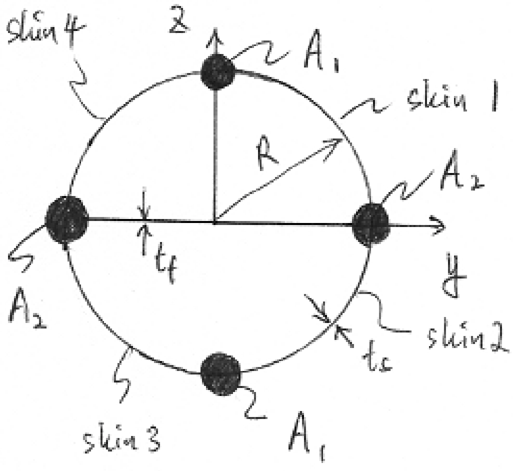
Home Assignment #9 : Solutions



Stringers : modulus E

skin : modulus G.

We need to find the axial stresses in the stringers, shear stre
in the skins and floor, and the cross-section properties for the five different load cases. The axial stresses can be found usin
the equation (from unit #15, p10)

$$\sigma_{xx} = \left\{ \frac{F^{TOT}}{A} - \frac{M_z^{TOT}}{I_z} y - \frac{M_y^{TOT}}{I_y} z \right\} \quad \text{——— ①}$$

*Note : for the cross-section given, $I_{yz} = 0$, and there are no temp effects in the current problem, so some terms have been elimi

The shear stress is found by using

$$\sigma_{xs} = \frac{q}{t} \quad \text{——— ②}$$

where

$$q_{out} - q_{in} = \frac{S_y Q_z}{I_z} + \frac{S_z Q_y}{I_y} \quad \text{——— ③}$$

at each node. To find the shear flow, q, we need to use the following relations.

torque equivalence : $\sum T_{internal} = \sum T_{applied}$ —— ④

no twist condition : $\oint \frac{q}{t} ds = 0$ —— ⑤
(for applied shear)

equal twist condition : $\frac{d\alpha}{dx}$ = equal in each cell —— ⑥
(for applied torsion)

To use equations ① and ③, we need the second moments of inertia values of the cross-section. Since we know that the centroid is at the center due to symmetry, the second moments of inertia are calculated at this point.

$$I_y = \sum A_i z_i^2 = A_1 R^2 + A_2 \cdot 0^2 + A_1(-R)^2 + A_2 \cdot 0^2$$

$$\Rightarrow \boxed{I_y = 2A_1 R^2}$$

$$I_z = \sum A_i y_i^2 = A_1 \cdot 0^2 + A_2 R^2 + A_1 0^2 + A_2(-R)^2$$

$$\Rightarrow \boxed{I_z = 2A_2 R^2}$$

$$I_{yz} = \sum A_i y_i z_i = A_1 \cdot 0 R + A_2 \cdot R \cdot 0 + A_1 \cdot 0 \cdot (-R) + A_2 \cdot (-R) \cdot 0$$

$$\Rightarrow \boxed{I_{yz} = 0}$$

Load Case 1 : moment M_y about y-axis

For this case, axial stress simplifies to

$$\sigma_{xx} = -\frac{M_y}{I_y} z$$

Plugging in our value of I_y, we get

$$\sigma_{xx} = -\frac{M_y}{2A_1R^2} z \qquad \text{——— (7)}$$

Thus, at each stringer, σ_{xx} is

Stringer	1	2	3	4
σ_{xx}	$-\frac{M_y}{2A_1R}$	0	$+\frac{M_y}{2A_1R}$	0

The applied moment does not cause any shear stresses, so

$$\boxed{\sigma_{xs} = 0}$$

<u>Load Case 2 : moment M_z about z-axis</u>

The axial stress in this case is

$$\sigma_{xx} = -\frac{M_z}{I_z} y$$

We can plug in our value of I_z and evaluate σ_{xx} in each stringer as in load case 1.

