5.4> A) Fully coupled Iteration

B) 3D IBLT

Reading: See reference list · Veldman, Drela.

6.1> Stability and Transition

A) Small Perturbation Theory.

B) Orr - Sommerfeld Eqn.

Reading: Sch 449 – 483, White 335 – 355

A) Fully Coupled Iteration

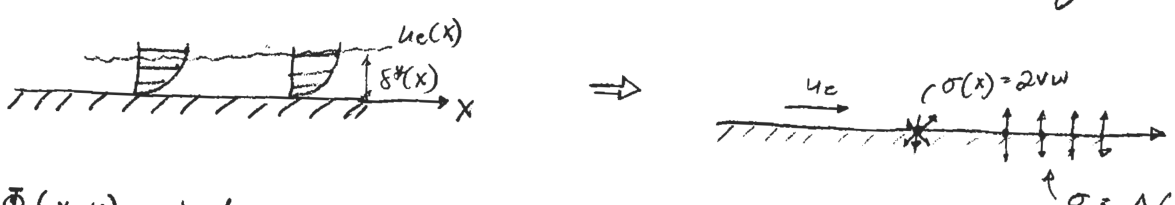
· Last lecture, we showed that loosely coupled viscous-inviscid iteration is unstable — need suitable interaction laws

Example Q1D flow

IBLT: $u_e = \dfrac{m/\rho}{h-\delta^*}$ Classical: $u_e = \dfrac{m/\rho}{h}$

Solve 3x3 system for θ, δ^*, and u_e

· In 2D IBLT, local interaction law is not strictly correct, since outer flow may be elliptic => $u_e(x)$ depends on $\delta^*(x)$ everywhere. One approximate solution is Hilbert integral from thin airfoil theory. Applies rigourously to flat plate



$$\sigma = \Delta(\vec{v}\cdot\hat{n}) = 2v_w = 2\frac{d}{dx}(u_e\delta^*)$$

$$\Phi(x,y) = \frac{1}{2\pi}\int \sigma(x_0)\ln\sqrt{(x-x_0)^2+y^2}\,dx_0$$

$$u(x,y) = \frac{1}{2\pi}\int \sigma(x_0)\frac{x-x_0}{(x-x_0)^2+y^2}\,dx_0 \text{ . at } y=0 \quad u(x,0) = u_0 = \frac{1}{2\pi}\int\frac{\sigma(x_0)}{x-x_0}dx_0 = 2\frac{d}{dx}(m/\rho)$$

$$\nabla \cdot (\phi_1 \nabla \phi_2) - \nabla \cdot (\phi_2 \nabla \phi_1)$$

$$\cancel{\nabla \phi_1 \cdot \nabla \phi_2} + \phi_1 \nabla^2 \phi_2 - \cancel{\nabla \phi_2 \cdot \nabla \phi_1} - \phi_2 \nabla^2 \phi_1$$

$$\Phi = u_\infty(x + \varphi)$$

$$\frac{\partial \Phi}{\partial x} = u_e = u_\infty + \varphi_x$$

$$u_e = u_\infty + \frac{1}{2\pi}\int \frac{\sigma(x_0)\, dx_0}{x - x_0}$$ ← Veldman.

$$= u_\infty + \frac{1}{\pi}\int \frac{d(M/\rho)}{x - x_0}$$

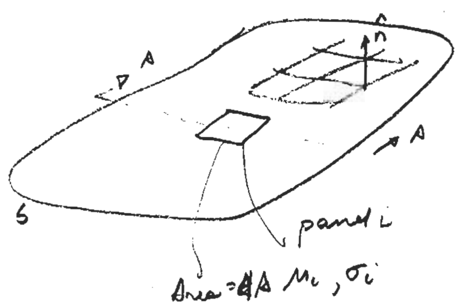
$u_e(x)$ is influenced by $\sigma(x) \leftrightarrow \delta^*$ distribution at upstream and downstream points. (1) Effect dies off as $1/r$, (2) Neglects "high sensitivity" regions like T-E.

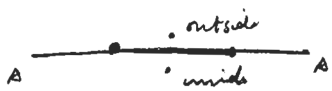
Numerical implementation:

$$u_e^{n+1}(x) - u_e^{n}(x) = \frac{1}{\pi}\int \frac{d\left[(M/\rho)^{n+1} - (M/\rho)^{n}\right]}{x - x_0}$$

incremental update.

"Exact" interaction law for 2-D and 3-D flows is developed using panel methods (20 - 30) + BL method (usually integral)





$\varphi_o - \varphi_i = \mu$ doublet strength

$\left.\frac{\partial \varphi_o}{\partial n}\right|_{out} - \left.\frac{\partial \varphi}{\partial n}\right|_{in} = \sigma$ source strength.

