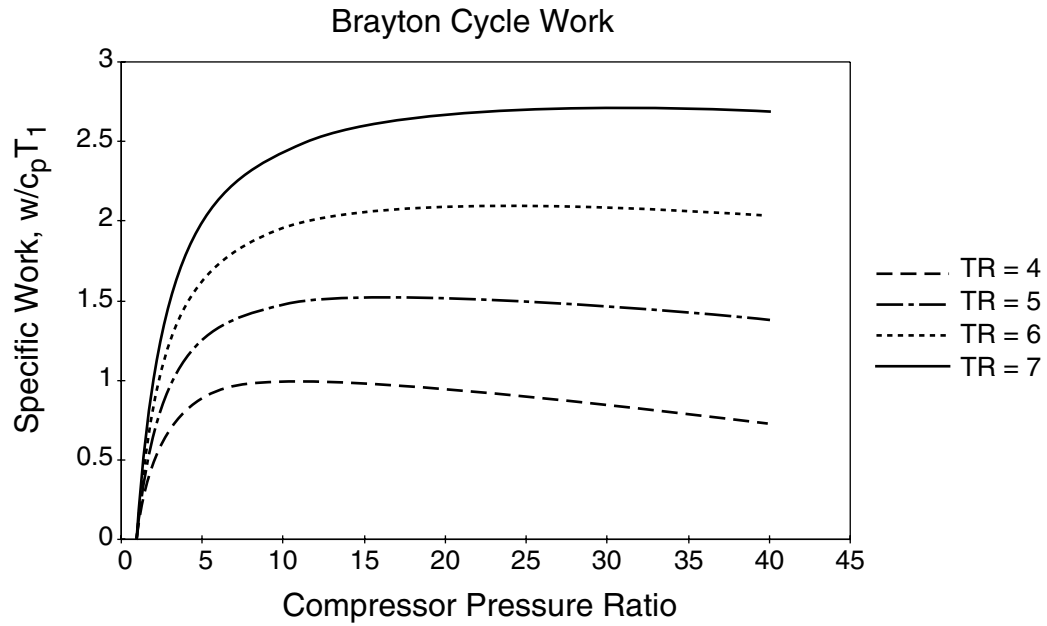
<u>and for the efficiency</u>

$$\eta = 1 - \left(\frac{T_4 - T_1}{T_3 - T_2} \right) \qquad = 1 - \frac{T_1}{T_2} \left[\frac{\frac{T_4}{T_1} - 1}{\frac{T_3}{T_2} - 1} \right] \qquad \text{but from above}$$
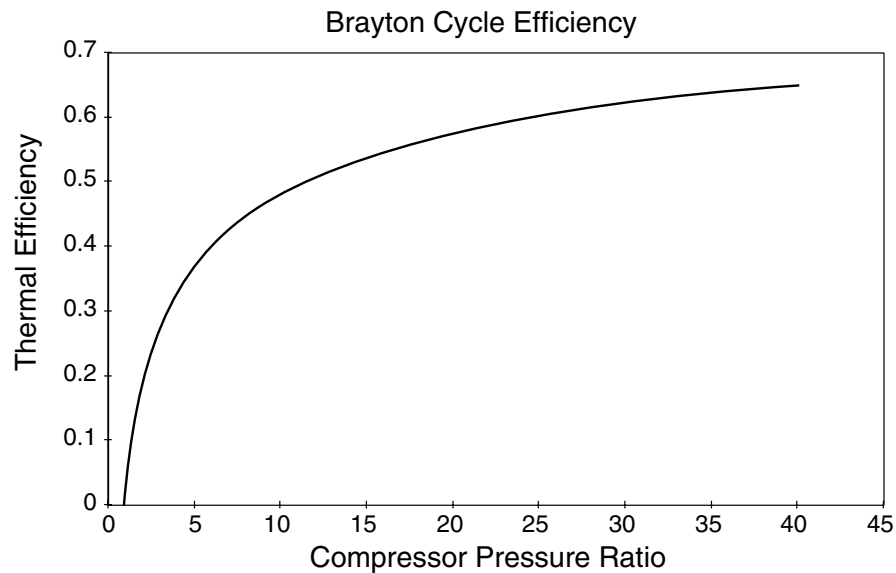
$$\frac{T_4}{T_1} = \frac{T_3}{T_2}$$

so

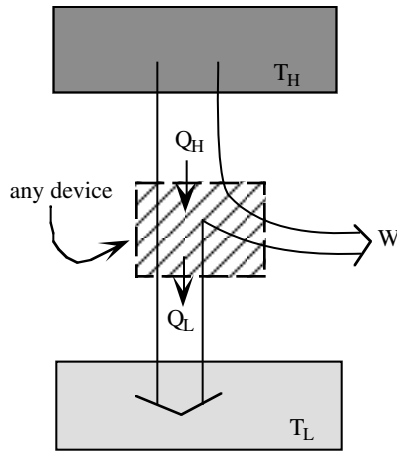$$\eta_{\text{Brayton}} = 1 - \frac{T_1}{T_2} = 1 - \frac{1}{\left(\dfrac{p_2}{p_1}\right)^{\gamma-1/\gamma}}$$

e) Performance plots

Brayton Cycle Work

In the plot above, $TR = T_3/T_1$. Note that for a given turbine inlet temperature, T_3, (which is set by material limits) there is a compressor pressure ratio that maximizes the work.

Brayton Cycle Efficiency

C. Generalized Representation of Thermodynamic Cycles *(VW, S & B: 6.1)*

Note that **heat engines** can be represented generally as:

a transfer of heat from a high temperature reservoir to a device
+
a rejection of heat from a device to a low temperature reservoir
+
net work done on surroundings

D. To obtain data for an operating gas turbine engine (the one that provides power to MIT!) checkout the following links: http://cogen.mit.edu/index.htm and http://cogen.mit.edu/unified/