$$\sigma_{xx} = \frac{M_z}{2A_2R^2} y \qquad \text{——— (8)}$$

Stringer	1	2	3	4
σ_{xx}	0	$-\frac{M_z}{2A_2R}$	0	$\frac{M_z}{2A_2R}$

The applied moment does not cause any shear stresses, so

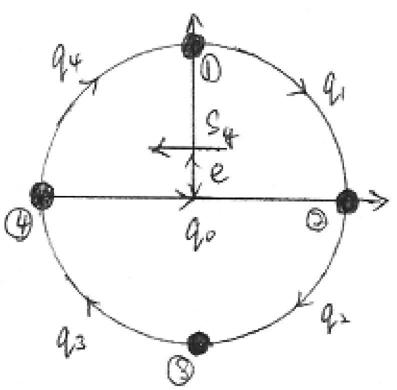
$$\boxed{\sigma_{xs} = 0}$$

<u>Load Case 3 : Shear resultant S_y in y-direction</u>

A pure shear force applied on a cross-section does not cause any moments. So,

$$\boxed{\sigma_{xx} = 0}$$

To find the shear stress, we begin by labeling each stringer and calculating the shear flow in each web by applying eq.



e : vertical distance between shear cen and centroid.

* Note : $Q_z = A y$.

Node ① : $q_1 - q_4 = \dfrac{S_y A \cdot 0}{2 A_2 R^2} = 0$

$\Rightarrow q_1 - q_4 = 0$ ——— ⑨

Node ② : $q_2 - q_1 - q_0 = \dfrac{S_y \cancel{A_2} \cdot \cancel{R}}{2 \cancel{A_2} R^{\cancel{2}}}$

$\Rightarrow q_2 - q_1 - q_0 = \dfrac{S_y}{2R}$ ——— ⑩

Node ③ : $q_3 - q_2 = \dfrac{S_y \cdot A_1 \cdot 0}{2 A_2 R^2} = 0$

$\Rightarrow q_3 - q_2 = 0$ ——— ⑪

Node ④ : will not yield an independent equation.

Now, we apply the torque equivalence condition in equation ④.

$$\begin{cases} \sum T_{internal} = \sum q_i \,(\text{skin length})(\text{moment arm}) & \text{——— ⑫} \\ T_{applied} = S_y e. & \text{——— ⑬} \end{cases}$$

Summing the torque contribution in each skin about the centroid of the cross-section, we get:

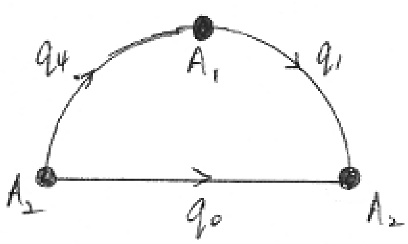
$$-q_1\left(\pi \frac{R}{2}\right)R - q_2\left(\pi \frac{R}{2}\right)R - q_3\left(\pi \frac{R}{2}\right)R - q_4\left(\pi \frac{R}{2}\right)R = S_y e$$

$$\Rightarrow \quad \frac{\pi R^2}{2}\left(q_1 + q_2 + q_3 + q_4\right) = -S_y e \qquad \text{——— ⑭}$$

Finally, we apply the no-twist condition in equation ⑤.

$$\oint \frac{q}{t} ds = 0 \quad \Rightarrow \quad \sum \frac{q_i}{t_i}(\text{skin length}) = 0$$

for each cell. For the top cell,

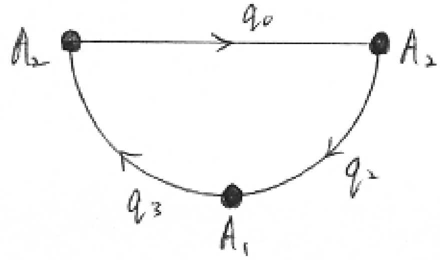


$$+\circlearrowright \quad \frac{q_1}{t_s}\left(\frac{\pi R}{2}\right) + \frac{q_4}{t_s}\left(\frac{\pi R}{2}\right) - \frac{q_0}{t_f}(2R$$

$$\Rightarrow \quad \frac{q_1}{t_s}\cdot\frac{\pi}{2} + \frac{q_4}{t_s}\cdot\frac{\pi}{2} - \frac{q_0}{t_f}2$$