Green's theorem:

$$\Phi(P) = \frac{1}{4\pi}\oiint_A \left(\frac{1}{r}\nabla\varphi - \varphi\nabla \frac{1}{r}\right)\cdot \hat{n}\, dA + \varphi_\infty$$

Value of Φ at any point in terms of $\frac{\partial \varphi}{\partial n}$ at boundary and φ

$$\Phi = \oiint_S \left[\mu \nabla \tfrac{1}{r}\cdot\hat{n} + \frac{1}{r}\sigma \right]\frac{dA}{4\pi} + \Phi_\infty$$

$$4\pi \bar{\bar{I}}\vec{\mu} = \bar{\bar{a}}\vec{\mu} + \bar{\bar{b}}\vec{\sigma} + 4\pi \bar{\bar{I}}\Phi_\infty$$

$$\bar{\bar{A}}\vec{\mu} = \bar{\bar{b}}\vec{\sigma} + 4\pi\bar{\bar{I}}\Phi_\infty$$

$$\vec{\mu} = \bar{\bar{b}}'\vec{\sigma} + 4\pi\bar{\bar{A}}^{-1}\Phi_\infty$$

$$\vec{\Phi} = \bar{\bar{b}}'\left(\frac{\partial\vec{m}}{\partial s}\right) + 4\pi\bar{\bar{A}}^{-1}\Phi_\infty$$

↑ streamwise
gradient of mass def. ?

$$\vec{u}_e = \bar{\bar{b}}'\vec{m} + \bar{\bar{c}}\,\Phi_\infty$$

↳ insert in BL eqn.

Fully Coupled Iteration

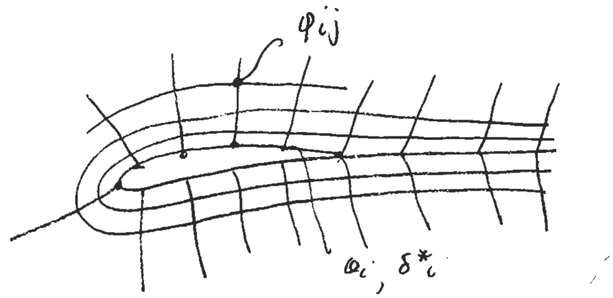
Combine Φ, θ, δ^* into one state vector. The governing equations are

$$\nabla^2\Phi = 0$$

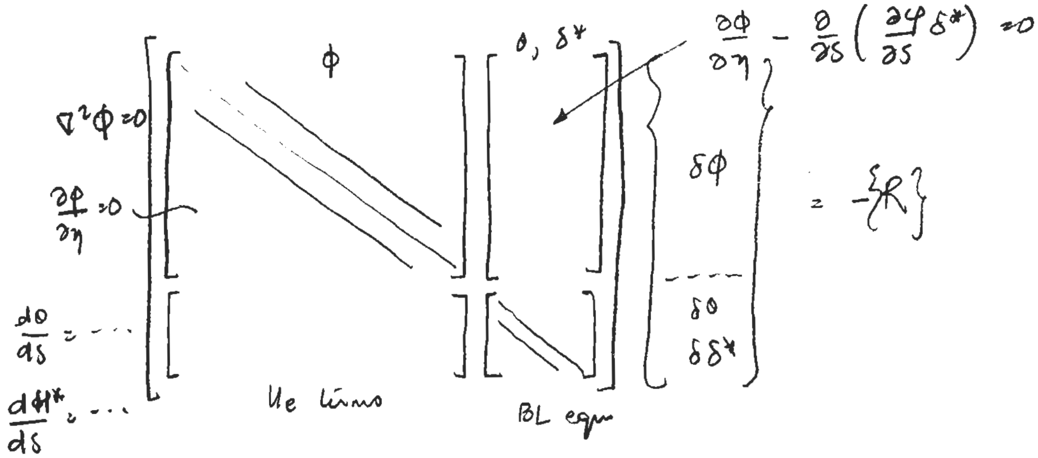
$$\frac{\partial\Phi}{\partial n} = V_w = \frac{\partial}{\partial s}\left(\frac{\partial\Phi}{\partial s}\cdot\delta^*\right)$$

$$\frac{d\theta}{ds} = \ldots$$

$$\frac{dH^*}{ds} = \ldots$$



Resulting in one overall Newton system:



- Direct/inverse iteration neglects off-diagonal matrices
- Finite-diff method can also be used, but at significant increase in computational cost. (F_{ij}, U_{ij}, S_{ij})
- Integral method natural choice for IBLT approach.

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