——— (

For the bottom cell,



$$+\circlearrowright \quad \frac{q_2}{t_s}\left(\frac{\pi R}{2}\right) + \frac{q_3}{t_s}\left(\frac{\pi R}{2}\right) + \frac{q_0}{t_f}(2R) =$$

$$\Rightarrow \quad \frac{q_2}{t_s}\left(\frac{\pi}{2}\right) + \frac{q_3}{t_s}\left(\frac{\pi}{2}\right) + \frac{q_0}{t_f}(2)$$

So now, we have 6 equations (⑨, ⑩, ⑪, ⑭, ⑮ and ⑯) and 6 unknowns ($q_0 \sim q_4$, e). In matrix form,

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 & -1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ \frac{\pi R^2}{2} & \frac{\pi R^2}{2} & \frac{\pi R^2}{2} & \frac{\pi R^2}{2} & 0 & -S_y \\ \frac{\pi}{2t_s} & 0 & 0 & \frac{\pi}{2t_s} & -\frac{2}{t_f} & 0 \\ 0 & \frac{\pi}{2t_s} & \frac{\pi}{2t_s} & 0 & \frac{2}{t_f} & 0 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_0 \\ e \end{Bmatrix} = \begin{Bmatrix} 0 \\ \frac{S_y}{2R} \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

Solving the equation by some suitable means, we get

$$q_1 = \frac{-t_s S_y}{(4t_s + \pi t_f)R} \qquad q_2 = \frac{t_s S_y}{(4t_s + \pi t_f)R}$$

$$q_3 = \frac{t_s S_y}{(4t_s + \pi t_f)R} \qquad q_4 = \frac{-t_s S_y}{(4t_s + \pi t_f)R}$$

$$q_0 = \frac{-t_f \pi S_y}{2(4t_s + \pi t_f)R} \qquad e = 0$$

← shear center is located at centroi[d]; this is expected fr[om] symmetry.

To get the shear stresses, use equation ②.

skins 1 & 4 : $\sigma_{xs} = \dfrac{-S_y}{(4t_s + \pi t_f)R}$

skins 2 & 3 : $\sigma_{xs} = \dfrac{S_y}{(4t_s + \pi t_f)R}$

skin 0 : $\sigma_{xs} = -\dfrac{\pi S_y}{\ldots}$

* As a check, u[se] that

$\int q(y\text{-direction})\,dy$

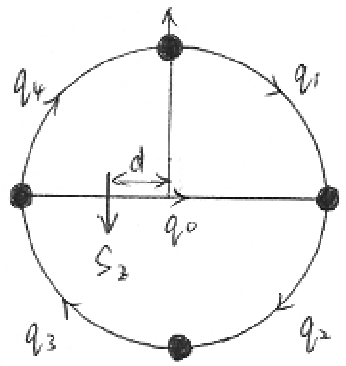
$\int q(z\text{-direction})\,dz$

Load Case 4: Shear resultant S_z in the z-direction

Again, the axial stresses are zero.

$$\boxed{\sigma_{xx} = 0}$$

For the shear stresses, use same procedure as in load case 3.



d = horizontal distance between shear center and centroid.

* Note : $Q_y = A \cdot z$.

Node ① : $q_1 - q_4 = \dfrac{S_z A_1 R}{2A_1 R^2} = \dfrac{S_z}{2R}$ —

Node ② : $q_2 - q_0 - q_1 = \dfrac{S_z A_2 \cdot 0}{2A_1 R^2} = 0$ —— ⑱

Node ③ : $q_3 - q_2 = \dfrac{S_z A_1 \cdot (-R)}{2A_1 R^2} = -\dfrac{S_z}{2R}$ —— ⑲

Now, we apply the torque equivalence equation, noting that

$$T_{applied} = S_z d$$

and that $\Sigma T_{internal}$ is identical to the expression found in loa

case 3. Thus, from equation ⑭,

$$\frac{\pi R^2}{2}(q_1 + q_2 + q_3 + q_4) = -S_z d \quad \text{—— ②}$$

Finally, we apply the no twist condition. The equations for the no twist conditions from equation ⑤ are also identical to load case

$$\frac{q_1}{t_s}\frac{\pi}{2} + \frac{q_4}{t_s}\frac{\pi}{2} - \frac{q_0}{t_f}\cdot 2 = 0 \qquad \text{—— (21)} \quad (=(15))$$

$$\frac{q_2}{t_s}\frac{\pi}{2} + \frac{q_3}{t_s}\frac{\pi}{2} + \frac{q_0}{t_f}\cdot 2 = 0 \qquad \text{—— (22)} \quad (=(16))$$

Summarizing and expressing in matrix form, we get

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 & -1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ \frac{\pi R^2}{2} & \frac{\pi R^2}{2} & \frac{\pi R^2}{2} & \frac{\pi R^2}{2} & 0 & S_z \\ \frac{\pi}{2t_s} & 0 & 0 & \frac{\pi}{2t_s} & -\frac{2}{t_f} & 0 \\ 0 & \frac{\pi}{2t_s} & \frac{\pi}{2t_s} & 0 & \frac{2}{t_f} & 0 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_0 \\ d \end{Bmatrix} = \begin{Bmatrix} \frac{S_z}{2R} \\ 0 \\ -\frac{S_z}{2R} \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

Solving, we get

$$q_1 = \frac{S_z}{4R} \qquad q_2 = \frac{S_z}{4R}$$

$$q_3 = -\frac{S_z}{4R} \qquad q_4 = -\frac{S_z}{4R}$$

$q_0 = 0 \leftarrow$ (floor carries no shear in z-direction) $\qquad d = 0 \leftarrow$ (shear center at centroid)

Calculating shear stress, we get

skins 1 & 2 : $\sigma_{xs} = \dfrac{S_z}{4t_sR}$

skins 3 & 4 : $\sigma_{xs} = \dfrac{-S_z}{4t_sR}$

* As a check,

$\int q(\text{y-direction})\,dy = 0$

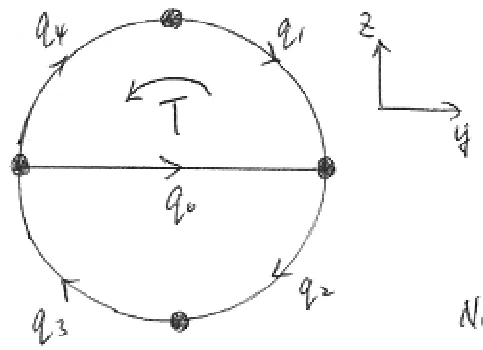
$\int q(\text{z-direction})\,dz = S_z$

Load Case 5: Torque T about the x-axis

An applied torque does not cause any axial stress, so,

$$\boxed{\sigma_{xx} = 0}$$

To find the shear stresses, use the same procedure used for load cases 3 and 4.



* Since we have pure torque,

$$q_{out} - q_{in} = 0$$

at each node ($S_y = S_z = 0$)

Node ①: $q_1 - q_4 = 0$ —— ㉓

Node ②: $q_2 - q_1 - q_0 = 0$ —— ㉔

Node ③: $q_3 - q_2 = 0$ —— ㉕

Next, apply the torque equivalence. Since $\Sigma T_{internal}$ is identical to load ca

and 4, i.e.

$$\underbrace{\frac{\pi R^2}{2}(q_1 + q_2 + q_3 + q_4)}_{\Sigma T_{internal}} = \underbrace{-T}_{T_{applied}} \quad \text{——} \; (2$$

Finally, we need to apply the condition of equal twist in the two cells. Using equation ⑥,

$$\frac{d\alpha}{dx} = \frac{1}{2A_{lm} G} \oint \frac{q}{t} ds = \frac{1}{2A_{\dots} G} \oint \frac{q}{t} ds \quad \text{——} \; (2$$

where $\begin{cases} A_{top} = \text{area enclosed by the top cell} = \dfrac{\pi R^2}{2} \\ A_{bot} = \text{area enclosed by the bottom cell} = \dfrac{\pi R^2}{2} \end{cases}$

We can evaluate $\frac{d\alpha}{dx}$ in the top part from equation (21) as follows.

$$\frac{d\alpha}{dx} = \frac{1}{2A_{top}G}\oint \frac{q}{t}\,ds = \frac{1}{2A_{top}G}\sum \frac{q_i}{t_i}(\text{skin length})$$

$$= \frac{1}{2A_{top}G}\left(\frac{q_1}{t_s}\frac{\pi R}{2} + \frac{q_4}{t_s}\frac{\pi R}{2} - \frac{q_0}{t_f}(2R)\right) \quad — (28)$$

Similarly, for the bottom part,

$$\frac{d\alpha}{dx} = \frac{1}{2A_{bot}G}\oint \frac{q}{t}\,ds = \frac{1}{2A_{bot}G}\sum \frac{q_i}{t_i}(\text{skin length})$$

$$= \frac{1}{2A_{bot}G}\left(\frac{q_2}{t_s}\frac{\pi R}{2} + \frac{q_3}{t_s}\frac{\pi R}{2} + \frac{q_0}{t_f}(2R)\right) \quad — (29)$$

Equating equations (28) and (29), we get,

$$\frac{1}{\cancel{2}\cancel{A_{top}}\cancel{G}}\left(\frac{q_1}{t_s}\frac{\pi \cancel{R}}{2} + \frac{q_4}{t_s}\frac{\pi \cancel{R}}{2} - \frac{q_0}{t_f}(2\cancel{R})\right) = \frac{1}{\cancel{2}\cancel{A_{bot}}\cancel{G}}\left(\frac{q_2}{t_s}\frac{\pi \cancel{R}}{2} + \frac{q_3}{t_s}\frac{\pi \cancel{R}}{2} + \frac{q_0}{t_f}(2\cancel{R})\right)$$

* $A_{top} = A_{bot} = \dfrac{\pi R^2}{2}$

$$\Rightarrow \frac{\pi q_1}{2t_s} + \frac{\pi q_4}{2t_s} - \frac{2q_0}{t_f} = \frac{\pi q_2}{2t_s} + \frac{\pi q_3}{2t_s} + \frac{2q_0}{t_f} \quad — (30)$$

Now we have 5 equations ((23), (24), (25), (26), (30)) and 5 unknowns.

Writing in matrix form, we get

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ -1 & 1 & 0 & 0 & -1 \\ 0 & -1 & 1 & 0 & 0 \\ \frac{\pi R^2}{2} & \frac{\pi R^2}{2} & \frac{\pi R^2}{2} & \frac{\pi R^2}{2} & 0 \\ \frac{\pi}{2t_s} & -\frac{\pi}{2t_s} & -\frac{\pi}{2t_s} & \frac{\pi}{2t_s} & -\frac{4}{t_f} \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ -T \\ 0 \end{Bmatrix}$$

Solving for the q's, we get

$$q_1 = -\frac{T}{2\pi R^2} \qquad q_2 = -\frac{T}{2\pi R^2}$$

$$q_3 = -\frac{T}{2\pi R^2} \qquad q_4 = -\frac{T}{2\pi R^2}$$

$$q_0 = 0$$

Calculating the shear stress, $\left(\sigma_{xs} = \frac{q}{t_s}\right)$

skins 1, 2, 3, 4	:	$\sigma_{xs} = \frac{-T}{2\pi R^2 t_s}$
skin 0	:	$\sigma_{xs} = 0$

* As a check,

$$\int q\,dy = 0$$

$$\int q\,dz = 0$$

A summary of all the axial and shear stresses in both the stringers and the skins are shown on the next page.

Now, we will calculate the remaining section properties, A_{yy}, A_{zz}, A_{yz} and J.

from unit #15 p. 29

$\bar{q}_y$ = shear flow due to unit S_y

$\bar{q}_z$ = shear flow due to unit S_z

$$A_{yy} = \frac{1}{\int \frac{\bar{q}_y^2}{t} ds} = \frac{1}{\sum \frac{\bar{q}_{yi}^2}{t_i} (\text{skin length})} \quad \text{—— } (31)$$

$$A_{zz} = \frac{1}{\int \frac{\bar{q}_z^2}{t} ds} = \frac{1}{\sum \frac{\bar{q}_{zi}^2}{t_i} (\text{skin length})} \quad \text{—— } (32)$$

$$A_{yz} = \frac{1}{\int \frac{\bar{q}_y \bar{q}_z}{t} ds} = \frac{1}{\sum \frac{\bar{q}_{yi} \bar{q}_{zi}}{t_i} (\text{skin length})} \quad \text{—— } (33)$$

from unit #15 p. 30

$\bar{\bar{q}}$ = shear flow due to unit T

$$J = \frac{2A}{\oint \frac{\bar{\bar{q}}}{t} ds} = \frac{2A}{\sum \frac{\bar{\bar{q}}_i}{t_i} (\text{skin length})} \quad \text{—— } (34)$$

To calculate A_{yy} using equation (31), we need to get $\bar{q}_y$ from load case 3. Setting $S_y = 1$,

$$\bar{q}_{y1} = \frac{-t_s}{(4t_s + \pi t_f)R} \qquad \bar{q}_{y2} = \frac{t_s}{(4t_s + \pi t_f)R}$$

$$\bar{q}_{y3} = \frac{t_s}{(4t_s + \pi t_f)R} \qquad \bar{q}_{y4} = \frac{-t_s}{(4t_s + \pi t_f)R}$$

$$\bar{q}_{y0} = \frac{-t_f \pi}{2(4t_s + \pi t_f)R}$$

Therefore, A_{yy} is

Axial Stresses

Load	Stringer 1	Stringer 2	Stringer 3	Stringer 4
M_y	$\frac{-M_y}{2A_1R}$	0	$\frac{M_y}{2A_1R}$	0
M_z	0	$\frac{-M_z}{2A_2R}$	0	$\frac{+M_z}{2A_2R}$
S_y	0	0	0	0
S_z	0	0	0	0
T	0	0	0	0

Shear Stresses

Load	Skin 1	Skin 2	Skin 3	Skin 4	Skin 0
M_y	0	0	0	0	0
M_z	0	0	0	0	0
S_y	$\frac{-S_y}{(4t_s+\pi t_f)R}$	$\frac{S_y}{(4t_s+\pi t_f)R}$	$\frac{S_y}{(4t_s+\pi t_f)R}$	$\frac{-S_y}{(4t_s+\pi t_f)R}$	$\frac{-\pi S_y}{2(4t_s+\pi t_f)R}$
S_z	$\frac{S_z}{4t_sR}$	$\frac{S_z}{4t_sR}$	$\frac{-S_z}{4t_sR}$	$\frac{-S_z}{4t_sR}$	0
T	$\frac{-T}{2\pi t_sR^2}$	$\frac{-T}{2\pi t_sR^2}$	$\frac{-T}{2\pi t_sR^2}$	$\frac{-T}{2\pi t_sR^2}$	0

$$A_{yy} = \left[\overset{2}{\cancel{4}} \cdot \frac{1}{t_s} \left(\frac{t_s}{(4t_s + \pi t_f)R} \right)^2 \left(\frac{\pi R}{\cancel{2}} \right) + \frac{1}{t_f} \left(\frac{t_f \pi}{2(4t_s + \pi t_f)R} \right)^2 (2R) \right]^{-1}$$

$$\Rightarrow \quad A_{yy} = \left[\frac{2 t_s \pi}{(4t_s + \pi t_f)^2 R} + \frac{t_f \pi^2}{2(4t_s + \pi t_f)^2 R} \right]^{-1}$$

$$= \left[\frac{1}{2(4t_s + \pi t_f)^2 R} \left(4 t_s \pi + t_f \pi^2 \right) \right]^{-1}$$

$$A_{yy} = \left[\frac{\cancel{(4t_s + t_f \pi)} \pi}{2(4t_s + \pi t_f)^{\cancel{2}} R} \right]^{-1}$$

$$\therefore \quad \boxed{A_{yy} = \frac{8 R t_s}{\pi} + 2 t_f R}$$

Similarly, for A_{zz}, using equation (32), where

$$\bar{q}_{z1} = -\frac{1}{4R} \qquad \bar{q}_{z2} = -\frac{1}{4R}$$

$$\bar{q}_{z3} = \frac{1}{4R} \qquad \bar{q}_{z4} = \frac{1}{4R}$$

$$\bar{q}_{z0} = 0$$

$$\Rightarrow \quad A_{zz} = \left[\overset{2}{\cancel{4}} \frac{1}{t_s} \left(\frac{1}{4\cancel{R}} \right)^2 \left(\frac{\pi \cancel{R}}{\cancel{2}} \right) \right]^{-1}$$

$$\Rightarrow \quad \boxed{A_{zz} = \frac{8}{\pi} t_s R}$$

For A_{yz}, define

$$B = \frac{t_s}{(4t_s + \pi t_f)R}, \qquad C = \frac{1}{4R}$$

Then, equation ㉝ is

$$A_{yz} = \left\{ \frac{\pi R}{2t_s}\Big[(\overset{\bar{q}_{y1}}{-B})(\overset{\bar{q}_{z1}}{-C}) + \cancel{(\overset{\bar{q}_{y2}}{B})(\overset{\bar{q}_{z2}}{-C})} + (\overset{\bar{q}_{y3}}{B})(\overset{\bar{q}_{z3}}{C}) + \cancel{(\overset{\bar{q}_{y4}}{-B})(\overset{\bar{q}_{z4}}{C})} \Big] + \frac{2R}{t_f}\underbrace{\left(\frac{-t_f \pi}{(4t_s + \pi t_f)2R}\right)}_{\bar{q}_{y0}} \underbrace{(0)}_{\bar{q}_{z0}} \right\}^{-1}$$

$$\Rightarrow \quad A_{yz} = \{ 0 \}^{-1}$$

$$\therefore \quad \boxed{A_{yz} = \infty}$$

← * Indicates that there is no coupling in shear in the y and z direction (which is expected due to symmetry)

Noting that for $T = 1$,

$$\bar{\bar{q}}_1 = -\frac{1}{2\pi R^2} \qquad \bar{\bar{q}}_2 = -\frac{1}{2\pi R^2}$$

$$\bar{\bar{q}}_3 = -\frac{1}{2\pi R^2} \qquad \bar{\bar{q}}_4 = -\frac{1}{2\pi R^2}$$

$$\bar{\bar{q}}_0 = 0$$

and using equation ㉞,

$$J = 2A\left[\frac{-4}{2\pi t_s R^2} \cdot \frac{\pi R}{2}\right]^{-1}$$

$$\Rightarrow \quad \boxed{J = -2\pi R^3 t_s}$$

* J is negative because we de[illegible] our shear flows as being in [illegible]

Section Properties

$$I_y = 2A_1R^2$$

$$I_z = 2A_2R^2$$

$$I_{yz} = 0$$

$$A_{yy} = \frac{8Rt_s}{\pi} + 2Rt_f$$

$$A_{zz} = \frac{8}{\pi}Rt_s$$

$$A_{yz} = \infty$$

$$J = -2\pi R^3 t_s$$